Fractions with Prime Factor Tiles™

by David Bernasconi

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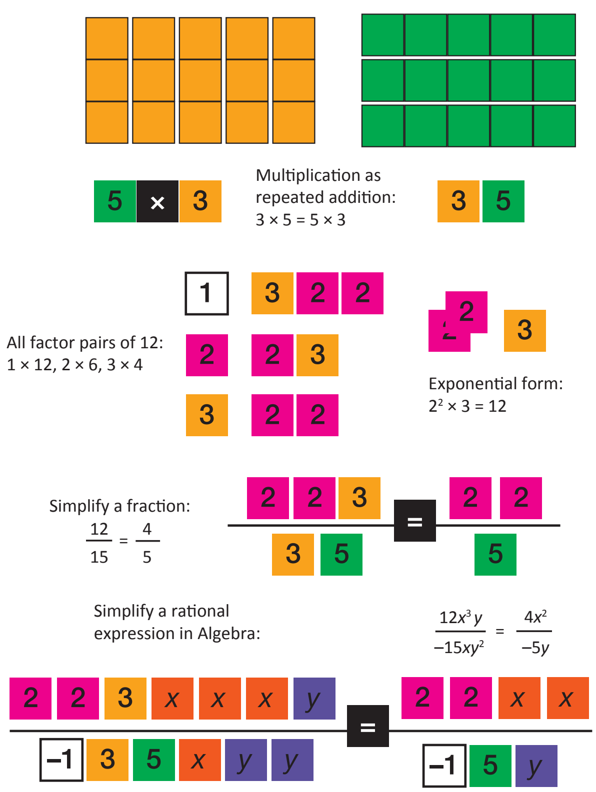
The Prime Factor Tiles System and Method

Prime Factor Tiles are a unique mathematics teaching aid consisting of prime numbers printed on color-coded tiles. Activities featuring Prime Factor Tiles enable students to see and manipulate the hidden factors of composite numbers in order to understand and apply traditional algorithms for performing fraction arithmetic operations. Students who learn to compute with fractions using Prime Factor Tiles develop a deep understanding of factors that translates naturally to algebraic concepts and operations that are similarly dependent upon recognizing and manipulating factors—variable and negative factors included.

Prime Factor Tiles™ System and Method



Why Prime Factor Tiles Are Needed

The consensus among mathematics educators and researchers is that fractions instruction in the lower and middle grades fails to prepare many students for learning higher-level math. "Early Predictors of High School Mathematics Achievement," an analysis of long-term, longitudinal studies from the US and UK first published in 2012, concludes that "elementary school students' knowledge of fractions and of division uniquely predicts those students' knowledge of algebra and overall mathematics achievement in high school." (http://journals.sagepub.com/doi/abs/10.1177/0956797612440101) The Final Report of the 2008 National Mathematics Advisory Panel identifies difficulty with fractions as a "pervasive problem" and a "major obstacle to further progress in mathematics, including algebra." According to this report, "the most important foundational skill not presently developed appears to be proficiency with fractions." (https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf)

Effective instructional strategies and teaching aids for developing conceptual knowledge of fractions are already in widespread use. Concrete learning experiences with fraction circles and fraction strips help students to understand the properties of fractions as they partition a whole into equal parts and make physical comparisons by size and by reference to the number line. The same cannot be said for developing procedural knowledge of fractions. Students are expected to become proficient with the four fraction arithmetic operations through memorized algorithms taught abstractly and without reference to physical models. Tricks like "keep-change-flip" substitute for sound mathematics, and the result is poorly understood rules that are routinely misapplied. The Prime Factor Tiles System and Method was specifically developed to make learning the algorithms for fraction operations less abstract and more concrete.

Why Prime Factor Tiles Work

Standard algorithms for fraction arithmetic operations require students to recognize and manipulate the factors comprising the numerator and denominator—specifically, factors common to both numbers. In The Prime Factor Tiles System and Method, a number is expressed exclusively as a product of its prime factors. With each prime factor corresponding to a specific color, primes common to the numerator and denominator become visually obvious. Moveable tiles allow individual prime factors to be introduced, canceled out, or combined in accordance with each problem-solving step in an algorithm.

With practice, the decision to move, remove, or introduce a factor becomes intuitive according to the color of the tile and its relative location within the fraction: Factors placed side by side in a horizontal row get multiplied, while any factor below the fraction bar cancels out a common factor above the fraction bar. Because Prime Factor Tiles teach the same algorithms for fraction operations that students are expected to use without the aid of any manipulative, there is a natural transition to pencil-and-paper methods. By emphasizing the role of factors in fraction operations, the use of Prime Factor Tiles builds a strong foundation for later algebraic operations involving rational expressions, polynomials, radicals, and more.

Using Prime Factor Tiles

The scripted activities in this workbook were written for teacher-directed instruction with a small group of students, but they can be modified for use in a whole-class or student-directed setting. Prime Factor Tiles are simple to use, but there is no substitute for reading through the background information and rehearsing each activity yourself prior to introducing it to students. As a new way for students to represent and manipulate numbers, it is critical that there is no confusion on the part of the instructor as to how Prime Factor Tiles are to be used to solve problems. Solving problems through physical manipulation of prime factors is the singular purpose of the tiles; they are not intended for representing all numbers or displaying both the problem and the result simultaneously.

Many of the activities for Prime Factor Tiles begin with one or more demonstrations involving concrete objects in order to activate prior knowledge developed through the use of hands-on activities and visual models in earlier grades. Simple fraction problems modeled with fraction strips, pizza slices, tokens, and so on, ensure that the steps of an algorithm can be acted out and final results can be confirmed physically. Modeling these same problems using Prime Factor Tiles has the explicit purpose of building procedural knowledge upon conceptual knowledge. Making reference to a physical model or real-world situation familiar to students when introducing an algorithm shifts the instructional focus from applying a rule to understanding how and why the rule works.

What's in the Kit

Each student set of Prime Factor Tiles consists of the following factors: 8 magenta #2 tiles, 6 yellow #3 tiles, 4 green #5 tiles, 3 blue #7 tiles, and two white #1 tiles with –1 on the reverse side. The colorful prime number tiles make it possible to represent every number on the 10 × 10 times table and create a large variety of fraction problems to solve. The white #1 tile is necessary as a numerator when other factors have canceled out, and the –1 on the reverse facilitates introduction to negative numbers and absolute value. The black arithmetic operation tiles are double-sided to reinforce the inverse relationships between addition and subtraction and between multiplication and division. Each student set includes 2 "+/–" and 2 "×/÷" tiles. One black "=/<" tile and 2 tiles with a parenthesis on one side and absolute value bars on the reverse are also included.

The Factor Finder

For many students, the inability to recall times-table facts from memory is the first roadblock to success with fraction operations. The Factor Finder is a temporary aid that makes it possible for these students to participate in the activities written for Prime Factor Tiles and attain computational fluency with fractions even as they work toward mastery of times-table facts. Facts from the 10 x 10 times table are arranged in numerical order, making it easy for students to quickly identify one or more factor pairs as a starting point to determining the prime factorization of a number.

The Fraction Mat

The reproducible fraction mat is intended to be cut in half, with each student receiving a single fraction bar strip. The same Prime Factor Tiles used to model the problem should be manipulated to produce a solution. The problem and solution are not to be displayed simultaneously.

Activity 1 Multiplication with Prime Factor Tiles

Objective

Students will:

1. Learn the rules for using the Prime Factor Tiles.
2. Multiply with three or more factors.
3. Apply the Commutative and Associative Properties of Multiplication.

Materials and Background Knowledge

- Prime Factor Tiles student sets

Students should understand multiplication as repeated addition.

Vocabulary

prime factor

composite factor

Commutative Property of Multiplication

Associative Property of Multiplication

partial product

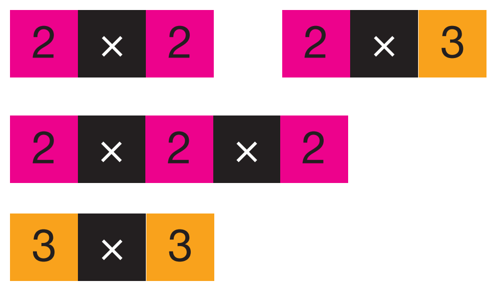
Overview

Students previously defined prime and composite numbers by using square tiles to physically represent area models for multiplication. Now, students will learn how Prime Factor Tiles are used to represent products of two or more prime factors. Later, students will use Prime Factor Tiles to represent any number on the 10 × 10 times table as a product of its prime factors.

Introduce the Activity

1. Introduce the students to the set of Prime Factor Tiles. *Distribute the student sets.* *Ask*: **What do you notice about the tiles?** (Tiles with the same number are the same color; the black tiles have operation symbols; there is no 0, 4, or 6.) *Ask*: **What do the numbers on the colored tiles all have in common?** (2, 3, 5, and 7 are all prime numbers.) *Ask*: **How are prime and composite defined?** (Prime: Exactly two factors. Composite: Three or more factors.) *Ask*: **1 is on the white tile. Is it prime or composite?** (1 is neither prime nor composite. 1 has only one factor.) *Say:* **The –1 on the reverse side will be used in future activities.**

2. Represent composite numbers. *Say:* **Prime Factor Tiles can be used to represent every number on the 10 × 10 times table as a product of prime factors.** *Ask:* **What single-digit numbers are missing from the set of tiles?** (4, 6, 8, 9) *Ask:* **Are these numbers prime or composite?** (composite) *Say:* **Please use the Prime Factor Tiles, including the multiplication tile, to represent 4, 6, 8, and 9 as products of prime factors.** (See diagram.)



3. Model the Commutative Property. *Ask:* **Did you represent 6 with 2 × 3 or with 3 × 2?** (Either is correct.) *Ask:* **Does the order of the factors matter?** (No.) *Ask:* **What property does this model?** (The Commutative Property of Multiplication.) *Say:* **The Commutative Property of Multiplication allows you to rearrange the order of the factors being multiplied.**

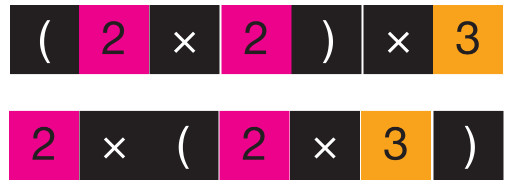
4. *Say:* **Please flip the magenta and yellow tiles upside down and make two rectangles, one with two rows of three yellow tiles and the second with three columns of two magenta tiles.** *Ask:* **How does this model multiplication as repeated addition?** (Two groups of three is 3 + 3 or 2 × 3. Three groups of two is 2 + 2 + 2 or 3 × 2.)

5. Reinforce partial products. *Say:* **Please represent the number 8 again.** *Ask:* **How do you multiply to get 8?** (2 × 2 = 4 and 4 × 2 = 8.) *Say:* **With three factors, you must multiply twice.** *Say:* **Insert black parentheses tiles to emphasize that 2 × 2 was multiplied first.** (See diagram.) *Say:* **The 2 × 2 in parentheses is called a *partial product*.**

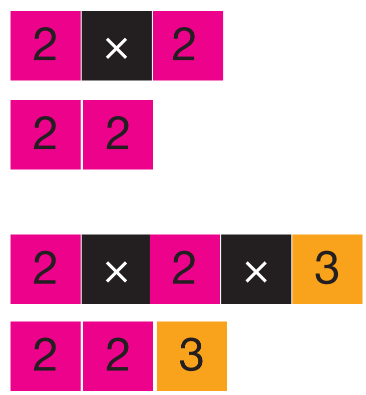
6. Introduce primes greater than 7. *Say:* **Please represent the number 10.** *Ask:* **Does the order of the factors matter?** (No.) *Say:* **By the Commutative Property of Multiplication, 2 × 5 = 5 × 2.** *Ask:* **Is it possible to represent the number 11?** Why or why not? (No, because 11 is not a multiple of 2, 3, 5, or 7.) *Say:* **Two 1 tiles would be 1 × 1, not 11. Remember, there is no place value with Prime Factor Tiles. The Prime Factor Tiles can represent products of 2, 3, 5, and 7 only and are not intended to represent every number.**

7. *Say:* **Please use Prime Factor Tiles to represent the number 12.** (See diagram.) *Ask:* **Should everyone have the same Prime Factor Tiles?** (Yes.) *Ask:* **Should everyone have their tiles in the same order?** (Not necessarily. The order does not matter because of the Commutative Property.) *Ask:* **Did anyone put the yellow #3 tile first? If so, what times-table fact for 12 did you recall?** (Probably 3 × 4 = 12.) *Say:* **The order of the factors will be different depending on the times-table fact recalled.**

8. Model the Associative Property. *Say:* **Please arrange your tiles as 2 × 2 × 3 and place parentheses tiles around 2 × 2.** *Ask:* **What is the partial product and times-table fact if you multiply 2 × 2 first?** (4; 4 × 3 = 12) *Ask:* **Can you place the parentheses tiles around 2 × 3 instead?** (Yes.) *Ask:* **What property is modeled by moving the parentheses?** (the Associative Property of Multiplication) *Say:* **The Associative Property of Multiplication allows you to group a different pair of factors within parentheses.** *Ask:* **What is the partial product and times-table fact if you multiply 2 × 3 first?** (6; 2 × 6 = 12) *Say:* **By the order of operations, we simplify within parentheses first.**



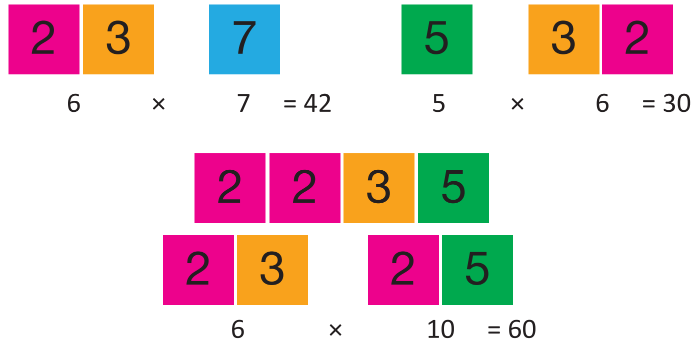
9. *Future use of the multiplication tile. Say:* **The Prime Factor Tiles are intended for use without the multiplication tiles between the factors. For this activity, the multiplication tile was used to introduce Prime Factor Tiles in a way that was familiar to you. In future activities, multiplication between factor tiles is implied and the multiplication tile is generally not used.** *Say:* **Please model 4 and 12 without the multiplication tile.**



10. Reinforce that Prime Factor Tiles are multiplied together even without the multiplication tile. *Say:* **The two 2s side-by-side is not 22, it is 4 because 2 × 2 = 4. The combination of 2-2-3 is not two hundred twenty-three, it is 12 because 2 × 2 × 3 = 12.**

11. *Independent practice.* Have students practice finding composite numbers represented with Prime Factor Tiles. *Say:* **Choose up to four tiles arranged in any order. Next, use the Commutative and Associative Properties**

of Multiplication to rearrange and/or regroup the tiles to make partial products that are familiar. Don't use the multiplication tiles and instead of parenthesis tiles, leave a space between grouped factors. Review student practice problems, emphasizing the use of the Properties of Multiplication to make partial products for convenient computations.



Activity 2 Prime Factorization of Composite Numbers

Objectives

Students will be able to:

1. Represent any composite number as a product of prime factors.

2. Model prime factorization with Prime Factor Tiles.

3. Create "factor hedges" as a superior alternative to factor trees.

Materials and Background Knowledge

- Prime Factor Tiles student sets; Factor Finder

Students should be familiar with prime and composite numbers; know the divisibility tests for 2, 3, 5, and 10; and be able to recall multiplication facts from the 10 × 10 times table.

Vocabulary

prime
composite
factor
product
factor pair
factor hedge

Overview

Students previously understood that different pairs of factors can be multiplied to give the same product. Now, students will learn that every composite number can be broken down into a product of prime factors. Later, students will multiply prime factors in different combinations to determine every factor pair of a composite number.

Introduce the Activity

1. Reintroduce students to the set of Prime Factor Tiles. Students should remember the following rules of use:

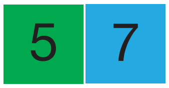
a. The prime factors 2, 3, 5, and 7 can be multiplied to generate every composite number on the 10 × 10 times table.

b. A composite number can be represented only as the product of its prime factors.

c. Operation symbols are not inserted between the Prime Factor Tiles when representing composite numbers.

d. Place value is not used in the system, and prime factors of a number can be listed in any order.

2. *Say:* **Place a green #5 tile and a blue #7 tile next to one another in a row to represent the number 35.** *Ask:* **Why do these tiles represent 35 and not 57?** (A green tile next to a blue tile represents 35 because 5 × 7 = 35.) *Ask:* **If we reverse the order of the green #5 tile and the blue #7 tile, does it represent a different number?** (No. The order of the factors does not matter since 7 × 5 = 35.)



5 × 7 = 35

7 × 5 = 35

3. Reinforce that the Commutative Property of Multiplication allows you to change the order of factors that are multiplied together and that there is no place value assigned.

4. Have students choose any combination of three or four Prime Factor Tiles arranged in a horizontal row and calculate the composite number that results from their product. Ask students to share what they did. For example: "Two yellow number three tiles and two magenta number two tiles represent 36 because 3 × 3 × 2 × 2 = 36."

No specific order of prime factors (i.e., least to greatest) should be emphasized at this point. The order in which prime factors are identified and listed is going to differ depending upon how times-table facts are recalled and whether or not divisibility tests are used.

3 × 3 × 2 × 2 = 36

9 × 4 = 36

When starting from a composite number, students must use times-table facts and/or divisibility tests to break the number down into its prime factors. Identification of prime factors is an important skill for operations involving fractions and for factoring in algebra.

5. *Ask:* **How can you determine the prime factorization if given the composite number first instead of its factors?** Listen for a variety of responses and discuss how simple or efficient a given strategy might be. (Possible answers: Use times-table facts in reverse, use divisibility tests, use guess and check.) Instant recall of times-table facts is preferred, but numbers off the 10 × 10 times table will typically require students to employ divisibility tests.

6. *Say:* **Show the prime factorization of 25 with your tiles.** Displaying 25 with two green #5 tiles should be easy for every student because 5 × 5 = 25 is a common times-table fact and the product of only two primes.

7. *Say:* **Model 42 using Prime Factor Tiles.** (one magenta #2 tile, one yellow #3 tile, and one blue #7 tile) *Ask:* **What strategy did you use?** ("From the times table, 6 times 7 equals 42, and 6 needs to be represented as 2 times 3 since it is a composite number." "Since 42 is even, it must be divisible by 2, and that leaves 21, which is 3 times 7.") *Ask:* **Could the order of tiles be 7-3-2 instead?** (Yes, because 7 × 3 × 2 = 42.) *Ask:* **What property does this model?** (The Commutative Property of Multiplication)

2 × 3 × 7 = 42

6 × 7 = 42

7 × 3 × 2 = 42

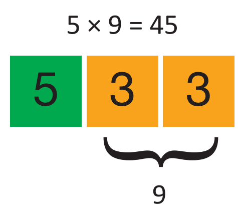
21 × 2 = 42

8. Introduce the Factor Finder as a resource for finding pairs of factors for any composite number on the 10 × 10 times table. The Factor Finder is a temporary aid for students who have not yet memorized the times table or until students have mastered recall of times-table facts "in reverse."

9. Practice using the Factor Finder by calling out products from the 10 × 10 times table and asking students to provide one or more of the

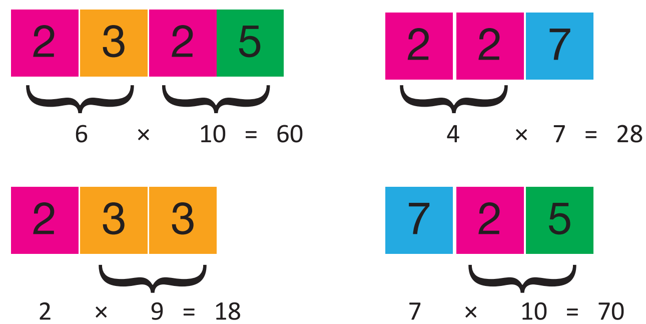
corresponding factor pairs as quickly as they are able. Make sure to call upon those students who lack quick recall and will be most likely to refer to the Factor Finder.

10. *Say:* **Model 45 using the Prime Factor Tiles.** Students should try to recall the times-table fact from memory and refer to the Factor Finder only as necessary. Students will need to recognize that one of the factors of 45, 9, is a composite number and needs to be modeled as 3 × 3 since only prime factors appear on the tiles.



11. Have the students work in pairs to represent the prime factorization of several times-table facts using the Prime Factor Tiles. They should take turns, with one student arranging the tiles and the second student multiplying the prime factors together to confirm the solutions.

Students should write out the prime factorization of the numbers they have chosen. Have several students share with the rest of the class by announcing the composite number, its prime factorization, and the related times-table fact that was employed. Some samples are as follows:



Activity 5 Generate Equivalent Fractions

Objectives

Students will be able to:

1. Understand that two or more fractions can have the same value even if they consist of different numbers.
2. Generate equivalent fractions by multiplying the numerator and denominator by the same (common) factor.
3. Apply the Identity Property of Multiplication to maintain equivalence.

Vocabulary

fraction
numerator
denominator
product
sector
equivalent fraction
Identity Property of Multiplication

Materials and Background Knowledge

- Prime Factor Tiles student sets; fraction mat (page 40); Factor Finder
- For teacher review demonstration: Set of fraction circles, three standard rulers marked off into thirds, sixths, and twelfths, assorted coins

Students should recognize that a fraction represents a part of a whole and also should possess a conceptual understanding of equivalent fractions through concrete experiences with fraction circles, fraction strips, rulers, money, and so on.

Overview

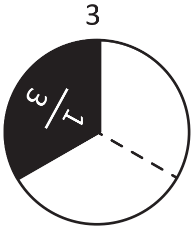
Students previously modeled equivalent fractions by size comparisons using fraction circles or fraction strips. Now, students will learn how to mathematically transform a single fraction into several equivalent forms through multiplication. Later, students will use equivalent fractions to compare, add, and subtract fractions that have unlike denominators.

Introduce the Activity

1. Briefly review the concept of a fraction with teacher demonstrations. Display the whole fraction circle for three-thirds. *Ask:* **Does anyone know what the “pie-shaped” pieces of a circle are called?** (sectors) Slide one of the three sectors off to the left so that one-third and two-thirds of the circle are displayed side by side.

“one third”

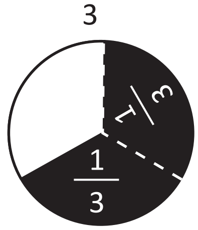
$\frac{1}{3}$



numerator: 1 sector
denominator: 3 sectors in one whole

“two thirds”

$\frac{2}{3}$

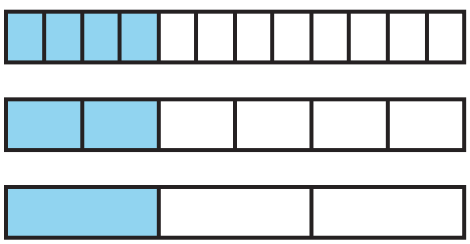


numerator: 2 sectors
denominator: 3 sectors in one whole

2. *Ask:* **What is the name of the fraction on the left? Can you identify the numerator and denominator and explain their meaning?** (1/3; The numerator is 1 because there is 1 part/sector represented, and the denominator is 3 because it takes 3 parts/sectors to make one whole circle.)
3. Represent a fraction of one dollar with coins. Display two dimes. *Ask:* **Who can name and describe the fraction represented by the two dimes by identifying the numerator and denominator?** (2/10; The numerator is 2 because there are two dimes, and the denominator is 10 because it takes 10 dimes to make one whole dollar.) Any mention of twenty cents or one-fifth by a student—or by the teacher—will naturally lead to a discussion of equivalent fractions.
4. Briefly review the concept of equivalent fractions with teacher demonstrations. Display equivalent fraction circles for 1/3,

2/6, and 4/12 side by side for a physical comparison by size. *Ask:* **Can you name the fractions displayed?** (1/3, 2/6, and 4/12) *Ask:* **Does each model represent the same fraction of one whole circle?** (Yes.) *Say:* **For each of the models, the sizes of the pieces are different, the number of pieces are different, and the number of pieces that make up one whole are different, but each model represents the same portion of one whole circle: 1/3 = 2/6 = 4/12. Equivalent fractions have equal value.**

5. Model the fractions 1/3, 2/6, and 4/12 with standard rulers: one marked off in thirds, another in sixths, and the third in twelfths.



Ask: **What fraction does each ruler represent?** (1/3, 2/6, and 4/12) *Ask:* **Are the shaded portions of the rulers each the same length?** (Yes.) *Say:* **Despite the different numbers in the numerators and denominators, the lengths are the same: 1/3 = 2/6 = 4/12. The fractions are equivalent.**

6. Display the two dimes again. *Ask:* **What is another way to represent this money value using other coins?** (4 nickels or 20 pennies) Display 4 nickels and 20 pennies. *Ask:* **What fractions do the nickels and pennies represent? Can you explain each numerator and denominator?** (4/20; 4 nickels out of 20 nickels to equal one dollar; 20/100; 20 pennies out of 100 pennies to equal one dollar.)

7. *Say:* **The dimes, nickels, and pennies in this example have the same value, so they are equivalent: 2/10 = 4/20 = 20/100.** *Ask:* **Can a single fraction represent 20 cents if 20 cents is composed of one dime, one nickel, and five pennies?** (No, a single fraction cannot be written to show 20 cents unless all of the parts are the exact same size or value.) (See below.)

8. Introduce the Identity Property of Multiplication. Call out several multiplication problems with 1 as a factor. Ask for unison response: 12 × 1, 1 × 25, 37.5 × 1, 1 × 2/3, fifty-seven million × 1. *Ask:* **What is the product of 1 times any number?** (The number itself.) *Say:* **The Identity Property of Multiplication can be used to change the**

10 dimes equal 1 dollar. 1 dime is $\frac{1}{10}$ of 1 dollar.

20¢ or $0.20

$\frac{2}{10}$

20 nickels equal 1 dollar. 1 nickel is $\frac{1}{20}$ of 1 dollar.

20¢ or $0.20

$\frac{4}{20}$

100 pennies equal 1 dollar. 1 penny is $\frac{1}{100}$ of 1 dollar.

20¢ or $0.20

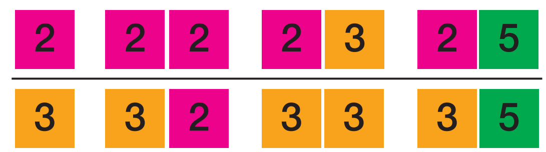
$\frac{20}{100}$

form of a number without changing its value, such as making a specific denominator.

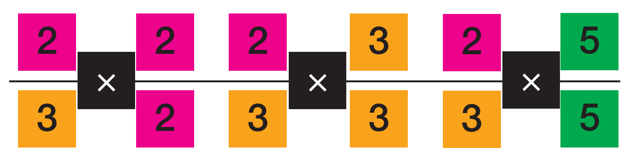
9. Call out a few fractions with the same number in the numerator and the denominator: 2/2, 7/7, 99/99, one-million one millionths. *Ask:* **What is the value of these fractions?** (1) *Ask:* **Who can explain why each of these fractions equals 1?** (If you have the same number of parts that make up the parts in one whole, you have one whole.)

10. *Ask:* **How does having a numerator that is the same as the denominator relate to division?** (The numerator is divided by the denominator, and any number divided by itself equals one 1.) Operations with fractions often involve "canceling out" factors that are common to the numerator and denominator, so it is critical that students understand that any number divided by itself is one, not zero, and that any number times 1 is the number itself. *Say:* **Any number divided by itself equals 1. If five students share five cookies, each student gets one cookie, not zero cookies.**

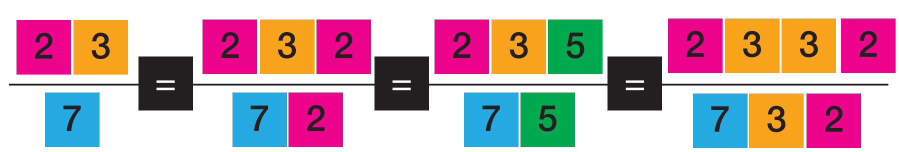
11. Display the fraction 2/3 using Prime Factor Tiles and the fraction mat. Display a few equivalent fractions alongside 2/3 by introducing the same factor in both the numerator and denominator. (2/3, 4/6, 6/9, 10/15)



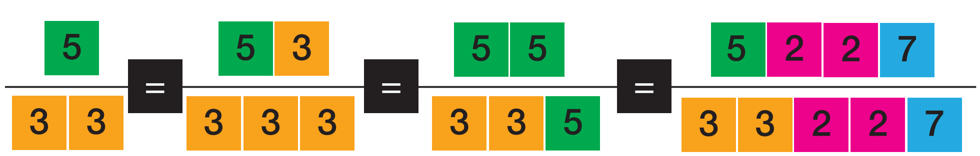
12. Emphasize multiplication of the given fraction by 1 by sliding the tiles apart to insert the black multiplication tile between the original fraction 2/3 and the factor over itself. *Say:* 4/6 = 2/3 × 2/2, 6/9 = 2/3 × 3/3, 10/15 = 2/3 × 5/5. *Say:* **The Identity Property of Multiplication makes it possible to change the numerator and denominator of a fraction without changing its value.**



13. Have students represent a fraction using only two or three Prime Factor Tiles, and then create three equivalent fractions alongside it. *Ask:* **Who would like to share their equivalent fractions with the group and describe each as a product?** (Each equivalent fraction is the product of the original fraction and a fraction having a value of one.) Have students write each equivalent fraction and sketch the tiles that represent it. (See examples below.)



6/7 = 12/14 = 30/35 = 36/42



5/9 = 15/27 = 25/45 = 140/252

Activity 9 Add and Subtract Fractions with Unlike Denominators

Objectives

Students will be able to:

1. Add and subtract fractions by generating equivalent fractions according to the lowest common denominator (LCD).
2. Understand and explain why fractions with unlike denominators cannot be added or subtracted.
3. Transition from Prime Factor Tiles to a paper-and-pencil method.

Vocabulary

numerator

denominator

like and unlike denominators

Materials and Background Knowledge

- Prime Factor Tiles student sets; fraction mat (page 40); Factor Finder
- For teacher demonstration: fraction circles and coins

Students should understand how to generate equivalent fractions and be able to write the prime factorization of composite numbers. Students should understand how to identify the value of any fraction relative to other fractions through comparisons between like numerators and comparisons between like denominators.

Overview

Students previously added and subtracted numerators of fractions having like denominators. Now, students add and subtract fractions by making equivalent fractions with like denominators. Later, students will add and subtract polynomials and rational expressions in algebra according to like terms.

Introduce the Activity

1. Activate prior conceptual understanding of fraction addition and subtraction with reference to familiar models. Using fraction circles, distribute two 1/5 sectors each to two students. *Ask:* **What fraction of the whole does each student have, and how could you calculate the fraction of the whole the two students have altogether?** (Each student has 2/5 and together they have 4/5, because 2/5 + 2/5 = 4/5.)

2. Next, display three quarters and give one of the coins to a student. *Ask:* **What fraction of a whole dollar did I start with, what fraction went to your classmate, and how could we calculate the fraction I have left?** (The teacher started with 3/4 and the classmate received 1/4. The teacher has 2/4 left because 3/4 – 1/4 = 2/4.)

3. *Ask:* **What does the numerator of a fraction represent and how is it affected by addition or subtraction?** (The numerator represents the number of the parts, and when the parts are all the same, we can simply add or subtract the numerators.) *Ask:* **What does the denominator of a fraction represent and how is it affected by addition or subtraction?** (The denominator represents the number of parts in one whole. Adding or subtracting parts yields a sum or difference of the same part, not some other part. Fifths stay fifths and fourths stay fourths unless you simplify.)

4. Model addition of fractions with unlike denominators using coins. One student already has one quarter; now give three nickels to a second student. *Ask:* **What fraction of one dollar does each student have?** (The first student has 1/4 dollar because it takes 4 quarters to make one dollar. The second student has 3/20 because it takes 20 nickels to make one dollar.)

5. *Ask:* **Can we add the numerators together to calculate the fraction of one dollar the two students have altogether?** (No, we cannot add

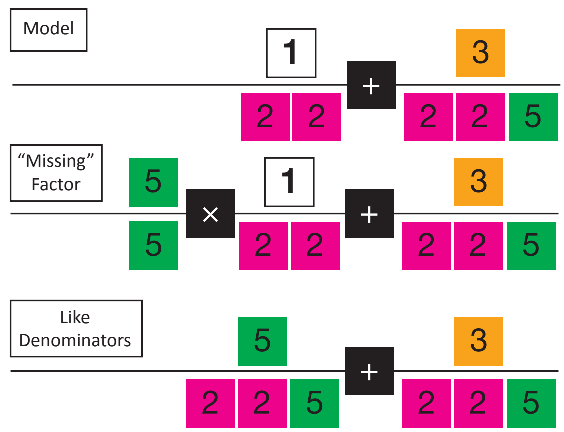
the numerators because the denominators are different. The coins have different names because they have different values.) *Ask:* **What is one way we could add the values of the coins together?** (One or more students will likely suggest adding up the cents: 25 + 15 = 40 cents.)

6. *Ask:* **What fraction of a dollar is one cent?** (One cent is 1/100 of a dollar because it takes 100 cents or pennies to make $1.) *Ask:* **How could you write 1/4 and 3/20 as fractions with denominators of 100?** (By making equivalent fractions! Multiply 1/4 × 25/25 to get 25/100 and multiply 3/20 × 5/5 to get 15/100. Now we can add the fractions by summing the numerators: 25/100 + 15/100 = 40/100 and 40/100 is 40 cents.)

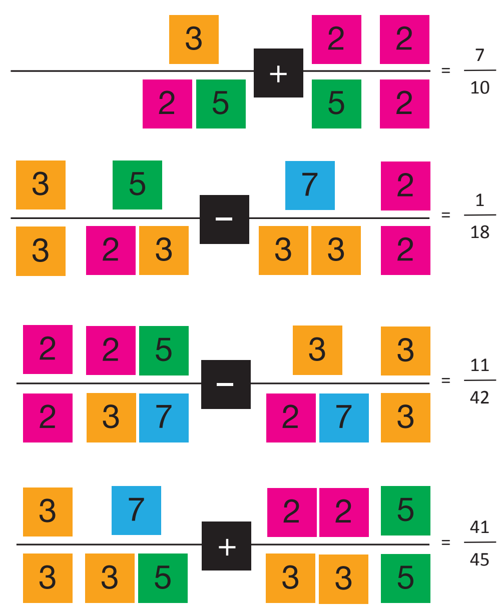
7. Model addition and subtraction of fractions with Prime Factor Tiles. Say: **Please model the problem 1/4 + 3/20 using Prime Factor Tiles and a fraction mat.** (Confirm correct modeling with the graphic, right column.) *Ask:* **What do you notice about the factors in the denominators, and what do you remember about making equivalent fractions with like denominators?** (Both denominators have a pair of magenta #2 tiles. Like denominators must have the exact same factors, and if you multiply the denominator by any factor, you have to multiply the numerator by the same factor.)

8. *Reinforce:* **Any factor divided by itself is 1, and 1 times any number is that number according to the Identity Property of Multiplication.** *Ask:***What prime factor is present in 20 but not in 4?** (5 is a factor of 20 but not 4.) *Ask*: **Are any factors of 4 not factors of 20?** (No, 4 is a factor of 20.) *Ask:* **What is the lowest common denominator or LCD of the fractions and what is necessary to make the denominators alike?** (The LCD is 20 because 20 has all the factors of both 4 and 20. 1/4 needs to be multiplied by 5/5 to make 5/20, nothing needs to be done to 3/20.)

9. *Say:* **Please model how to make equivalent fractions with like denominators, then calculate the sum of the fractions.** (Confirm modeling with the graphic below. The sum is 8/20, which can be reduced to 2/5. 40/100, 8/20, and 2/5 are all equivalent fractions. Modeling the solution is not necessary or recommended since some solutions cannot be modeled as products of 2, 3, 5, and 7.)



10. Assign practice problems from the following list: 3/10 + 2/5, 5/6 – 7/9, 10/21 – 3/14, 7/15 + 4/9. Students should model with Prime Factor Tiles in three successive steps as introduced above. Models and solutions are below.



Fraction Mat

Fraction Mat

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