

# the Algebra Game

## Trig Functions



By Catheryne Draper



395 Main Street  
Rowley, MA 01969  
[www.didax.com](http://www.didax.com)

# Contents

---

<b>Correlations to the Math Standards</b>	<b>v</b>
<b>Getting Started</b>	<b>ix</b>
Working with <i>The Algebra Game: Trig Functions</i>	x

## ACTIVITIES

---

### Lesson 1

Degrees, Radians, and Right Triangle Patterns	1
Activity 1.1: Toothpick Ratios	1
Activity 1.2: Pipe Cleaner Measures	2

---

### Lesson 2

Sine, Cosine, and Tangent	9
Activity 2.1: Graphs of Sine, Cosine, and Tangent	9

---

### Lesson 3

Cotangent, Secant, and Cosecant	16
Activity 3.1: Graphs of Cotangent, Secant, and Cosecant	16

---

### Lesson 4

Tracing Trig Function Relationships	23
Activity 4.1: Mapping Trig Function Relationships	23

---

### Lesson 5

Coefficient Influence	29
Activity 5.1: Doubling and Halving Trig Graphs Coefficients	29
Activity 5.2: Effects of Other Coefficients on Trig Graphs	33
Activity 5.3: Effects of Coefficient Changes on Asymptote and Amplitude	35
Activity 5.4: Effects of Coefficient Changes on Period	35

---

**Game 1**

Going Fishing! 43

---

**Game 2**

Match Match 44

---

**Game 3**

Rummy 44

---

**Using the Write-Your-Own Activity**

Write-Your-Own Activity 48

Discussion Questions 49

Student Notes 50

**EXTRA MASTERS**

Conjecture Worksheet 51

Trig Graph Grid Worksheet 52

Trig Graph Record Worksheet 53

Role Cards 55

Repair Cards 57

**Answers 61**

# Getting Started

## Welcome to *The Algebra Game: Trig Functions*

Congratulations on your purchase of *The Algebra Game: Trig Functions!* By using these decks and making connections, your middle school and high school students will visualize, describe, and analyze the associations of the basic concepts of trigonometric functions. The instructional result is that students reinforce techniques learned in Algebra 2 while actively building new knowledge from experience.

*The Algebra Game: Trig Functions* enables students to make sense of trigonometric equations by matching period and amplitude or asymptote cards to cards for corresponding graphs, equations, and coordinate pairs tables.

Decks are organized to match students' skill level, starting with beginners and progressing to more advanced learners. For example, students can study the effects of changing amplitude on the graph of a trigonometric equation separately from the effects of the changes in period. Then they can move on to studying the interaction of the amplitude and period changes together.

## *The Algebra Game: Trig Functions* Supports Different Learning Styles

The flexibility of the card decks in *The Algebra Game: Trig Functions* allows teachers to enhance the various learning styles and strengths that students bring to the classroom.

- Students are engaged visually, kinesthetically, and logically as they match, trace, and classify cards in activities and games that focus on the relationships among the characteristics of trigonometric graphs and equations.
- Sorting mats provide the opportunity for students to see how all of the cards fit together.
- Discussion Questions and a Conjecture Worksheet allow students to collaborate and describe the ideas shared in groups.

- Space is provided on all worksheets for students to write about their observations during card activities and games, either independently or as a group.

In addition, teachers can also use the optional Write-Your-Own Activity templates to tailor lesson activities to their individual classroom needs.

## *The Algebra Game: Trig Functions* and Math Standards

*The Algebra Game: Trig Functions* supports the Math standards, focusing on Grade 8 and High School Content Standards involving trigonometric graphs and functions. Each Activity is correlated to one or more Math Content Standards, which are identified at the top of each Activity sheet. Correlation tables for Math Content Standards also appear at the front of the book.

*The Algebra Game: Trig Functions* also supports the Math Process standards, which are represented as the eight Mathematical Practices listed below. Each Process Standard is aligned with one or more Activities, which are identified in the Process Standards correlation table at the front of the book.

### Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Common Core State Standards for Mathematics* (pp. 6–8),  
© Copyright 2010 National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.

# Working with *The Algebra Game: Trig Functions*

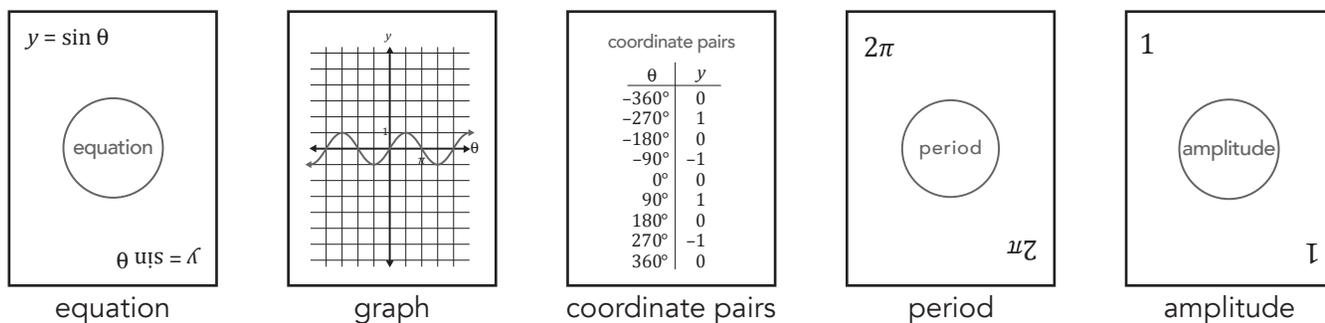
## How to Use the Decks

**Deck Organization** Each of the four decks contains 12 matching sets of 5 types of cards. Each matching set consists of a card for a unique trig graph and a card for its related equation, as well as for the corresponding coordinate pairs table, period, and amplitude or asymptote. For example, the set of cards shown below is for the trigonometric equation  $y = \sin \theta$ . The **Repair Cards** Extra Masters at the back of the book can be copied, cut out, and filled in to replace any misplaced cards or to create a custom set of matching cards.

The four decks are organized by level of difficulty:

- Deck A has only three trig functions: sine, cosine, and tangent.
- Deck B includes the reciprocal functions for Deck A.
- Deck C incorporates different angle measures.
- Deck D has a mixture of angle changes and origin shifts.

The table below specifies the number of matching sets in each deck based on the characteristics of each graph and its related equation.



The set of matching cards for  $y = \sin \theta$ , from left to right, includes the Equation, Graph, Coordinate Pairs, Period, and Amplitude or Asymptote cards.

**Trig Functions Card Deck Specifications**  
(Number of Matching Sets per Deck)

Features	Deck A	Deck B	Deck C	Deck D
sin	4	0	2	2
cos	4	0	2	2
tan	4	0	2	2
cot	0	4	2	2
sec	0	4	2	2
csc	0	4	2	2
Reciprocal function	0	12	6	6
Translated origin	0	0	12	6
Amplitude	8	0	4	4
Asymptote	4	12	8	8
Period $> 2\pi$	2	2	0	9
Period $< 2\pi$	5	5	5	1
Period $= 2\pi$	5	5	7	2
Double angle	3	3	1	0
Half-angle	3	3	0	5
Third-angle	0	0	0	1
Quarter-angle	0	0	0	5

**Arranging the Students into Groups** By solving problems and discussing solutions within groups, students reinforce their understanding of the subject matter, resulting in greater retention of material for assessments.

When arranging groups for *The Algebra Game: Trig Functions*, you can introduce the cards before the Activities and Games start. For example, if four students are in each group, you can arrange groups by using four matching cards such as Equation, Graph, Coordinate Pairs, and Period. Select as many matching sets as needed and distribute the cards to all students in the class. Then, one by one, students with Equation cards can announce their equations to the class or write them on the board. Students with corresponding Graph, Coordinate Pairs, and Period cards can then self-identify, and groups can be seated together.

Similar strategies can be followed using two matching cards for two partners or use three or more matching cards for making larger groups. You may want to use matching cards that represent an earlier lesson so that students can have a quick review of what happened earlier or absent students can have a chance to catch up.

### **Determining the Number of Decks and Levels**

Boxed sets of *The Algebra Game: Trig Functions* include 4 decks of 60 cards each, for a total of 240 cards. The number of decks needed depends on how many decks are used and how they are divided between groups of students.

- Using four decks with the Activities: Separate each deck into two sets so that you have 8 packages for groups of four students each (32 students in class).
- Using three decks: Separate the decks into 6 sets (two from each deck level) resulting in enough cards for 5 students in each group.

The recommended decks are listed for each Lesson, as well as every Activity and Game. If you would like all groups to have the exact same cards, then you may purchase additional boxed sets for the student groups in your classroom by going to [didax.com/the-algebra-game](http://didax.com/the-algebra-game).

## **Before Class Preparation**

### **1. Duplicate Activity Instructions and Worksheets**

Activity sheets, Discussion Questions, and some worksheets are located in the lesson itself, while generic worksheets, called “Extra Masters”, are located in the back of the book. Make enough copies so that there is one Activity sheet and one Discussion Questions worksheet per group for each activity. Requirements for additional materials are listed at the beginning of each lesson.

**2. Assign Student Roles** Arrange the students into groups and assign roles. The Activity instructions describe tasks for each role in the group. For groups with more than four members, additional cards can be created and include Time Keeper, Observer, and/or Encourager.

Make copies of the Role cards in the back of the book and give each student a corresponding card to display in front of themselves. Students should keep track of the role that they have been assigned so as not to repeat the same role in a given session.



**3. Managing the Groups** The following management ideas are suggestions by teachers for using the cards with groups of students.

- a. Designate group member roles.
- b. Assemble the materials for each group (decks, Activity directions, and so on) in baskets or other similar-size containers. Distribute the baskets, one per group, after general instructions have been presented to the class. Color-code the containers for easy identification.
- c. Work within specific time limits. Use a kitchen timer or a clock. If increased time is needed for any given task, use 5-minute increments.
- d. If anyone in the group has a question, encourage the member to ask all group members first before the group asks the teacher or outside resource.
- e. Predetermine the signals for students to stop talking and refocus back to the teacher—for example, dim the lights, clap your hands three times, or raise your hand and students raise theirs.

#### 4. Collect Any Extra Materials You May Need

Every lesson has a list of materials needed for the specific activities plus some optional items. Some collaborative activities or games may require extra worksheets for recording information or keeping score. The Extra Masters at the end of this book include many additional worksheets for collaborative activities.

To organize and display materials, a pocket chart can be useful. Pocket charts can be found in most educational catalogs. The pockets should be at least 3 inches wide and allow for at least six matching cards or more in a display.

Many teachers have incorporated graphing calculators and computer software when using these decks. Some teachers provide one graphing

calculator for each group, and other teachers allow access to several graphing calculators or computers for the whole class to use as needed. Because the decks provide the opportunity for students to make initial decisions about the graph and trigonometric relationships, using simple text editing or slide show software can be helpful for extending any conjectures or exploring new assumptions in mathematics.

### Lesson Quickstart

The steps below provide an outline of how a lesson can be organized. Start the class with an activity Launch in which students match cards to make groups with 4 or 5 members each.

**Launch the Activity** Give each student one card. Tell the students to locate the other members of their team for today by finding the matching cards. For start-up activities, use cards that match for the current day's lesson. For reinforcement, use cards that represent skills learned in an earlier lesson. Some suggested cards for a launch, based on selected lesson topics, are listed in the chart below.

Suggested Cards for the Launch
<b>Right Triangle Patterns:</b> Select Coordinate Pairs cards and Graph cards from all decks.
<b>Sine, Cosine, Tangent:</b> Select Amplitude or Asymptote cards and Graph cards from Deck A.
<b>Cotangent, Secant, Cosecant:</b> Select Asymptote cards and Graph cards from Deck B.
<b>Tracing Trig Functions:</b> Select Equation, Asymptote or Amplitude, and Coordinate Pairs cards from all decks.
<b>Coefficient Influence:</b> Select Period and Equation cards from all decks.

---

As an alternative to a Launch with cards, each lesson includes various worksheets that you can copy and distribute at the beginning of the lesson, allowing students to write their observations as an initial start-up or warm-up class activity. Students can then bring these worksheets with their own observations to the group for discussion.

If you have arranged the materials in baskets or other containers before class for each group then it is appropriate at this time for the Materials Manager in each group to go get the container for their group. You also have the option of distributing the baskets yourself or allowing one student to distribute all of the baskets to the different groups.

**Complete the Activity** Once the groups have been formed and roles have been assigned, students complete the activity by following the instructions on the Activity sheet. Each student takes on their assigned role for that Activity and performs their assigned tasks. Note that if students are working in pairs, you will need to make some minor adjustments to the assigned student roles.

You may notice that all of the Activities start with students dividing the cards and sorting them into stacks by type. One reason for this practice is so that students can have an overview of which cards they will be working with on any given activity. Also, sorting by the type of card allows students to practice their trigonometry vocabulary.

**Set Time Limits** Many or most of the Activities, including task and discussion worksheets, can be completed in 15 to 30 minutes. Designate a specific time frame for students to work within so that there is enough time for students to report their findings to the rest of the class. Time

intervals can vary depending on the difficulty of the Activity. For the easier tasks, allow 15 minutes for the task and worksheet completion.

**Review Discussion Worksheets** Always allow students to discuss the group's responses to the Discussion Questions in a public classroom forum either during the same day or on the following day. All groups should have the opportunity to present their findings and to ask their questions. You may want to appoint a general Classroom Recorder to record all of the groups' findings or collect the group worksheets before the students leave.

**Extension Activities** As an extension of each lesson topic, students may complete an Extension Activity using the Write-Your-Own Activity sheet on page 48, with Discussion Questions on page 49. In addition, there is an option to compare the cards in pairs and write your own description on page 50. By comparing all of the cards according to their own descriptions, students can explore relationships between trigonometric equations and graphs.

**Conclusion** By using the different decks of cards with the activities and games in *The Algebra Game: Trig Functions*, students have the opportunity to discover how equations, amplitudes, asymptotes, periods, and coordinate pairs tables all connect to unique graphs. Whether you use the guidelines in this manual or develop your own structures for using the Trig Functions decks, your students will develop a deeper understanding of basic trigonometric concepts by actively engaging with *The Algebra Games: Trig Functions!*

## LESSON

2

# Sine, Cosine, and Tangent

## Introducing the Basic Trigonometric Functions

The graphs of the three basic trig functions (sine, cosine, and tangent) each have recognizable features such as periodicity and amplitude or asymptotes. In these activities, students sort the graph cards by inspection and identify the function.

You can use the blank templates at the end of the manual to write your own investigation activities. Or encourage your students to use them to create their own activities.

Remember to make sufficient copies of all worksheets for students.

**Recommended decks:** Deck A. Refer to the outline of the Trig Function decks on page x to help you choose the cards for the lesson.

### Materials

- Deck A
- One Activity sheet per group
- Discussion Questions Worksheet, Trig Sorting Mat, Group Notes Worksheet (one per group), Literacy Worksheets for Sine, Cosine, and Tangent (one per student)
- **Optional:** graphing calculators

## Graphs of Sine, Cosine, and Tangent

### Activity

2.1

**HSF-IF.B.4** ...Sketch graphs showing key features given a verbal description of the relationship. Also **HSF-TF.B.5** and **HSF-IF.C.7e**.

**Materials Manager:** Give each group member a Sine, Cosine, and Tangent Literacy Worksheet.

**All:** On the Literacy Worksheets, note any relationships between the graphs.

**Recorder:** Transfer the group results to the Group Notes worksheet.

**Materials Manager:** Divide the cards between group members for sorting.

**All:** Sort the cards into 6 stacks: Graph, Amplitude, Asymptote, Equation, Coordinate Pairs, and Period. Pool the stacks so there are 6 stacks for the whole group.

**Facilitator:** Keep the Graph, Coordinate Pairs, and Equation cards and put aside the others. Place the Trig Sorting Mat in the center of the group table.

**Materials Manager:** Divide the Graph and Equation cards between group members.

**All:** Arrange the Graph and Equation cards on the Trig Sorting Mat based on your notes made on the Literacy Worksheets. Next, match the Equation cards to the Graph cards.

**Recorder:** Read the Discussion Questions to the group and write the responses that the whole group agrees on. Be sure to include responses for the new questions posed by the group.

**Presenter:** Report the group's responses to the class.

# Trig Sorting Mat

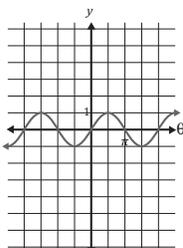
<b>Sin</b>		<b>Sin</b>
<b>Cos</b>		<b>Cos</b>
<b>Tan</b>		<b>Tan</b>

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Sine Literacy Worksheet

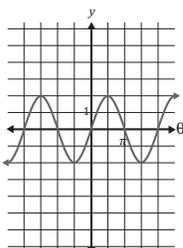
The graphs match the equations. Write notes in the space about any relationships you see between each equation and the matching graph. You may wish to use the actual Equation and Graph cards.

1.  $y = \sin \theta$

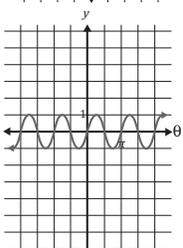


Notes

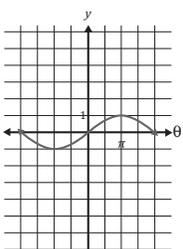
2.  $y = 2 \sin \theta$



3.  $y = \sin 2\theta$



4.  $y = \sin \frac{1}{2} \theta$



Describe connections you notice between the graph and the matching equation. Use the back of this sheet if you need more space.

---



---



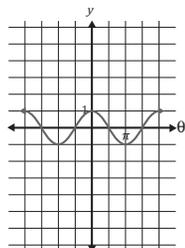
---

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Cosine Literacy Worksheet

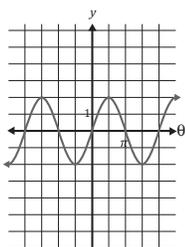
The graphs match the equations. Write notes in the space about any relationships you see between each equation and the matching graph. You may wish to use the actual Equation and Graph cards.

1.  $y = \cos \theta$

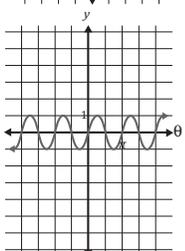


Notes

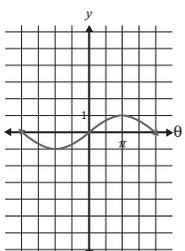
2.  $y = 2 \cos \theta$



3.  $y = \cos 2\theta$



4.  $y = \cos \frac{1}{2} \theta$



Describe connections you notice between the graph and the matching equation. Use the back of this sheet if you need more space.

---



---



---



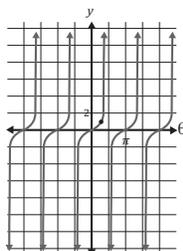
---

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## Tangent Literacy Worksheet

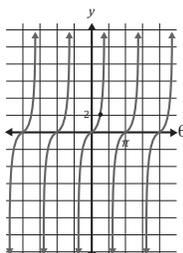
The graphs match the equations. Write notes in the space about any relationships you see between each equation and the matching graph. You may wish to use the actual Equation and Graph cards.

1.  $y = \tan \theta$

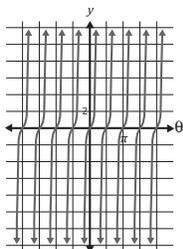


Notes

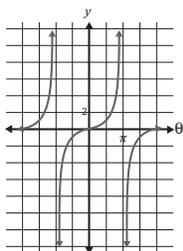
2.  $y = 2 \tan \theta$



3.  $y = \tan 2\theta$



4.  $y = \tan \frac{1}{2}\theta$



Describe connections you notice between the graph and the matching equation. Use the back of this sheet if you need more space.

---



---



---

Group Members: \_\_\_\_\_  
\_\_\_\_\_

## Group Notes Worksheet

Use as many descriptors as you need to help you recognize the graphs without the Literacy Worksheets.

Sine	Cosine	Tangent

Group Members: \_\_\_\_\_  
\_\_\_\_\_

## Discussion Questions for Activity 2.1

1. How are the sine graphs different from the cosine graphs? Describe all of the differences you see.

---

---

2. Compare the tangent graphs with the cosine graphs. Write all of the similarities and differences that you see. Use the back of the page if you need more room.

Similarities	Differences

3. Describe how to make a quick sketch of the basic sine, cosine, and tangent graphs by memorizing only a couple of key points on each curve. Use the back of the page if you need more room.

---

---

As a group, write two questions to ask the rest of the class. Before asking the questions, decide as a group on acceptable responses.

**Question 1.** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Question 2.** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_