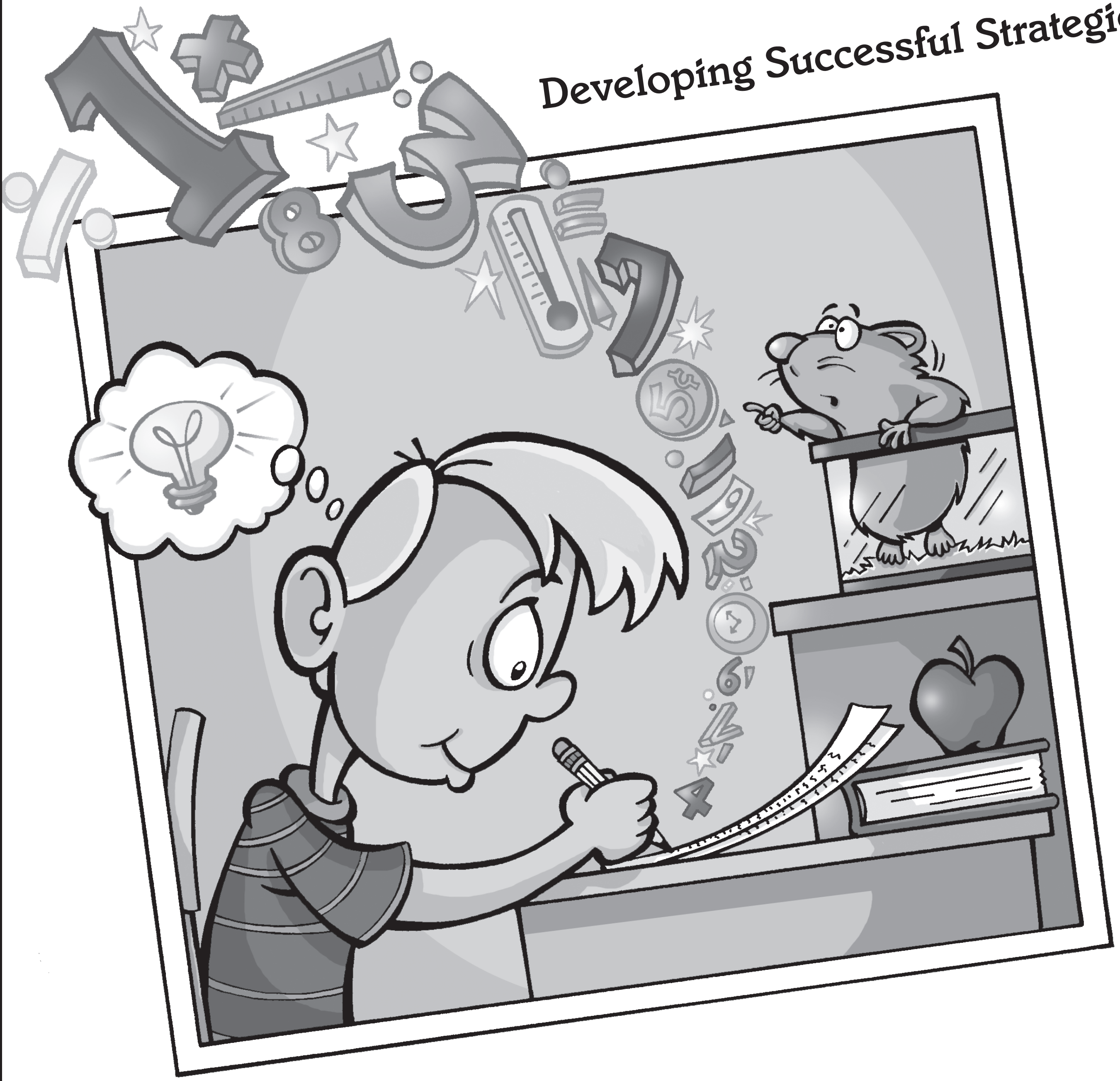
Grades 1–2

Math PROBLEM-SOLVING Skills

Developing Successful Strategies



Printed in the United States of America.

This book is printed on recycled paper.

Order Number 211020
ISBN 978-1-58324-319-9

C D E F G 16 15 14 13 12

Didax

AEP
The Association of Educational Publishers

395 Main Street
Rowley, MA 01969
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CONTENTS

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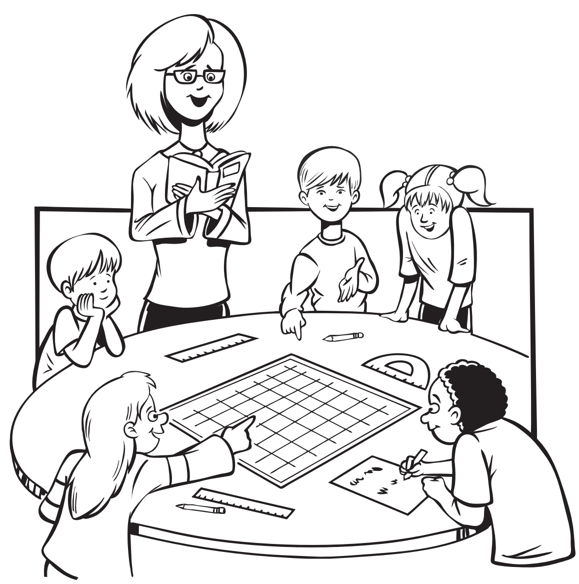
The ***Math Problem-Solving Skills*** series has been developed to provide a rich resource for teachers of students from the elementary grades through middle school. The series of problems, discussions of ways to understand what is being asked, and means of obtaining solutions presented in these books aim to improve the problem-solving performance and persistence of all students. The authors believe it is critical that students and teachers engage with a few complex problems over an extended period rather than spend a short time on many straightforward problems or exercises. In particular, it is essential to allow students time to review and discuss what is required in the problem-solving process before moving to another and different problem. This series includes extensive ideas for extending problems and solution strategies to help teachers implement this vital aspect of mathematics in their classrooms. The problems have been constructed and selected over many years of experience with students at all levels of mathematical talent and persistence, as well as in discussions with teachers in classrooms and professional learning and university settings.

Problem solving does not come easily to most people, so learners need many experiences engaging with problems if they are to develop this crucial ability. As they grapple with problem meaning and find solutions, students will learn a great deal about mathematics and mathematical reasoning. This leads to a focus on organizing what needs to be done rather than simply looking to apply one or more strategies.

Student and Teacher Pages

The student pages present problems chosen with a particular problem-solving focus and draw on a range of mathematical understandings and processes. For each set of related problems, teacher notes and discussion are provided. Answers to the more straightforward problems and detailed solutions to the more complex problems ensure appropriate explanations and suggest ways in which problems can be extended. Related problems occur on one or more pages that extend the problem's ideas, the solution processes, and students' understanding of the range of ways to come to terms with what the problems are asking.

At the top of each teacher page, a statement highlights the particular thinking that the problems will demand, together with an indication of the mathematics that might be needed, a list of materials that can be used in seeking a solution, and the NCTM standards addressed. Each book is organized so that when a problem requires complicated strategic thinking, two or three problems occur on one page (supported by a teacher page with detailed discussion) to encourage students to find a solution together with a range of means that can be followed. More often, problems are grouped as a series of three interrelated pages where the level of complexity gradually increases, while the associated teacher page examines one or two of the problems in depth and highlights how the other problems might be solved in a similar manner.



Each teacher page concludes with two further aspects critical to the successful teaching of problem solving. A section on likely difficulties points to reasoning and content inadequacies that experience has shown may well impede students' success. In this way, teachers can be on the lookout for difficulties and be prepared to guide students past these potential pitfalls. The final section suggests extensions to the problems to enable teachers to provide related experiences that build a rich array of experiences with particular solution methods.

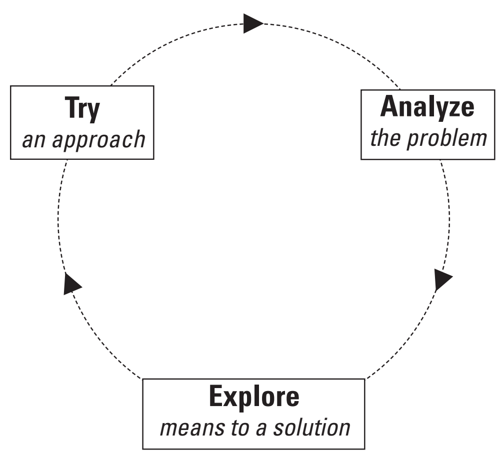
Mathematics and Language

The difficulty of the mathematics gradually increases over the series, largely in line with what is taught at the various grade levels, although problem solving both challenges at the point of the mathematics that is being learned and provides insights and motivation for what might be learned next. For example, the computation required gradually builds from additive thinking, using addition and subtraction separately and together, to multiplicative thinking, where multiplication and division are connected concepts. More complex interactions of these operations build over the series as the operations are used to both come to terms with the meaning of problems and to achieve solutions.

Similarly, two-dimensional geometry is extended to more complex uses over the range of problems and then joined by interaction with three-dimensional ideas. Measurement also extends across the series from length to perimeter, and from area to surface area and volume, drawing on the relationships among these concepts to organize solutions as well as give an understanding of the customary and metric systems. Investigations related to mass rely on both the concept itself and practical measurements.

The language in which the problems are expressed is relatively straightforward, although this too increases in complexity across the series in terms of both the context in which the problems are set and the mathematical content that is required. It will always be a challenge for some students to "unpack" the meaning from a worded problem, particularly as the problems' context, information, and meanings expand. This ability is fundamental to the nature of mathematical problem solving and must be built up with time and experiences rather than diminished or left out of problem situations. It is suggested that students work in groups so that they can help each other tackle the ideas in complex problems through discussion, rather than simply leaping into the first ideas that come to mind (leaving the full extent of the problem unrealized).

An Approach to Solving Problems



The careful, gradual development of an ability to analyze problems for meaning, organize the information to make it meaningful, and make connections among problems to suggest a way forward to a solution is fundamental to the approach taken with this series, from the first book to the last. At first, materials are used explicitly to aid these meanings and connections; however, in time they give way to diagrams, tables, and symbols as students' understanding of and experience with solving complex, engaging problems increases.

Not only is this model for the problem-solving process helpful in solving problems, but it also provides a basis for students to discuss their progress and solutions and determine whether or not they have fully answered a question. At the same time, it guides teachers' questions of students and provides a means of seeing underlying mathematical difficulties and ways in which problems can be adapted to suit particular needs and extensions. Above all, it provides a common framework for discussions between a teacher and group or whole class that focus on the problem-solving process rather than simply on the solution of particular problems. Indeed, as Alan Schoenfeld, in Steen, L. (Ed.) *Mathematics and Democracy* (2001), states so well, in problem solving:

Getting the answer is only the beginning rather than the end. … An ability to communicate thinking is equally important.

We wish all teachers and students who use these books success in fostering engagement with problem solving and building a greater capacity to come to terms with and solve mathematical problems at all levels.

George Booker and Denise Bond

INTRODUCTION

Problem Solving and Mathematical Thinking

> *By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.*
>
> **—NCTM, Principles and Standards for School Mathematics (2000, p. 52)**

Problem solving lies at the heart of mathematics. New mathematical concepts and processes have always grown out of problem situations, and students' problem-solving capabilities develop from the very beginning of mathematics learning. A need to solve a problem can motivate students to acquire new ways of thinking as well as come to terms with concepts and processes that might not have been adequately learned when first introduced. Even those who can calculate efficiently and accurately are ill-prepared for a world where new and adaptable ways of thinking are essential if they are unable to identify which information or processes are needed.

On the other hand, students who can analyze the meaning of problems, explore means to a solution, and carry out a plan to solve mathematical problems have acquired deeper and more useful knowledge than simply being able to complete calculations, name shapes, use formulas to make measurements, or determine measures of probability and data analysis. It is critical that mathematics teaching focuses on enabling all students to become both able and willing to engage with and solve mathematical problems.

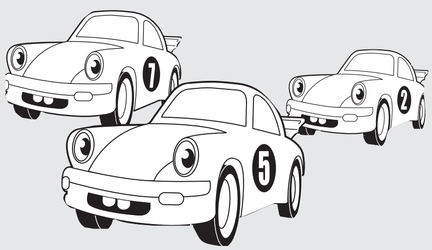
Well-chosen problems encourage deeper exploration of mathematical ideas, build persistence, and highlight the need to understand thinking strategies. They also reveal the central role of ***sense making*** in mathematical thinking—not only the need to assess the reasonableness of an answer or solution, but also the need to consider the interrelationships among the information provided in a problem situation. This may take the form of number sense, allowing numbers to be represented in various ways and operations to be interconnected; through spatial sense, allowing the visualization of a problem in both its parts and whole; to a sense of measurement across length, area, volume, and probability and data analysis.

Problem Solving

A problem is a task or situation for which there is no immediate or obvious solution, so ***problem solving*** refers to the processes used when engaging with this task. When problem solving, students engage with situations for which a solution strategy is not immediately obvious, thus drawing on their understanding of concepts and processes they have already met and often developing new understandings and ways of thinking as they move toward a solution. It follows that a task that is a problem for one student may not be a problem for another and that a situation that is a problem at one level will only be an exercise or routine application of a known means to a solution at a later time.

As the world in which we live becomes ever more complex, the level of mathematical thinking and problem solving needed in life and in the workplace has increased considerably.

To enable students to thrive in this changing world, attitudes that help them to deal with new or unfamiliar tasks are essential. Such attitudes must be developed from the beginning of mathematics learning as students form beliefs about meaning, the notion of taking control over the activities they engage with, and the results they obtain, and as they build an inclination to try different approaches. In other words, students must see mathematics as a way of thinking rather than a means of providing answers to be judged right or wrong by a teacher, textbook, or some other external authority.



In a car race, Jordan started in fourth place. During the race, he was passed by six cars. How many cars must he pass to win the race?

To solve this problem, it is not enough to simply use the numbers given. Rather, an analysis of the race situation is needed first to see that when Jordan started, there were 3 cars ahead of him. When another 6 cars passed him, there were now 9 ahead of him. If he is to win, he needs to pass all 9 cars. The 4 and 6 implied in the problem were not used at all! Rather, a diagram or the use of counters or other materials is needed first to interpret the situation and then to see how a solution can be obtained.

However, many students feel inadequate when they encounter problem-solving questions. They seem to have no idea of how to go about finding a solution and are unable to draw on the competencies they have learned in number, geometry, and measurement. Often these difficulties stem from underdeveloped concepts for the operations, spatial thinking, and measurement processes. Students may also have an underdeveloped capacity to read problems for meaning and a tendency to be led astray by the wording or numbers in a problem situation. Their approach may then simply be to try a series of guesses or calculations rather than consider using a diagram, materials, or other systematic approach to come to terms with what the problem is asking. It is this ability to analyze problems that is the key to problem solving, enabling decisions to be made about which mathematical processes to use, which information is needed, and which ways of proceeding are likely to lead to a solution.

Making Sense in Mathematics

Making sense of the mathematics being developed and used must be seen as the central concern of learning. Making sensible interpretations of any results and determining which of several possibilities is more or equally likely is critical in problem solving.

Number sense, which involves being able to work with numbers comfortably and competently, is important in many aspects of problem solving: in making judgments, interpreting information, and communicating ways of thinking. It is based on a full understanding of numeration concepts such as zero, place value, and the renaming of numbers in equivalent forms so that 207 can be seen as 20 tens and 7 ones as well as 2 hundreds and 7 ones (or that $\frac{5}{2}$, 2.5, and $2\frac{1}{2}$ are all names for the same fraction amount). Automatic, accurate access to basic facts also underpins number sense, not as an end in itself, but rather as a means of combining with numeration concepts to allow manageable mental strategies and fluent processes for larger numbers. Well-understood concepts for the operations are essential in allowing relationships within a problem to be revealed and taken into account when framing a solution.

Number sense requires:

- understanding relationships among numbers
- appreciating the relative size of numbers
- a capacity to calculate and estimate mentally
- fluent processes for larger numbers and adaptive use of calculators
- an inclination to use understanding of and facility with numeration and computation in flexible ways

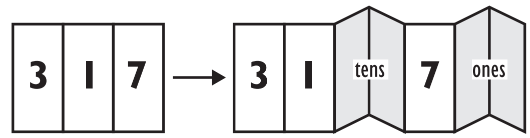
The following problem highlights the importance of these understandings.

There were 317 people at the New Year's Eve party on December 31. If each table could seat 5 couples, how many tables were needed?

Reading the problem carefully shows that each table seats five couples, or 10 people. At first glance, this problem might be solved using division; however, this would result in a decimal fraction, which is not useful in dealing with people seated at tables:

$10\overline{)317}$ is 31.7

In contrast, a full understanding of numbers allows 317 to be renamed as 31 tens and 7 ones:



This provides for all the people at the party, and analysis of the number 317 shows that there have to be at least 32 tables for everyone to have a seat and allow partygoers to move around and sit with others during the evening. Understanding how to *rename* a number has provided a direct solution without any need for computation. It highlights how coming to terms with a problem and integrating this with number sense provides a means of solving the problem more directly and allows an appreciation of what the solution might mean.

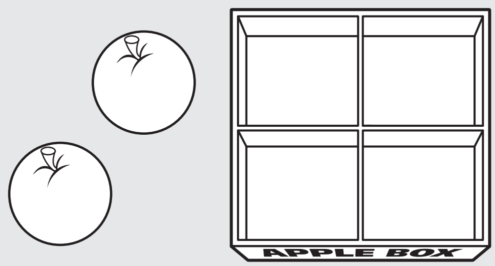
Spatial sense is equally important, since information is frequently presented in visual formats that must be interpreted and processed, while the use of diagrams is often essential in developing conceptual understanding across all aspects of mathematics. Using diagrams, placing information in tables, or enlisting a systematic way of dealing with the various possibilities in a problem help students to visualize what is happening.

Spatial sense involves:

- a capacity to visualize shapes and their properties
- determining relationships among shapes and their properties
- linking two-dimensional and three-dimensional representations
- presenting and interpreting information in tables and lists
- an inclination to use diagrams and models to visualize problem situations and applications in flexible ways

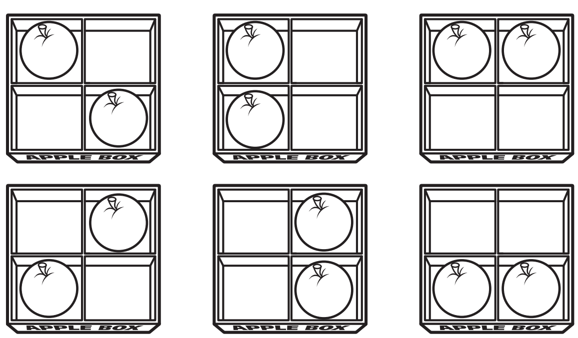
INTRODUCTION

The following problem shows how these understandings can be used.



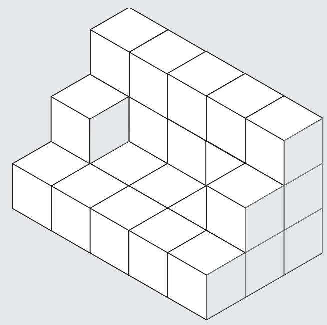
Cathy has 2 apples and 1 box. In how many different ways can she place the apples in the box?

Reading the problem carefully shows that only two spaces in the box can be used each time and that no use of the spaces can be duplicated. A systematic approach, placing one apple in a fixed position and varying the other spaces, will provide a solution; however, care will be needed to see that the same placement has not already occurred:



There are six possible arrangements. The placement of objects on the diagram has provided a solution, highlighting how coming to terms with a problem and integrating this with spatial sense allow a systematic analysis of all the possibilities.

Similar thinking is used with arrangements of two-dimensional and three-dimensional shapes and in visualizing how they can fit together or be taken apart.



How many cubes are needed to make this shape?

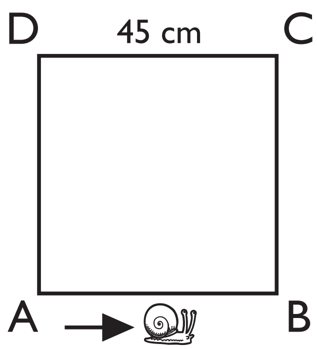
Measurement sense is dependent on both number sense and spatial sense, since attributes that are one-, two-, or three-dimensional are quantified to provide both exact and approximate measures and allow comparison. Many measurements use aspects of geometry (length, area, volume), while others use numbers on a scale (time, mass, temperature). Money can be viewed as a measure of value and uses numbers more directly, while practical activities such as map reading and determining angles require a sense of direction as well as gauging measurement. The coordination of the thinking for number and geometry, along with an understanding of how the metric system builds on place value, zero, and renaming, is critical in both building measurement understanding and using it to come to terms with and solve many practical problems.

Measurement sense involves:

- understanding how numeration and computation underpin measurement
- extending relationships from number understanding to the customary and metric systems
- appreciating the relative size of measurements

- a capacity to use calculators and mental or written processes for exact and approximate calculations
- an inclination to use understanding and facility with measurements in flexible ways

The following problem shows how these understandings can be used.



A snail crawls 3 m 15 cm around a square garden.

At which corner will it stop?

Carefully reading the problem shows that the snail will travel 45 cm as it moves along each side of the square. To come to terms with what is needed, 3 m 15 cm must be renamed as 315 cm. The distances the snail travels along each side can then be totaled until 315 cm is reached. It can also be inferred that the snail will travel along some sides more than once, since the distance around the outside of the square is 180 cm. At this point, the snail will be back at A. Traveling a further 45 cm will take it to B, a distance of 225 cm. At C it will have traveled 270 cm, and it will have traveled 315 cm (or 3 m 15 cm) when it reaches D for the second time.

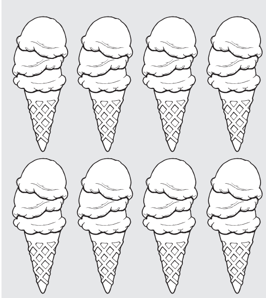
By using a diagram, an understanding of the problem situation has been integrated with a knowledge of meters and centimeters and a capacity to calculate mentally using addition and multiplication to provide an appropriate solution. Both spatial sense and number sense have been used to understand the problem and suggest a means to a solution.

Data sense is an outgrowth of measurement sense and refers to an understanding of the way number, spatial, and measurement sense work together to deal with situations where patterns must be discerned among data or when likely outcomes must be analyzed.

Data sense involves:

- understanding how numeration and computation underpin the analysis of data
- appreciating the relative likelihood of outcomes
- a capacity to use calculators or mental and written processes for exact and approximate calculations
- presenting and interpreting data in tables and graphs
- an inclination to use understanding and facility with number combinations and arrangements in flexible ways

The following problem shows how these understandings can be used.



You are allowed 3 scoops of ice cream: 1 chocolate, 1 vanilla, and 1 strawberry. How many different ways can the scoops be placed on a cone?

There are six possibilities for placing the scoops of ice cream on a cone. Systematically treating the possible placements one at a time highlights how the use of a diagram can account for all possible arrangements.

This problem also shows how *patterning* is another aspect of sense-making in mathematics. Often a problem calls for discerning a pattern in the placement of materials, the numbers involved in the situation, or the possible arrangements of data or outcomes to determine a likely solution. Being able to see patterns is also very helpful in getting an immediate solution or understanding whether or not a solution is complete. Allied to patterning are notions of symmetry, repetition, and extending ideas to more general cases. All of these aspects of mathematical sense-making are critical to developing the thinking on which problem solving depends, as well as solving problems per se.

As more experience in solving problems is gained, an ability to see patterns in what is occurring will help students see the relationship between a new problem and one that has been solved previously. It is this ability to relate problem types, even when the context appears to be quite different, that often distinguishes a good problem solver from one who is more hesitant.

Building a Problem-Solving Process

While the teaching of problem solving has often centered on the use of particular strategies that can apply to various classes of problems, many students are unable to access and use these strategies to solve problems outside of the teaching situations in which they are introduced. Rather than acquire a process for solving problems, they may attempt to memorize a set of procedures and view mathematics as a set of learned rules in which success follows the use of the right procedure to the numbers given in the problem. Any use of strategies may be based on familiarity, personal preference, or recent exposure rather than through a consideration of the problem to be solved. A student may even feel it is sufficient to have only one strategy and that the strategy should work all of the time—and if it doesn't, then the problem can't be solved.

In contrast, observation of successful problem solvers shows that their success depends more on an analysis of the problem itself—what is being asked, what information might be used, what answer might be likely, and so on—so that a particular approach is used only after the intent of the problem is determined. Using collaborative groups when problem solving, rather than tasks assigned individually, is an approach that helps students to develop this disposition.

Managing a Problem-Solving Program

Teaching problem solving differs from many other aspects of mathematics in that collaborative work can be more productive than individual work. Students who may be tempted to quickly give up when working on their own can be encouraged to see ways of proceeding when discussing a problem in a group, therefore building greater confidence in their capacity to solve problems and learning the value of persisting with a problem to tease out what is required. What is discussed with their peers is more likely to be recalled when other problems are met, while the observations made in the group increase the range of approaches that a student can access. Thus, time has to be allowed for discussion and exploration rather than insisting that students spend "time on task" as for routine activities.

Correct answers that fully solve a problem are always important, but developing a capacity to use an effective problem-solving process must be the highest priority. A student who has an answer should be encouraged to discuss his or her solution with others who believe they have a solution, rather than tell

his or her answer to another student or simply move on to another problem. In particular, explaining to others why he or she believes an answer is reasonable, as well as why it provides a solution, gets other students to focus on the entire problem-solving process rather than just quickly getting an answer.

These aspects of the teaching of problem solving should then be taken further as particular groups discuss their solutions with the whole class and all students are invited to participate in the discussion of the problem. In this way, problem solving as a way of thinking comes to the fore, rather than focusing on the answers as the main aim of mathematical activities.

Questions posed by the teacher must encourage students to explore possible means to a solution and try one or more of them, rather than point to a particular procedure. Questions can also help students see how to progress in their thinking rather than getting stuck in a loop where the same steps are repeated over and over. While posing too many questions that focus on the way to a solution may end up removing the problem-solving aspect from the question, posing too few may cause students to become frustrated with the task and think that it is beyond them. Students must experience the challenge of problem solving and gain pleasure from working through the process that leads to a full solution.

Taking time to listen to students as they try out their ideas, without comment or without directing them to a particular strategy, is also important. Listening provides a sense of how students' problem solving is developing, as assessing this aspect of mathematics can be difficult. After all, solving one problem will not necessarily lead to success on the next problem, nor will difficulty with a particular problem mean that the problems that follow will also be as challenging.

The teacher also may need to extend or adapt a given problem to ensure the problem-solving process is understood and can be used in other situations, instead of moving on to a different problem in another area of mathematics learning. This can help students to see how a way of thinking can be adapted to other, related problems. Having students engage in this process of problem posing is another way of both assessing them and bringing them to terms with the overall process of solving problems.

Using a Problem-Solving Model

The cyclical model *Analyze–Explore–Try* provides a very helpful means of organizing and discussing possible solutions. However, care must be taken that it is not seen simply as a procedure to be memorized and then applied in a routine manner to every new problem. Rather, it must be carefully developed over a range of different problems, highlighting the components that are developed with each new problem.

Analyze

- As students read a problem, the need to first read for the *meaning* of the problem can be stressed. This may require reading more than once and can be helped by asking students to state in their own words what the problem is asking them to do.
- Further reading will be needed to sort out which information is needed, whether some is not needed, if other information must be gathered from the problem's context (for example, data presented within the illustration or table accompanying the problem), and whether the students' mathematical understandings must be used to find other relationships among the information. As the form of the problems becomes more complex, this thinking will be extended to incorporate further ways of dealing with the information; for example, measurement units,

Expanding the Problem-Solving Process

- Put the solution back into the problem.
- Does the answer make sense?
- Does it solve the problem?
- Is it the only answer?
- Could there be another way?

- Read carefully.
- What is the problem asking?
- What is the meaning of the information? Is it all needed? Is there too little? Too much?
- Which operations will be needed and in what order?
- What sort of answer is likely?
- Have I seen a problem like this before?

- Use materials or a model.
- Use a calculator.
- Use pencil and paper.
- Look for a pattern.

- Use a diagram or materials.
- Work backwards or backtrack.
- Put the information into a table.
- Try and adjust.

fractions, and larger numbers might need to be renamed to the same mathematical form.

- Thinking about any processes that might be needed and the order in which they are used, as well as the type of answer that could occur, should also be developed in the context of new levels of problem structure.

- Developing a capacity to look beyond the problem's expression or context to identify similarities between it and other problems that already have been met is a critical way of building expertise in coming to terms with and solving problems.

Explore

- Some problems will require the use of materials (blocks, counters, and so on) to think through the whole of the problem's context. Others will demand the use of diagrams to show what is needed. Still others will require the systematic analysis of the problem situation using diagrams, lists, or tables. As these ways of thinking about the problem are understood, they can be included in the cycle of steps.

Try

- Many students often try to guess a result. This can even be encouraged by talking about "guess and check" as a means to solve problems. Changing to "try and adjust" is more helpful in building a way of thinking and can lead to a very powerful way of finding solutions.

- When materials, a diagram, or a table has been used, another means to a solution is to look for a pattern in the results. When these have revealed what is needed to try for a solution, it may also be reasonable to use pencil and paper or a calculator.

Analyze

- The point in the cycle at which an answer is assessed for reasonableness (for example, whether it provides a solution, is only one of several solutions, or there may be another way to solve the problem) also must be brought to the fore as different problems are met.

The Role of Calculators

When calculators are used, students devote less time to basic calculations, freeing up time that might be needed to either explore a solution or find an answer to a problem. In this way, attention is shifted from computation, which the calculator can do, to thinking about the problem and its solution—work that the calculator cannot do. It also allows a larger number of problems to be addressed in problem-solving sessions. In these situations, a calculator serves as a tool rather than a crutch, requiring students to think through the problem's solution in order to know how to use the calculator appropriately. Calculator use also underpins the need to make sense of the steps along the way and any answers that result, since keying in incorrect numbers, operations, or orders of operations quickly leads to results that are not appropriate.

Choosing, Adapting, and Extending Problems

In selecting problems for students to solve, it is recommended that teachers examine the problems to see if students already have an understanding of the underlying mathematics required and that the problem's expression can be meaningfully read by the group of students who will be attempting the solution—though not necessarily by *all* students in the group. The problem itself should be neither too easy nor too difficult. It should engage the interests of the students and also be able to be solved in more than one way.

As a problem and its solution is reviewed, posing similar problems—where the numbers, shapes, or measurements are changed—focuses attention back on what was approaches were used in analyzing the problem and in exploring the means to a solution. Extending these processes to more complex situations shows how a particular approach can be extended to other situations and how patterns can be analyzed to obtain more general methods or results. It also highlights the importance of a systematic approach when conceiving and discussing a solution and can lead to students approach asking themselves further questions about the situation and posing problems of their own as the significance of the problem's structure is uncovered.

Problem Structure and Expression

When analyzing a problem for appropriateness to a certain grade or ability level, it is possible to discern critical aspects of the problem's form and relate these to the appropriate level of mathematics and problem expression. A problem of first-level complexity uses simple mathematics and simple language. A second-level problem may incorporate simple language and more difficult mathematics or more difficult language and simple mathematics, while a third-level problem incorporates yet more difficult language and mathematics. Within a problem, the processes that must be used may be more or less obvious, the information that is required for a solution may be too much or too little, and strategic thinking may be needed in order to come to terms with what the problem is asking.

Level	processes obvious	processes less obvious	too much information	too little information	strategic thinking
increasing difficulty with problem's expression and mathematics required	simple expression, simple mathematics				
	more complex expression, simnple mathematics simple expression, more complex mathematics				
	complex expression, complex mathematics				

The Varying Levels of Problem Structure and Expression

(i) The processes to be used are relatively obvious, as these problems are comparatively straightforward and contain all the information necessary to find a solution.

(ii) The processes required are not immediately obvious, as these problems contain all the information necessary to find a solution but demand further analysis to sort out what is wanted, and students may need to reverse what initially seemed to be required.

(iii) The problem contains more information than is needed for a solution, as these problems contain not only all the information needed to find a solution but also additional information in the form of times, numbers, shapes, or measurements.

(iv) Further information must be gathered and applied to the problem to find a solution. These problems do not contain all the information necessary to find a solution but do contain a means of obtaining the required information. The problem's setting, the student's mathematical understanding, or the problem's wording must be searched for the additional material.

(v) Strategic thinking is required to analyze the question and determine a solution strategy. Deeper analysis, often aided by the use of diagrams or tables, is needed to come to terms with what the problem is asking so a means to a solution can be determined.

This analysis of the nature of problems can also serve as a means of evaluating the problems provided in a mathematics program. In particular, it can ensure that a full range of problems are provided across all problem forms, with the mathematics and expression suited to the level of the students.

Assessing Problem Solving

Assessment of problem solving requires careful and close observation of students working in a problem-solving setting. These observations can reveal the range of problem forms and the level of complexity in the expression and underlying mathematics that a student is able to confidently deal with. Further analysis of these observations can show to what extent the student is able to analyze the question, explore ways to a solution, select one or more methods to try, and then analyze any results obtained. It is the combination of two fundamental aspects—the types of problem that can be solved and the manner in which solutions are carried out—that will give a measure of a student's developing problem-solving abilities, rather than a one-off test in which some problems are solved and others are not.

Assessment informs:

What?
- *Problem form*
- *Problem expression*
- *Mathematics required*

How?
- *Analyze*
- *Explore*
- *Try*

Observations based on this analysis have led to a categorization of many of the possible difficulties that students experience with problem solving as a whole, rather than the misconceptions they may have about particular problems. These often involve inappropriate attempts at a solution based on little understanding of the problem.

A major cause of difficulties is the ***lack of a well-developed plan*** of attack, leading students to focus on the ***surface level*** of problems. In such cases, students:

- locate and manipulate numbers with little or no thought about their relevance to the problem
- try a succession of different operations if the first ones attempted do not yield a (likely) result
- focus on keywords for an indication of what might be done without considering their significance within the problem as a whole
- read problems quickly and cursorily to locate the numbers to be used
- use the first available word cue to suggest the operation that might be needed.

Problem	*Likely Causes*
Student is unable to make any attempt at a solution.	• not interested • feels overwhelmed • cannot think of how to start to answer question • needs to reconsider complexity of steps and information
Student has no means of linking the situation to the implicit mathematical meaning.	• needs to create diagram or use materials • needs to consider separate parts of question and then bring parts together
Students uses an inappropriate operation.	• misled by word cues or numbers • has underdeveloped concepts • uses rote procedures rather than real understanding
Student is unable to translate a problem into a more familiar process.	• cannot see interactions between operations • lack of understanding means he/she unable to reverse situations • data may need to be used in an order not evident in the problem statement or in an order contrary to that in which it is presented

Other difficulties result from a focus on being quick, which leads to:

- no attempt to assess the reasonableness of an answer
- little perseverance if an answer is not obtained using the first approach tried
- not being able to access strategies to which they have been introduced.

When the approaches to problem processing developed in this series are followed and the specific suggestions for solving particular problems or types of problems are discussed with students, these difficulties can be minimized, if not entirely avoided.

A Final Comment

If an approach to problem solving can be built up using the ideas developed in this series, students will develop a way of thinking about and with mathematics that will allow them to readily solve problems and generalize from what they already know to understand new mathematical ideas. They will engage with these emerging mathematical conceptions from their very beginnings, be prepared to debate and discuss their own ideas, and develop attitudes that will allow them to tackle new problems and topics. Mathematics can then be a subject that is readily engaged with and can become one in which the student feels in control, instead of one in which many rules devoid of meaning have to be memorized and applied. This early enthusiasm for learning and the ability to think mathematically will lead to a search for meaning in new situations and processes that will allow mathematical ideas to be used across a range of applications in school and everyday life.

MEETING THE NCTM STANDARDS

Numbers and Operations	
1.1 Understand numbers, ways of representing numbers, relationships among numbers, and number systems	pp. 20, 24, 28, 32, 36, 44, 52, 56, 86, 90, 92, 94, 100
1.2 Understand meanings of operations and how they relate to one another	pp. 24, 32, 26, 40, 44, 48, 52, 56, 86, 100
1.3 Compute fluently and make reasonable estimates	pp. 24, 32, 36, 40, 48, 52, 56, 100

Algebra	
2.1 Understand patterns, relations, and functions	pp. 20, 60, 64, 94, 100
2.2 Represent and analyze mathematical situations and structures using algebraic symbols	
2.3 Use mathematical models to represent and understand quantitative relationships	
2.4 Analyze change in various contexts	

Geometry	
3.1 Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	pp. 68, 72, 78
3.2 Specify locations and describe spatial relationships using coordinate geometry and other representational systems	pp. 82, 86
3.3 Apply transformations and use symmetry to analyze mathematical situations	pp. 68, 72, 78
3.4 Use visualization, spatial reasoning, and geometric modeling to solve problems	pp. 64, 68, 72, 78, 86

Data Analysis and Probability	
4.1 Formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them	pp. 90, 92, 100
4.2 Select and use appropriate statistical methods to analyze data	pp. 104
4.3 Develop and evaluate inferences and predictions that are based on data	pp. 96, 100, 104
4.4 Understand and apply basic concepts of probability	

Problem Solving, Reasoning and Proof, Communications, Connections, Representation	all activities

Problem Solving

To investigate patterns and order and make predictions based on these

Materials

- Colored blocks (red, yellow, blue)
- Crayons or colored pencils

NCTM Standards

- Number and Operations 1.1
- Algebra 2.1
- Problem Solving

Focus

These pages introduce ordinal numbers as an aspect of forming and describing patterns. Students are required to manipulate items (both real and drawn) to fit particular criteria and determined patterns, including how they relate to ordinal place.

Discussion

Page 21 – Patterns

Some students may have to put additional blocks out and count them to find their solutions, while others may be able to predict the block's color from the pattern they can see. Similarly, some students may be able to think of a pattern in their mind for the second problem and then arrange the blocks accordingly.

Page 22 – Ordinal Patterns

These activities require students to work backwards or "backtrack"to form different patterns. Ask students to check the blocks other classmates have arranged. Students have to see if the arrangements do show a pattern and then come up with a way of describing the pattern.

Page 23 – Car Race

The car race problem provides a very engaging way of consolidating the use of patterns and the manner in which ordinal numbers are used to describe them. If students have difficulty organizing the data, have them use color cubes to represent the cars and then transfer the results to the drawn cars. They may also need to do this first when planning their own race.

Possible Difficulties

- Moving blocks around indiscriminately
- Focusing on the particular positions—for example, forming a line where the fourth block is red and the seventh block is blue, but without a pattern in between—blocks are just in a line and no pattern is evident
- Difficulty in repeating a consistent pattern

Extension

- Have students make patterns in which three blocks are specified.
- In pairs, have one student decide the criterion for a pattern, while the other student makes the pattern.
- In pairs, play a game where one student forms a pattern behind a barrier that stops the other from seeing the pattern. The first student then describes the pattern, using colors and ordinal numbers, to the other student. The second student forms this pattern, as he/she understands it to be, on his/her side of the barrier. When the second pattern has been completed, the barrier is removed and the two patterns are compared.
- Have students use a table to find the results of their car races.

1.1 PATTERNS

Take 3 blue, 3 red, and 3 yellow blocks. Line them up like this:

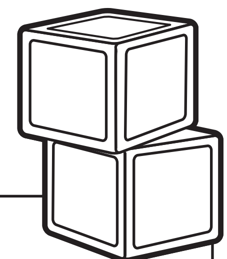
blue, red, yellow, blue, red, yellow

Continue the pattern.

What color would the 10th block be? ________________

What color would the 14th block be? ________________

Make a pattern that starts with 2 blue blocks.



Draw your pattern.

What color is the 8th block? ________________

What color would the 16th block be? ________________

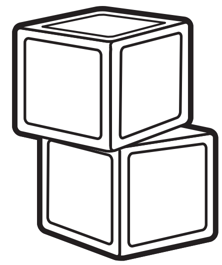
What about the 24th block? ________________

1.2 ORDINAL PATTERNS

Make a pattern where the 6th block is green.

Make and draw other patterns where the 6th block is green.

How many patterns did you make? ____________



Make a pattern where the 9th block is red.

Make five patterns where the 9th block is red.

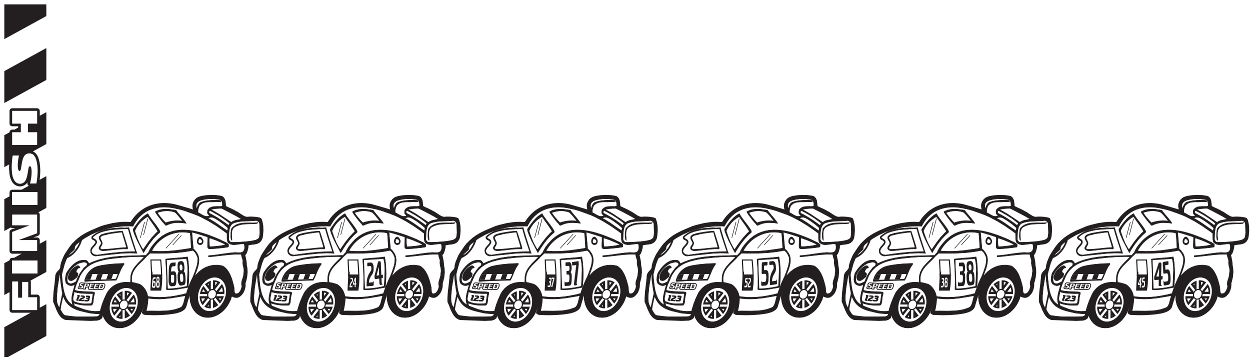
Make a pattern where the 8th and 9th blocks are red.

Make a pattern where the 4th block is red and the 7th block is green.

Make and draw other patterns where the 4th block is red and the 7th block is green.

1.3 CAR RACE

Use the clues to color the cars.



The yellow car is placed second.

The 6th car is black.

The red car is in front of the yellow car.

The orange car is between the black car and the blue car.

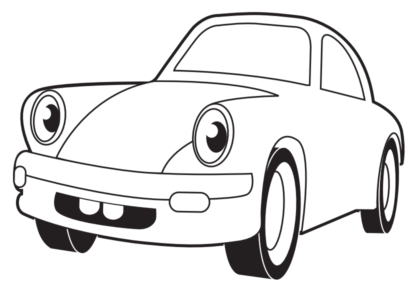
The green car is behind the yellow car.

Use the drawing to fill in which place each car is positioned.

1st ____________ 4th ____________

2nd ____________ 5th ____________

3rd ____________ 6th ____________



Draw your own car race and write clues to match.

Problem Solving

To read, interpret, and analyze information

Materials

- Counters, if needed

NCTM Standards

- Number and Operations 1.1, 1.2, 1.3
- Problem Solving

Focus

These pages explore relationships among numbers and then use this analysis to find a number that matches specific criteria. This process encourages students to disregard numbers that are not possible rather than simply looking for ones that are likely to work. Some students will use the given information to discard numbers until only the correct number remains, while other students may prefer to try each number in turn against all of the criteria until they find a number that answers all conditions.

Discussion

Page 25 – Guess My Number

The criteria listed allow numbers to not only be selected but also to be ruled out; for example, "more than 68" rules out 20, 39, and 53. Having "3 in the ones place" means the number must be 73. Some students may work down the list of conditions, while others may read all of the conditions and then decide where to start.

Page 26 – What's My Number?

Some students will use the listed information to discard numbers until only the correct number remains, while other students may prefer to try each number in turn against all of the criteria until they find a number that answers all conditions. The criteria listed allow numbers to be selected as well as ruled out; for example, "less than 76 tens" rules out 763, 776, and 795. Some students may work down the list of conditions, while others may read all the conditions and then decide where to start. This concept has been extended by having students think of their own number and write specific criteria for others to identify it.

Page 27 – What's My Age?

A number of mathematical terms must be taken into consideration in analyzing these problems—for example, *twice, half, in three years' time.* Some problems require students to start with the age given at the end of the problem and then backtrack to find a solution. If needed, counters can be used to help.

Possible Difficulties

- Selecting a number that matches only the first criterion
- Not matching the number against all criteria
- Not considering terms such as *twice* and *half*

Extension

- Students think of a number and make up criteria to match it, then give it to other students to solve.
- A one-digit number could be tried first, then a two-digit number.
- Conditions used can involve terms such as *between*, *more than*, *less than,* and *place value*.
- Students can also be encouraged to come up with mathematical criteria of their own.

2.1 GUESS MY NUMBER

62	73	20	53	78	39

My number:

- *is between 65 and 95*
- *is less than 75*
- *has a 3 in the ones place*
- *is more than 68*

1. My number is ______________.

76	34	84	63	39	25

My number:

- *is between 21 and 72*
- *is more than 31*
- *has a 4 in the ones place*
- *is less than 35*

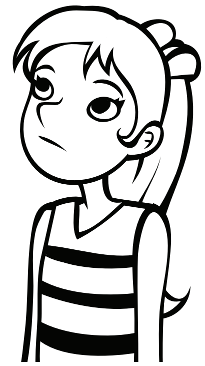
2. My number is ______________.

2.2 WHAT'S MY NUMBER?

My number ...

763	776	729	757	739	795

- is between 69 tens and 79 tens
- is more than 73 tens
- has 7 in the hundreds place
- does not have 3 in the ones place
- is less than 76 tens
- has a digit larger than 8 in the ones place



1. My number is __________.

My number ...

356	563	653	536	635	365

- is between 30 tens and 60 tens
- is less than 57 tens
- uses the digits 3, 5, 6
- does not have 3 in the hundreds place
- has a digit larger than 5 in the ones place

2. My number is __________.

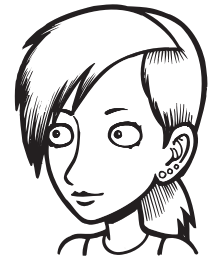
3. Make up your own number puzzle and give it to a friend to solve.

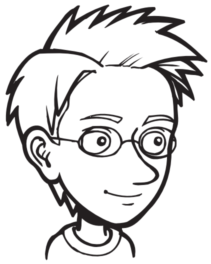
My number ...

2.3 WHAT'S MY AGE?

1. My sister is twice my age. She will be 15 next year.

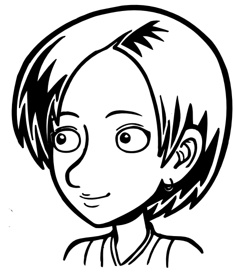
   How old am I? ___________





2. I am five years younger than my brother. I will be 12 next year.

   How old is he? ___________



3. My cousin is three times my age. She was 14 last year.

   How old am I? ___________

4. My older brother will be 16 in four years' time. I am half his age.

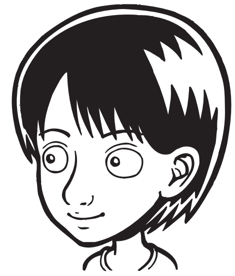
   How old am I now? ___________

5. My cousin is twice my age. She was 11 last year.

   How old am I? ___________

6. I will be nine in three years' time. My sister is twice my age.

   How old is she? ___________



7. My little sister is six. I will be twice her age in three years' time.

   How old am I now? ___________

Problem Solving

To interpret and organize information in a series of interrelated problem statements

Materials

- Crayons or colored pencils

NCTM Standards

- Number and Operations 1.1
- Problem Solving

Focus

These pages explore ways to arrange given numbers in problem situations so that the resulting stories make sense. Students must read the stories carefully to work out which number goes where. The numbers are not listed in the order in which they are used in the story. To extend this concept, some questions require students to think of their own numbers to fit problem situations.

Discussion

Page 29 –Missing Numbers
The first problem must be read and interpreted so that the students use the numbers provided to produce a story that makes sense. The second problem uses the same story structure but asks students to think of their own numbers. The last problem extends this to a different situation, one in which students have considerable leeway in choosing numbers that would fit the story. An understanding of the story context is needed, however; for example, to have 100 students in the class would not be likely, while 35 might be more reasonable.

Page 30 – Make Your Own Story
Here the idea of students choosing their own numbers to pose problems is extended to a more complex situation for which a table is provided to organize and analyze information.

Page 31 – More Missing Numbers
These problems build on the thinking used in the preceding pages. They require students to read and interpret the problem situations and then arrange the numbers that are provided so that the resulting stories make sense. The numbers are not listed in the order in which they are used in the stories. The last problem asks students to write their own missing number stories and give them to another student to solve.

The various stories students write should be discussed and contrasted. The leading concept that should be understood by students is that a range of numbers can be used in each sentence (there is no one right answer).

Possible Difficulties

- Placing the numbers in a story in the order in which they are listed rather than considering if they make sense
- Not taking into account the context of the story when selecting numbers
- Students having difficulty when trying to think of their own numbers for a story to make sense

Extension

- Students should be encouraged to write missing number stories of their own and swap them with others in the class to solve.

3.1 MISSING NUMBERS

Some numbers have been left out of the story. Put in the numbers given so that the story makes sense.

7	**12**	**3**

1. Mandy's school is __________ miles from her house.

   Her class has lunch at __________ o'clock.

   She is __________ years old.

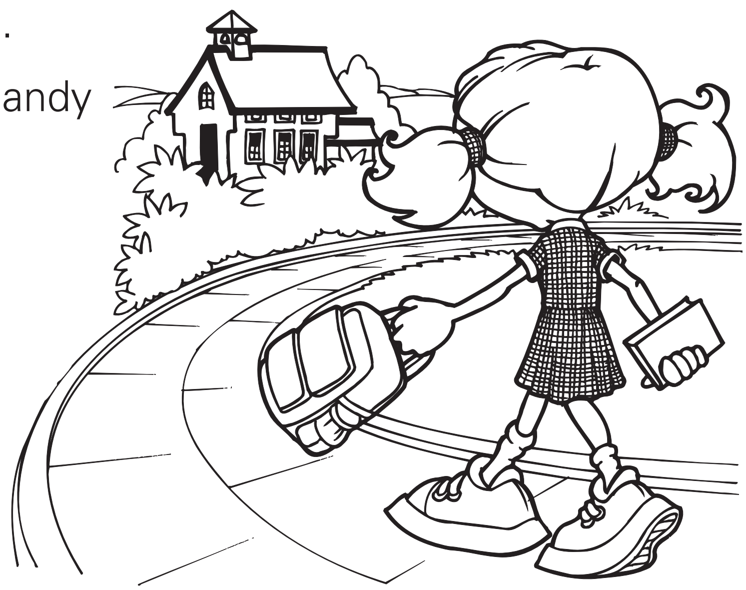
Now use your own numbers so that the story makes sense.

2. Mandy's school is __________ miles from her house.

   Her class has lunch at __________ o'clock.

   She is __________ years old.

   What is the youngest age Mandy could be? __________



Choose your own numbers so that this story makes sense.

3. Haani's mother works __________ miles from their house.

   She works as a teacher and has __________ students in her class.

   Her students go home at __________ o'clock.

3.2 MAKE YOUR OWN STORY

Make up your own numbers to use in the story. Draw a picture of your story when you have finished.

1. Linda and Lisa are sisters.

   Linda is __________ years old.

   She is in grade __________ at school.

   Lisa is her older sister. Lisa is __________ years old.

   Lisa is in grade __________ at school.

   Linda and Lisa have a brother, Lenny.

   Lenny is younger than Lisa but older than Linda.

   Lenny is __________ years old.

2. Record how old each person is in the table below.

Linda	Lisa	Lenny

3. Draw a picture of your story.

3.3 MORE MISSING NUMBERS

Some numbers have been left out of the stories below. Put in the numbers so the stories make sense.

4	43	8	5	10

1. Jake's mother drives to work. She leaves at __________ o'clock in the morning. She stops work at __________ o'clock and collects Jake from after-school care. They arrive home at about __________ o'clock. She is __________ years old. Jake is __________ years old.

6	7	9	12	1

Joseph, Jacob, and Jonathan are brothers. Joseph is older than Jonathan. Jonathan is older than Jacob. Jonathan is in 4th grade at school.

2. Record the age and grade level of each boy.

	Joseph	Jacob	Jonathan
Age			
Grade level			

3. Write your own "missing numbers" story and give it to a friend to solve.

____

____

____

____

____

Problem Solving

To identify and use information in a problem

NCTM Standards

- Number and Operations 1.1, 1.2, 1.3
- Problem Solving

Focus

These pages explore the reading and interpreting of information to solve problems involving numeration (no addition or subtraction is needed to solve the first two pages). Students analyze the problem to locate the required information, decide which information is not needed, and then use comparison rather than addition or subtraction to obtain solutions.

Discussion

Page 33 – How Many? 1
Anne skipped 10 more times than Maree, who skipped 42 times. In the next problem, students must read what happened on Saturday and the next day to infer that the next day was Sunday. The last problem is about marbles, with the additional information (about stickers) not required. Although Aaron has the largest number, these are stickers and not marbles. Jane has the largest number of marbles.

Page 34 – Trading Cards
In the first problem, Julie has no (or zero) trading cards, so technically she has the smallest number of cards. This can lead to a discussion about whether the problem means the smallest actual number or should include only the people who actually have cards. The next problem introduces the idea of organizing information in a table to help find a solution. The question does not specify how many cards John lost or how many Carla won. The information in the table will show that John has the least number of cards and Carla the most. As John loses some cards, he still has the least number, while Carla winning some means she still has the most.

Page 35 – Magic Squares 1
This page introduces the concept of magic squares. Simple three-by-three magic squares have been used to enable students to come to terms with the idea that each row, column, and diagonal adds to the magic number.

Possible Difficulties

Students are unable to interpret the information in the problems, including:

- thinking that Maree skipped 42 times and Anna skipped only 10 times
- not being able to figure out that the next day is Sunday
- not realizing that having no cards means 0 (zero)
- not seeing that having the least number of cards and then losing more will always mean you will have the least cards, while having the most cards and winning more will mean you always have the most

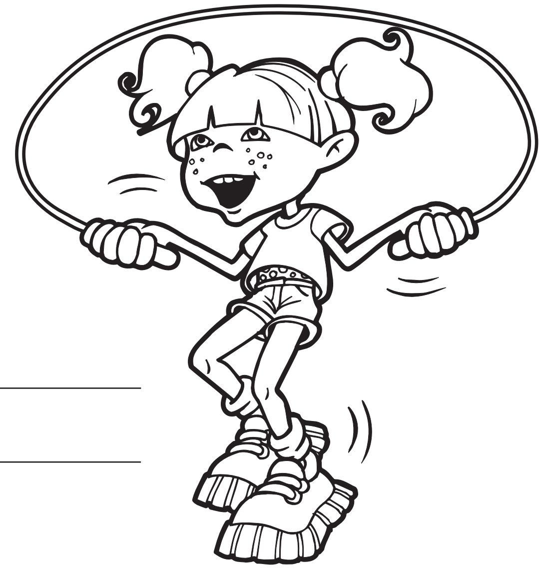
Students have an inadequate understanding of numbers:

- Not understanding that zero, as a number, shows none of something
- Counting or adding to find 10 more rather than using place value
- Using comparison to determine least and greatest tens place value is not developed

Extension

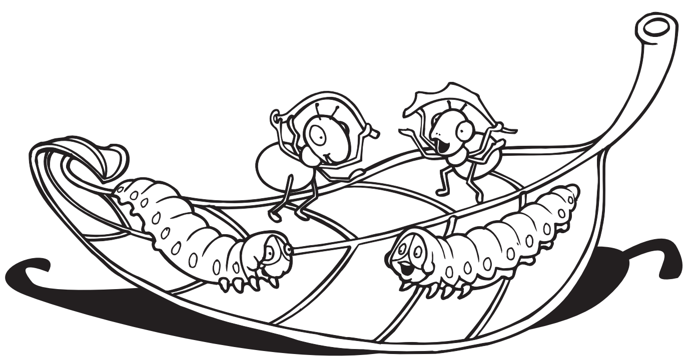
- Use a table to record information for the other problems and discuss how this helps in solving them.
- Investigate other magic squares.

4.1 HOW MANY? 1



**Maree and Anne were skipping.
Maree skipped 42 times.
Anne skipped 10 more times.**

1. Who skipped the most? ____________________
2. How many times did Maree skip? __________



On Saturday, Lee counted 46 ants and 23 caterpillars. The next day she counted 58 ants and 25 caterpillars.

3. How many ants did Lee count on Sunday?

   ____

4. On which day did she count the most caterpillars?

   ____

**Jane had 32 marbles.
Aaron had 48 stickers.
Scott had 29 marbles.**

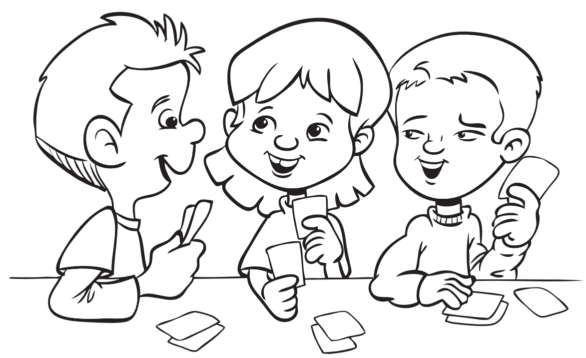
5. Who had the most marbles? ____________________

4.2 TRADING CARDS

Kerry has 42 trading cards.
Julie has no trading cards.
Valda has 35 trading cards.

1. Who has the smallest number of cards? ______________________

Mark has 53 trading cards.
John has 10 less than Mark.
Carla has 72 trading cards.



2. Use the table to record how many cards each person has.

Mark	John	Carla

The next day, John lost some cards and Carla won some cards.

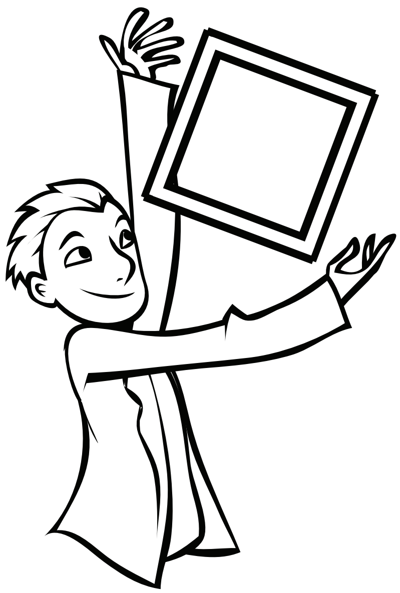
3. Who has the most cards? ______________________

4. Who has the least cards? ______________________

4.3 MAGIC SQUARES 1

In a magic square, every row, column, and diagonal adds up to the same number.

2	7	6
9	5	1
4	3	8



In this magic square, all the rows, columns, and diagonals add up to **15**. Its magic number is **15**.

Look at the magic squares below. Remember, each row, column, and diagonal must to add up to the same number.

1. Figure out what the magic number of each square is.
2. Fill in the missing numbers in each square.

6		8
7	5	3
	9	

(a) Magic number: __________

	1	6
3		7
		2

(b) Magic number: __________

Problem Solving

To analyze and use information in addition problems

Materials

- Counters or blocks

NCTM Standards

- Number and Operations 1.1, 1.2, 1.3
- Problem Solving

Focus

These pages explore word problems that require addition. Students must determine what the problem is asking in order to find a solution. Analysis of the problem reveals that different items may need to be added, which is more complex that just adding two or more of the same together. Other problems contain information that will not be needed.

Counters or blocks can be used to assist, since these problems are about reading for information and determining what the problem is asking, rather than just solving addition or basic fact problems.

Discussion

Page 37 – In the Garden

The stories must be read first to see what information is needed. In the first three problems the word *altogether* explicitly suggests addition. In the last problem, an understanding of the context is required to see that addition of the two numbers is needed.

Page 38 – How Many? 2

Some information (ordinal numbers, days of the week) is not needed to find a solution, while addition of several numbers is required. For the last problem, an understanding of the context shows that the numbers are added (despite the use of "lost").

Page 39 – How Many? 3

Careful reading of the information in the stories is needed to sort out which numbers must be added. In the first problem, all the ducks must be added despite their different locations. In the other problems, analyzing the question shows which information is to be used and which is to be discarded.

Possible Difficulties

- Unable to identify that addition must be used to find a solution
- Confusion about the use of ordinal numbers and which numbers must be added
- Adding all the numbers in a problem rather than identifying the additional information that is not needed
- Unable to add readily and accurately

Extension

- Have students write problems for other students to solve. Use the problems on these pages as a model.

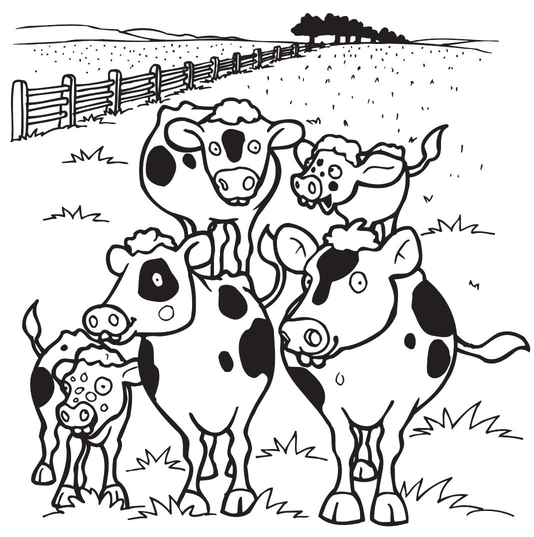
5.1 IN THE GARDEN

1. In the garden, a caterpillar ate 23 leaves in the morning and 6 leaves in the afternoon. How many leaves did it eat altogether?

____

2. There are 16 kookaburras in the trees. 8 more fly in to join them. How many kookaburras are there altogether?

____



3. The farmer has 26 cows in one paddock and 39 cows in another paddock. How many cows are there altogether?

____

4. Daniel picked 49 mangoes from one tree and 37 mangoes from another tree. How many mangoes does he have?

____

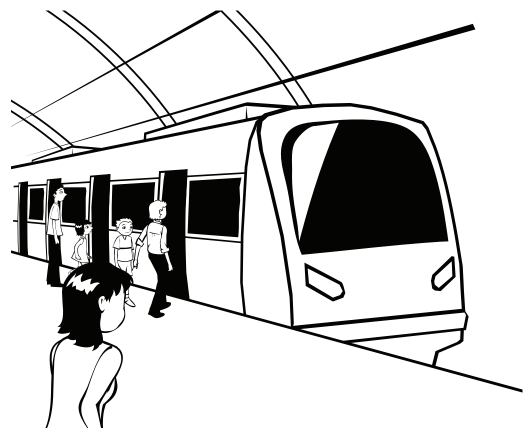
5.2 HOW MANY? 2

1. Amy went to the library. While there, she looked at 2 books on the first shelf, 4 books on the second shelf, and 1 book on the third shelf. How many books did she look at?

____

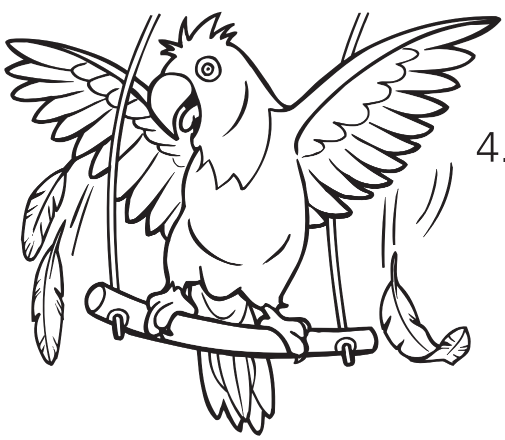
2. George went shopping for clothes. On the first floor he bought 5 shirts, on the second floor he bought 3 shirts, on the third floor he bought no shirts, and on the fourth floor he bought 2 shirts. How many shirts did he buy?

____



3. At the train station 19 people got into the first car, 17 people got into the second car, and 12 people got into the third car. How many people got on the train?

____



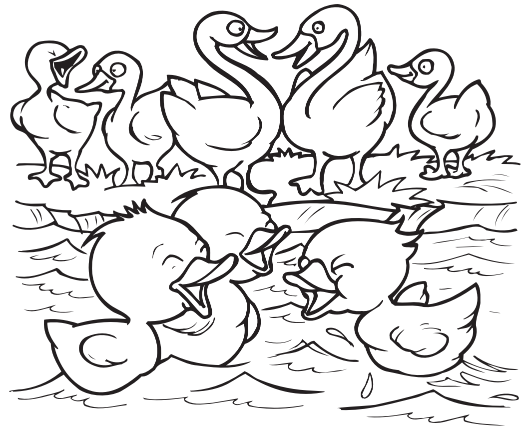
4. Claire's pet parrot has red, blue, and green feathers. He lost 7 red feathers on Monday, 2 blue feathers on Tuesday, and 5 red feathers on Wednesday. How many feathers did he lose?

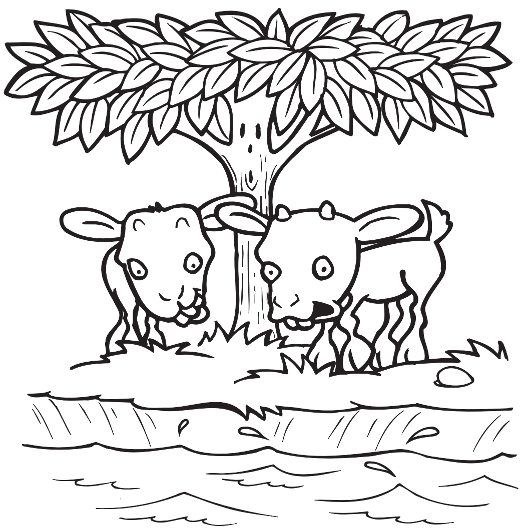
____

5.3 HOW MANY? 3

1. There were 23 ducks on the lake and 8 swans on the shore. 9 more ducks came to land on the lake. How many ducks were there altogether?

____





2. There are 16 black goats by the river and 13 white goats eating grass. Another 7 black goats are under the trees. How many black goats are there?

____

3. Matthew gave 6 guinea pigs to the first-grade class. He also gave 5 chicks and 8 guinea pigs to the second-grade class. How many guinea pigs did he give away?

____

4. There are 8 kangaroos sleeping under the gum trees and 5 koalas sleeping in the gum trees. Another 15 kangaroos are drinking at the water hole. How many kangaroos are there altogether?

____

Problem Solving

To identify and use information in subtraction problems.

Materials

- Place value chart or calculator, if needed

NCTM Standards

- Number and Operations 1.2, 1.3
- Problem Solving

Focus

These pages explore word problems that require subtraction. Analysis of the problems reveals that numbers must be identified and then subtracted correctly. Some problems may contain information that will not be needed or involve more than one subtraction to provide a solution. Problems involving subtraction as a comparison include the word *more*, which may lead to the problem being interpreted as addition.

Discussion

Page 41 – Word Problems 1

The stories must be read carefully to see what information is needed. In the first three problems, the use of the word *left* explicitly suggests subtraction. The fourth problem requires further interpretation of the language used to see what is needed, while the last problem involves a comparison of two different animals.

Page 42 – Word Problems 2

Not all of the information is needed to find a solution. For example, in the last problem an understanding of the context shows that it is only the number of bikes that must be considered, not their color.

There are different ways this problem can be solved. The number of bikes the shop had and the number sold can be totaled and then the subtraction carried out—the bikes sold are 10, leading to a simple addition and subtraction. Alternatively, the number of red bikes sold and the number of black bikes sold could be found by subtraction and the remaining amounts totaled.

Page 43 – Roland's Roses/Angela's Plants

The first problem involves a larger amount of information, with a number of questions arising from it. A careful reading of the information is needed to sort out which numbers must be subtracted. In each case, the initial number is known, and the amounts can be subtracted in turn to obtain the answers.

When the first question states that three roses are picked each day, the focus of the problem changes, since the question is asking how many times this can happen.

The second problem asks how many different ways plants can be arranged in groups and also involves subtraction, since it is asking how many ways items can be subtracted from 8.

Possible Difficulties

- Unable to identify the need to subtract to find a solution
- Adding rather than subtracting, or simply stating one of the written numbers as the result
- Confusion about which numbers must be subtracted
- Unable to subtract readily and accurately

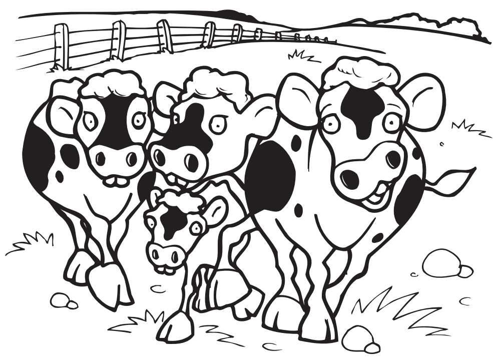
Extension

- Ask students to write problems for other students to complete, using the problems on the worksheets as a model.

6.1 WORD PROBLEMS 1

1. There were 16 jelly beans in the jar. During the day, 7 were eaten. How many jelly beans are left?

   ____

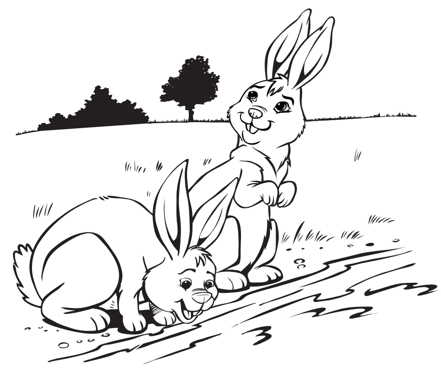


2. The farmer has 27 cows in a paddock. He moves 11 cows to another paddock. How many cows does he have left in the first paddock?

   ____

3. There are 27 rabbits drinking at the stream. 8 rabbits hop away. How many rabbits are left drinking at the stream?

   ____

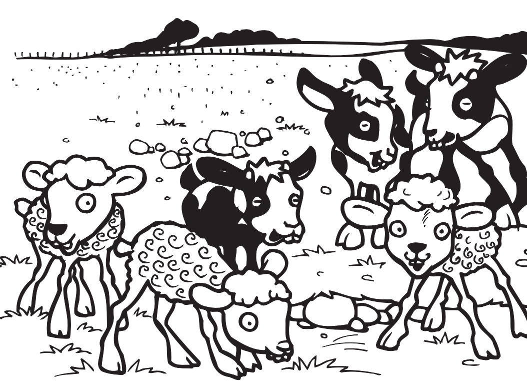


4. The last car on the train had 13 people. At the station, 7 people got off. How many people were still on the train?

   ____

5. The farmer has 11 sheep and 17 goats. How many more goats than sheep does the farmer have?

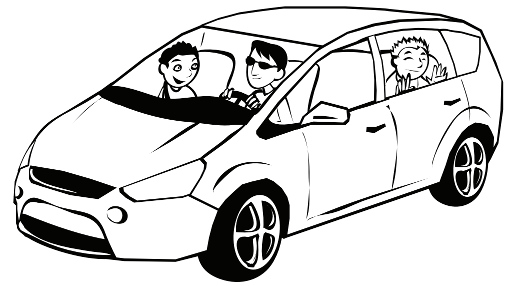
   ____



6.2 WORD PROBLEMS 2

1. The minivan has enough seats for 16 children and 1 driver. There are 9 children on the van. How many more children can fit in the van?

   ____

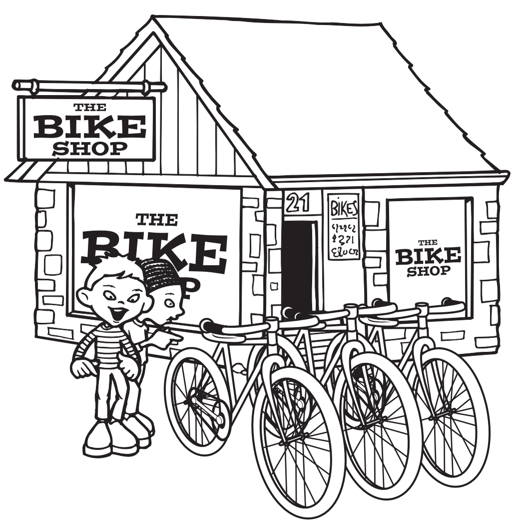


2. Craig has 24 stickers, Alex has 37 stickers, and Henry has 48 stickers. How many more stickers does Henry have than Craig?

   ____

3. Julie caught 31 fish and 4 crabs. She threw 14 small fish and 2 small crabs back into the water. How many fish did she keep?

   ____



4. The bicycle shop has 12 red bikes and 16 black bikes. It sells 3 red bikes and 7 black bikes. How many bikes are left?

   ____

6.3 ROLAND'S ROSES

The rosebush has 13 roses on it. Roland picked 2 roses one day, 4 roses the next day, and 3 roses on the third day.

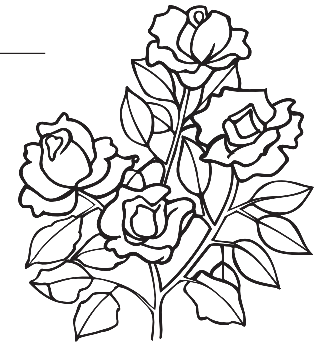
1. How many roses are left on the bush? ___________

   If he picks 2 roses each day for 3 days, how many are left on the bush? ___________

   If he picks 3 roses each day, for how many days can he pick roses? ___________

   If he picks a bunch of 6 roses, how many are left? ___________

   If he picks 2 bunches of 6 roses, how many are left? ___________



6.4 ANGELA'S PLANTS

Angela has 8 potted plants.

2. If 2 plants are kept outside, how many are kept inside?

   ____

3. List the different ways Angela can arrange her plants inside and outside.

Inside	Outside

Problem Solving

To analyze and use information in addition and subtraction problems

Materials

- Counters, blocks, or calculator, if needed

NCTM Standards

- Number 1.1, 1.2
- Problem Solving

Focus

These pages explore word problems that require addition and subtraction. Students must determine what the problem is asking and, in many cases, carry out more than one step to find a solution. Analysis of the problems reveals that different items may need to be added, while other problems contain additional information that is not needed.

Materials can be used to assist with the calculation if necessary, since these problems are about reading for information and determining what the problem is asking, rather than computation or basic facts.

Discussion

Page 45 – At the Fruit Store

Careful reading of each problem is needed to determine what the question is asking. In some cases, more information is provided than is needed, and some problems contain amounts that are not required to find the solution. Some problems require more than one step, and both addition and subtraction are needed at times. There are a number of ways to find a solution, and students should be encouraged to explore and try different possibilities of arriving at a solution.

Page 46 – Koala Corner

On this page, students are told that some problems involve addition and some involve subtraction and that only the subtraction problems must be solved. This requires a careful analysis of each story to determine what the problem is asking and whether addition or subtraction is needed to find a solution.

This activity shifts the focus from trying to find an answer to understanding the importance of determining what a problem is asking.

Page 47 – Bird Aviary

This investigation involves information about an aviary and a number of interrelated questions arising from it. The situation begins with a number of birds either flying around, in nests, or on the ground.

Using this information as base, students are required to keep track of new information and use it to answer the subsequent questions. The number of birds change to meet new criteria, with some birds landing and some birds flying.

Possible Difficulties

- Inability to identify when to add and when to subtract
- Confusion over the need to carry out more than one step to arrive at a solution
- Using the total amount given rather than just the numbers needed

Extension

- Students could write their own problems and give them to other students to solve.

7.1 AT THE FRUIT STORE

1. The fruit shop sold 46 boxes of strawberries on Monday, 62 boxes on Tuesday, and 73 boxes on Wednesday. How many boxes were sold altogether?

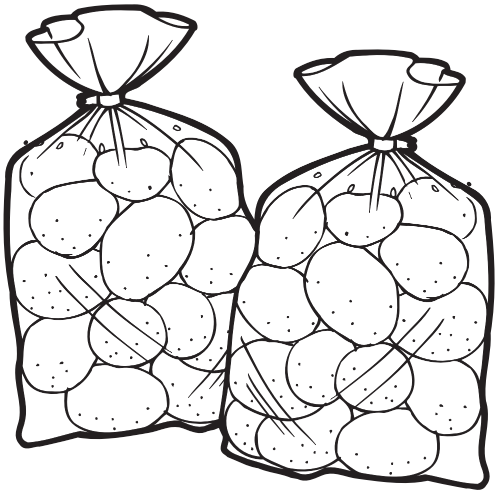
____

2. On Friday, 17 bags of oranges and 9 bags of lemons were sold. On Saturday, 19 bags of oranges and 6 bags of lemons were sold. How many bags of oranges were sold?

____

3. The fruit shop has 24 trays of mangoes. On Saturday, 6 trays were sold and, on Sunday, 8 trays were sold. How many trays were not sold?

____



4. On Tuesday, 24 bags of potatoes were sold. On Wednesday, 6 bags of onions and 19 bags of potatoes were sold. On Thursday, 12 bags of potatoes were sold. How many bags of potatoes were sold?

____

7.2 KOALA CORNER

Read the problems. Some involve addition and some involve subtraction. Solve ONLY the subtraction problems.

1. There are 27 koalas in a tree. 16 koalas are sleeping. How many koalas are not sleeping?

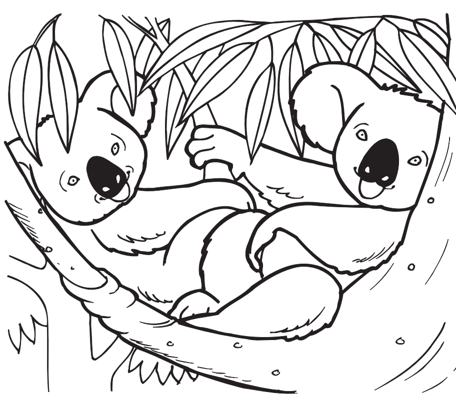
   ____

2. 18 koalas are eating gumleaves. 4 more koalas start eating leaves. How many koalas are eating gumleaves?

   ____

3. 35 koalas are sleeping in the tree. 9 koalas wake up. How many koalas are still sleeping?

   ____



4. 37 adult koalas are in the tree. 8 koalas have a baby on their back. How many koalas do not have a baby on their back?

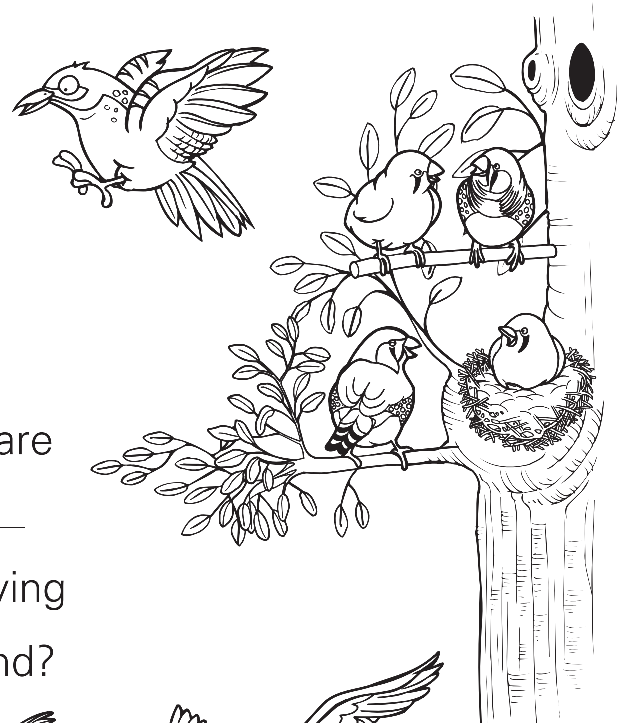
   ____

5. 26 koalas are sleeping in the tree. 9 more koalas go to sleep. How many koalas are sleeping?

   ____

7.3 BIRD AVIARY

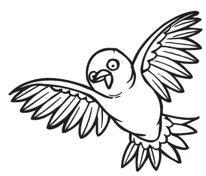
In the aviary, 43 birds are flying around, 19 birds are on the ground, and 13 birds are in their nests.

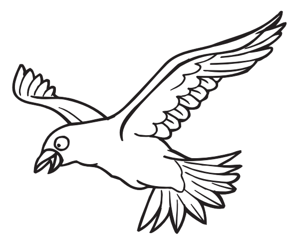


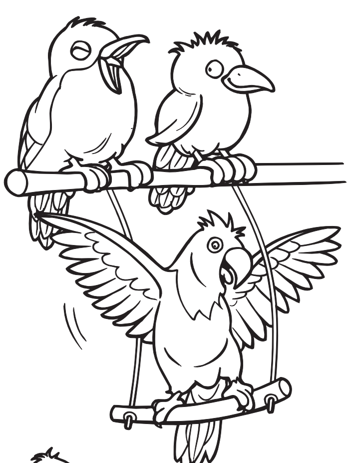
1. Altogether, how many birds are in the aviary? ______________

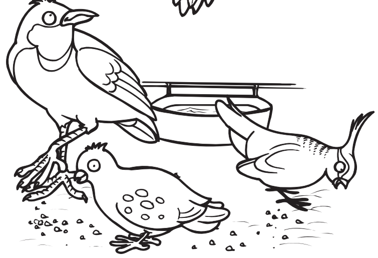
2. How many more birds are flying around than are on the ground?

____









3. If 9 of the birds flying around land on the ground, how many are now flying around?

   ____

4. How many are now on the ground?

   ____

5. All of the birds in their nests leave them to land on the ground to feed. How many birds are on the ground?

   ____

Problem Solving

To analyze and use information in multi-step word problems

Materials

- Place value chart, calculator

NCTM Standards

- Number and Operations 1.2, 1.3
- Problem Solving

Focus

These pages explore word problems that require addition and subtraction. Students must determine what the problem is asking and, in many cases, carry out more than one step to find the solution. Analysis of the problems reveals that unlike items may need to be added, which is more complex that just adding two or more like items together, while other problems contain additional information that is not needed.

If necessary, materials can be used to assist with the calculation, since these problems are about reading for information and determining what the problem is asking rather than just computation or basic facts.

Discussion

Page 49 – At the Beach

These problems require more than one step, and may involve addition as well as subtraction. The wording has been kept simple to assist with the problem-solving process. Students may choose a number of different ways to find a solution. For example, in the second problem, the people who go swimming (8) and walking (7) could be added together and then subtracted from the total of 42; or, alternatively, 8 could be subtracted from 42 and then 7 subtracted from 35 to obtain a solution. Students should be encouraged to try different ways of arriving at a solution.

Page 50 – At the Party

A careful reading of each question is needed to determine what the problem is asking. In some cases, there is more information than needed, and each problem contains numbers that are not required to find a solution. Some problems require more than one step, and both addition and subtraction are needed at times. Again, there are a number of ways to find a solution, and students should be encouraged to explore and try different possibilities of arriving at an answer.

Page 51 – Kangaroos

On this page students are given information that some problems require addition and some subtraction, but that only the subtraction problems are to be solved. This requires students to carefully analyze each story and determine what the problem is asking and whether addition or subtraction is needed to find the solution. This activity shifts the focus from having to find the answer to analysis of the problem and the importance of determining what the problem is asking.

Possible Difficulties

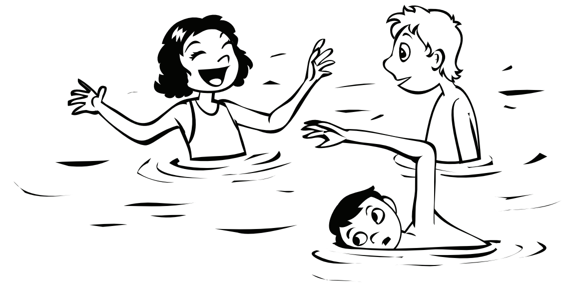
- Unable to identify the need to add or subtract
- Confusion over the need to carry out more than one step or type of calculation to arrive at a solution
- Using all the numbers listed in the problem rather than just the numbers needed

Extension

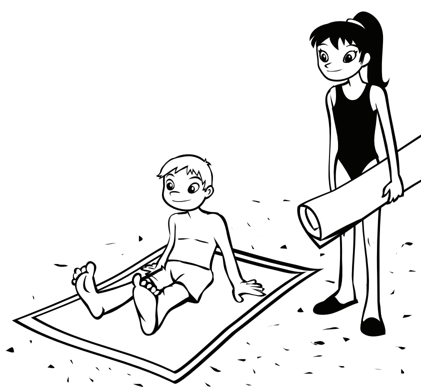
- Students could write their own problems and give them to other students to solve.

8.1 AT THE BEACH

1. 38 people are swimming in the ocean. 9 more people get in, but 13 get out. How many people are now swimming in the ocean?



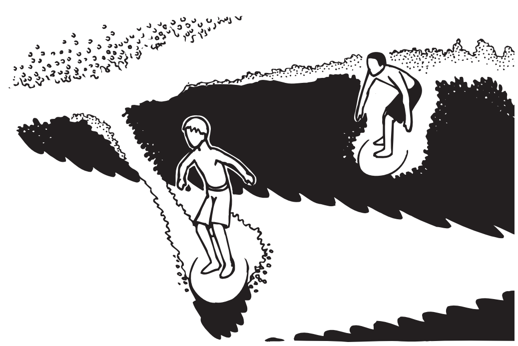
____



2. 42 people are lying on their towels. 8 people get up for a swim and 7 people leave for a walk. How many people are still lying on their towels?

____

3. 39 surfers are on their boards waiting for waves. 21 more surfers arrive, but 9 get out. How many surfers are now waiting for waves?



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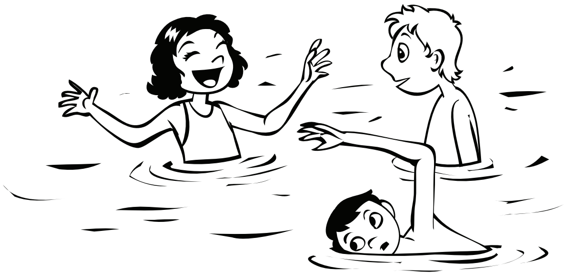


4. 47 people are swimming between the flags. 14 more arrive, but some get out. If there are now 42 people swimming, how many got out?

____

____

8.2 AT THE PARTY

1. There are 37 people at the party. On one table there are 43 sausage rolls, and on another table there are 39 sausage rolls. How many sausage rolls are there altogether?

   ____

2. There are 58 party pies on one tray and 46 party pies on another tray. During the party, 18 people eat 34 of the party pies. How many party pies were not eaten?

   ____

3. There are 20 spring rolls and 30 meatballs on one tray and 30 spring rolls and 40 meatballs on another tray. 37 meatballs and 42 spring rolls are eaten. How many more meatballs than spring rolls are left?

   ____

4. There are 56 soft drinks and 59 juice drinks on one table and 35 soft drinks and 25 juice drinks on another table. During the party, people drank 39 soft drinks and 43 juice drinks.

   (a) How many soft drinks are left? ______________

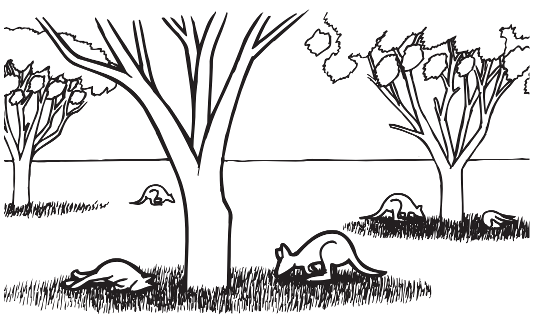
   (b) How many drinks were consumed? ______________

8.3 KANGAROOS

Look at the problems. Some use addition and some use subtraction. Solve only the subtraction problems.

1. There are 247 kangaroos at the waterhole. 119 kangaroos are drinking. How many kangaroos are not drinking?

   ____

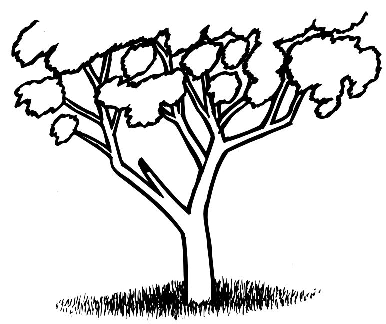


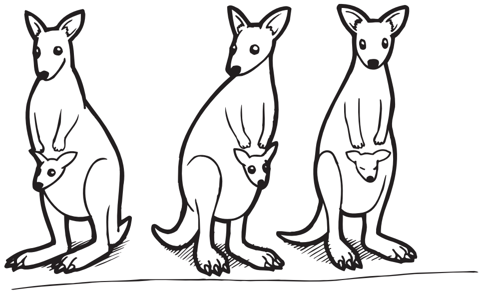
2. 135 kangaroos are eating grass and 164 kangaroos are sleeping under the trees. 59 more kangaroos start eating. How many kangaroos are eating?

   ____

3. 264 kangaroos are sleeping under the shady trees. 128 kangaroos wake up. How many kangaroos are still sleeping?

   ____





4. 172 adult kangaroos are by the river. 38 kangaroos have a baby in their pouch. How many kangaroos do not have a baby in their pouch?

   ____

Money

Problem Solving

To solve problems involving money and to make decisions based on particular criteria

Materials

- Counters, play money, or a calculator, if needed

NCTM Standards

- Number and Operations 1.1, 1.2, 1.3
- Problem Solving

Focus

These pages explore reading for information, obtaining information from another source (an illustration), and using it to find solutions. The problems are about using money, making decisions based on comparing amounts of money, rather than addition, subtraction, or mental facts. Solutions can be obtained by using materials and comparing amounts. The item amounts have been kept small to assist with the problem solving. Counters, blocks, play money, or a calculator can be used, if needed. This investigation lends itself to using a calculator and could be used to introduce calculator work or to extend work previously completed on a calculator.

Discussion

Page 53 – Toy Animals

Students read the items for sale and note how much each costs. Students who are not familiar with money can still do the activity with a calculator. This investigation involves the students obtaining information not only from the question but also from another source—the pictures. They must remember what they are buying and then work out how much it is, and in some cases add on to compare amounts, to see if they have enough money.

The last two questions have a number of possible solutions. Students might choose three items they would like and then add and compare only to discover they don't have enough money, while others may just choose the three cheapest items. Either way, they must compare money amounts and make decisions accordingly.

Page 54 – At the Toy Store

This page extends the investigation about money by giving students more money to spend. The concept of change (money left over) is introduced.

Page 55 – At the Deli

The same concepts are extended to a different situation—a deli menu. Students are asked to read the items on the menu and note how much each costs, and then add, subtract, or compare amounts to see if they have enough money.

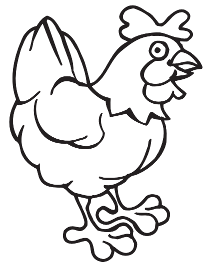
Possible Difficulties

- Confusion about the $ (dollar) symbol
- The concept of "enough money" as opposed to an "exact amount"
- Not buying different items when necessary
- Thinking the exact amount of $8, $7, $5, $20, or $10 has to be spent rather than not spending all that is available

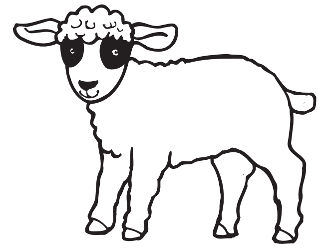
Extension

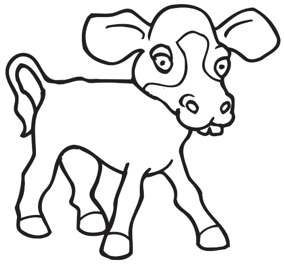
- Make a list of all the different possibilities for spending $8, $7, or $5 in the toy store in the first activity.
- Record and discuss the different ways students chose to spend their $10 in the deli.
- In pairs, ask students to write other questions about a toy store or deli and give them to another pair to solve.

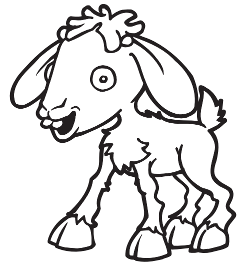
9.1 TOY ANIMALS

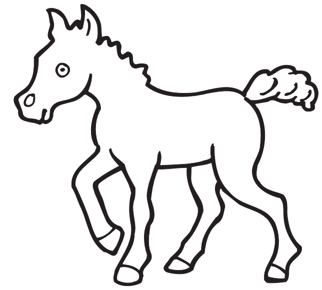

chicken $1

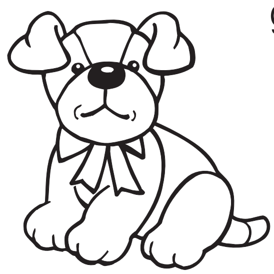

cat $2

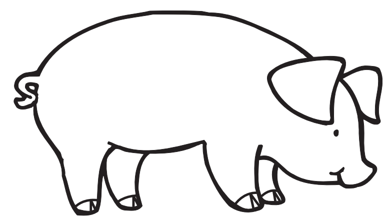

sheep $3


cow $2


goat $3


horse $4


dog $2


pig $3

Nancy has $4. Can she buy a chicken and a cat? ____________

Maria has $5. Does she have enough to buy the cow and the sheep? ____________

Daniel has bought a goat, a dog, and a pig. How much did he spend? ____________

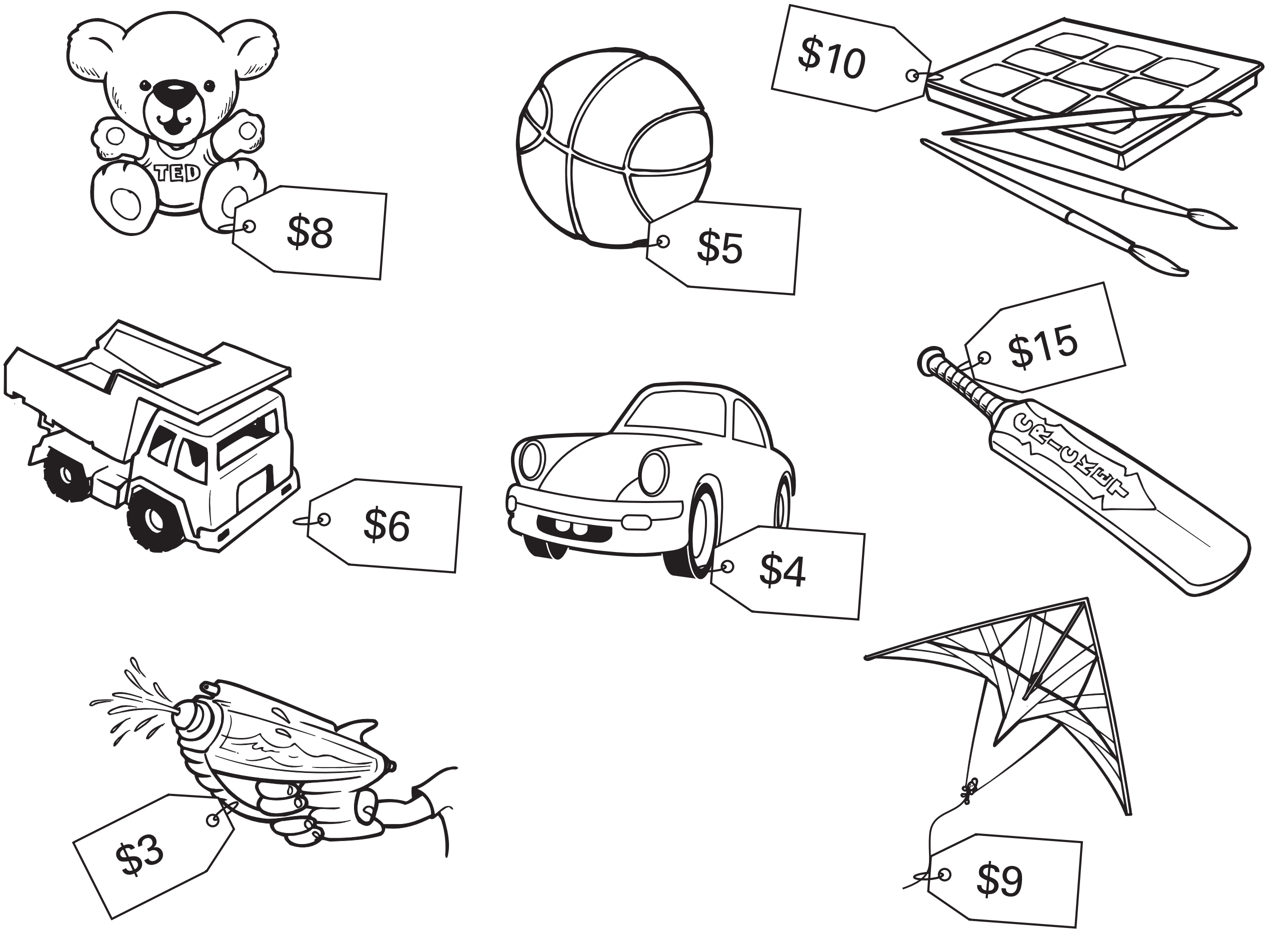
Mike has $8. He wants to buy 3 toy animals. Choose 3 different things he can buy with his money.

____

If you had $10, what would you buy?

____

9.2 AT THE TOY STORE



1. Nadia has $20. Does she have enough to buy the ball and the paint set? ____________
2. Nathan has $15. Can he buy 2 balls and a truck? ____________
3. Danielle has $30. She buys a cricket bat, a ball, and 2 water pistols. How much money does she have left? ____________
4. Minh has $15. He wants to buy 3 presents for his friends. Choose 3 things he can buy with his money.

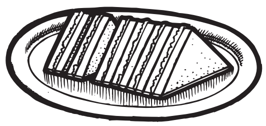
   ____

   ____

5. If you had $20, what would you buy? ____________________________

   ____

9.3 AT THE DELI



Tuckie's Deli

Sandwiches

Egg or cheese.......................$1
Tomato and cheese $2
Ham or chicken.................... $3

Rolls

Chicken or ham.................... $4
Salad $3
Tomato and cheese $2

Juice

Apple or orange.................... $1
Lemonade............................. $2

Milk

Plain $1
Chocolate $2

1. Kelly has $5. Does she have enough to buy a ham sandwich and a chocolate milk?

2. Kurt has $9. Can he buy two chicken rolls and a lemonade?

3. Kristy has $10. She buys a ham roll, a cheese sandwich, and a plain milk. How much money does she have left?

4. Kerry bought three chocolate milks for her brothers. How much change does she have from $10?

5. Kwan has $6. He needs to buy lunch for himself and his two sisters. Choose what he can buy with his money.

6. If you had $10, what would you buy?

Problem Solving

To identify and use number understanding

Materials

- Counters, blocks, or calculator, if needed

NCTM Standards

- Number and Operation 1.1, 1.2, 1.3
- Problem Solving

Focus

These pages explore the properties of magic squares and other shapes. Students must analyze the problems and locate the information necessary to find each magic number. With each square, once the magic number is known, it is then possible to proceed to finding the missing numbers.

Counters, blocks, or a calculator can be used by students to assist them, since the focus is the concept of magic squares and magic numbers rather than addition, subtraction, or basic facts.

Discussion

Page 57 – Magic Squares 2

This investigation extends the concept of magic squares introduced on page 35. Simple 3-by-3 magic squares are used to help students grasp the idea that each row, column, and diagonal adds to the same magic number.

Page 58 – Magic Squares 3

This page continues the previous concept. Again, simple 3-by-3 magic squares are used to help students understand the idea that each row, column, and diagonal adds to the same magic number. In this case, students investigate what happens to a magic square if they add 1 or 3 to each number in the square.

Page 59 – Magic Circles

In this situation, shapes other than squares are investigated. The first examples, using the digits 1 to 6, involve triangles, with each side of each triangle adding to make a magic number. The problems that follow involve more complex shapes, with four or five numbers in each of the lines adding to the same magic number.

Possible Difficulty

- Considering only rows or columns rather than rows, columns, and diagonals

Extension

- Discuss all possibilities the students discover for the magic shapes—are they really different from one another?
- Investigate other magic squares and magic numbers.

10.1 MAGIC SQUARES 2

Magic squares have rows, columns, and diagonals that all add to the same magic number.

7	0	5
2	4	6
3	8	1

1. This magic square has a magic number of _____.
2. Complete the magic squares. Remember, all rows, columns, and diagonals must add to the same magic number.

   Under each magic square, write the magic number.

(a)

8	1	6
	5	

(b)

11		9
	8	
		5

(c)

32	4	24
16		8

(d)

4		8
	7	
6		

10.2 MAGIC SQUARES 3

Complete the magic squares. Remember that all rows, columns, and diagonals must add to the same magic number.

1. The magic number is under each magic square.

9	5	
		6

15

15		
	9	
7		

27

10	3	
		9
	11	

21

2. What happens if you add one to each number in the above magic squares?

10	6	

Are they still magic squares? ____________

3. What happens if you add three to each number?

12	8	

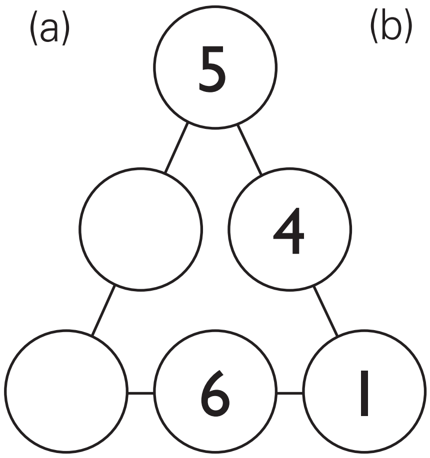
Are they still magic squares? ____________

10.3 MAGIC CIRCLES

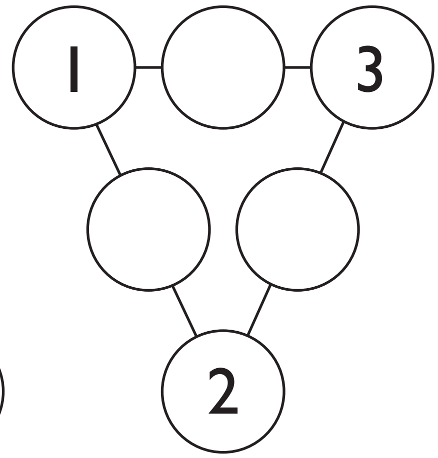
Use the digits from 1 to 6 to make each side or line add to the same total.

1. Write the magic number under each shape.

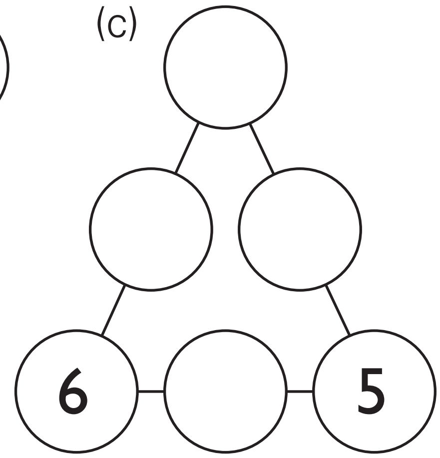
(a)



(b)

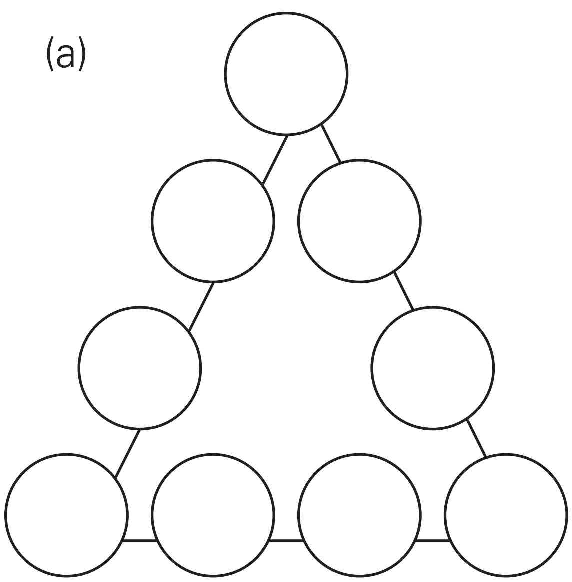


(c)



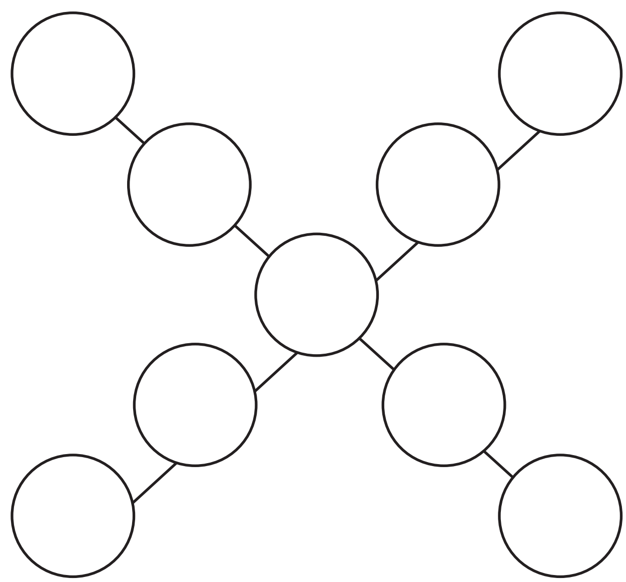
2. Now try these magic shapes, using the digits from 1 to 9. The magic number is written under each shape.

(a)



21

(b)



27

(c) Is there more than one way to arrange the numbers? ____________

Problem Solving

To reason logically, and to identify, create, and describe patterns

Materials

- Blocks such as Unifix™ Cubes or counters (for example, teddy bears) in four different colors

NCTM Standards

- Algebra 2.1
- Problem Solving

Focus

These pages explore making patterns, changing patterns, and using patterns and numbers. Students analyze what makes a pattern and make predictions based on their experiences.

Discussion

Page 61 – Blocks

The first activity involves taking four blocks (of different colors) and lining them up in different groups of two. It is important to explain to students that there can be two groups with the same two colors but they have to be lined up in a different order. The possible arrangements are:

blue	red	red	blue	yellow	blue	green	blue
blue	yellow	red	yellow	yellow	red	green	red
blue	green	red	green	yellow	green	green	yellow

The same concept is extended in the next activity, in which students now line up the blocks in groups of three. For example, a student might keep the same color first and then change the next two colors, or he/she might change the first color and then swap the following two colors. Encourage students to explore different arrangements. The possible arrangements are:

red	blue	green	red	yellow	blue	blue	red	green
red	blue	yellow	red	yellow	green	blue	red	yellow
red	green	yellow	blue	green	red	blue	yellow	green
red	green	blue	blue	green	yellow	blue	yellow	red
green	blue	red	green	yellow	red	yellow	blue	green
green	blue	yellow	green	yellow	blue	yellow	blue	red
green	red	blue	yellow	red	blue	yellow	green	red
green	red	yellow	yellow	red	green	yellow	green	blue

Page 62 – Grid Fun

This activity builds on the previous experience of lining up the blocks and interchanging the colors to make the pattern. The grid can be organized in a number of ways; however, each different way will always have the same diagonal with three blocks of the same color. Two grids are provided to enable students to explore different possible arrangements.

Page 63 – More Grid Fun

This activity builds on the previous experience of the three-by-three grid by extending the grid to four-by-four with an additional color. Again, the grid can be organized in a number of ways, and this time it is possible for both diagonals to have four blocks of the same color. Two grids are provided to enable students to explore different possible arrangements.

Possible Difficulties

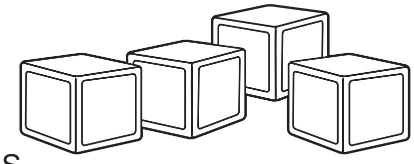
- Indiscriminately moving blocks around
- Unable to keep track of what has already been tried
- Content to find only one possibility
- Unable to consider both rows and columns

Extension

- Students can be encouraged to make and describe more complex patterns of their own.
- Extend the grid to five-by-five, using five different colors.

11.1 BLOCKS

Take 4 blocks, each a different color—
for example: red, blue, green, and yellow.



1. Line up and draw different groups of 2 blocks.

2. How many different groups can you make? ____________________

3. Now, line up and draw different groups of 3 blocks.

4. How many can you make? ____________________

11.2 GRID FUN

Make 3 groups of 3 blocks (9 blocks), with each group a different color.

1. Place the blocks on the grid, making sure that no row or column has the same color in it more than once.

2. What do you notice about the diagonals?

____

____

____

____

3. Try doing it a different way.

11.3 MORE GRID FUN

Make 4 groups of 4 blocks (16 blocks), with each group a different color.

1. Place the blocks on the grid, making sure that no row or column has the same color in it more than once.

2. What pattern do you notice? ____________________________________

____

3. Try doing it a different way.

Problem Solving

To use spatial visualization and logical reasoning to solve problems.

Materials

- Crayons or colored pencils (red, green, blue, and yellow)

NCTM Standards

- Algebra 2.1
- Geometry 3.4
- Problem Solving

Focus

These pages explore possible arrangements or combinations of objects in order to determine all possibilities in a situation. Spatial as well as logical thinking and organization are involved as students investigate all likely arrangements and make sure that they do not repeat combinations. Acquiring the skill of systematic thinking needed to solve these problems will also help students solve many other problems, not merely those that involve similar arrangements.

Discussion

Page 65– Window Panes

The panes can be organized in the windows in 24 possible ways. With 25 panes drawn on the page, some students may simply repeat a previous combination in order to fill all of the panes. The problem states that they use one of each color pane; therefore, each window needs one red pane, one green pane, one blue pane, and one yellow pane.

Page 66 – Egg Cartons

The example uses only one egg to show all of the possible arrangements that can be used. Extending this to placing four eggs is more complex, and there is both the likelihood of coming up with the same arrangement more than once or missing one or more possibilities. There are more egg cartons than needed.

Page 67– Spider Webs

Thinking and organization similar to those used on page 66 are needed to find the paths the spider can take. As in the egg carton problem, there are more webs drawn than are needed, but ensuring that a path is used only once may be more difficult than placing eggs differently. Students must travel in and out of the center of the web, as well as traverse the outer lines.

Possible Difficulties

- Not recognizing that an arrangement has been used more than once
- Using all of the windows, egg cartons, or spider webs, whether they were needed or not

Extension

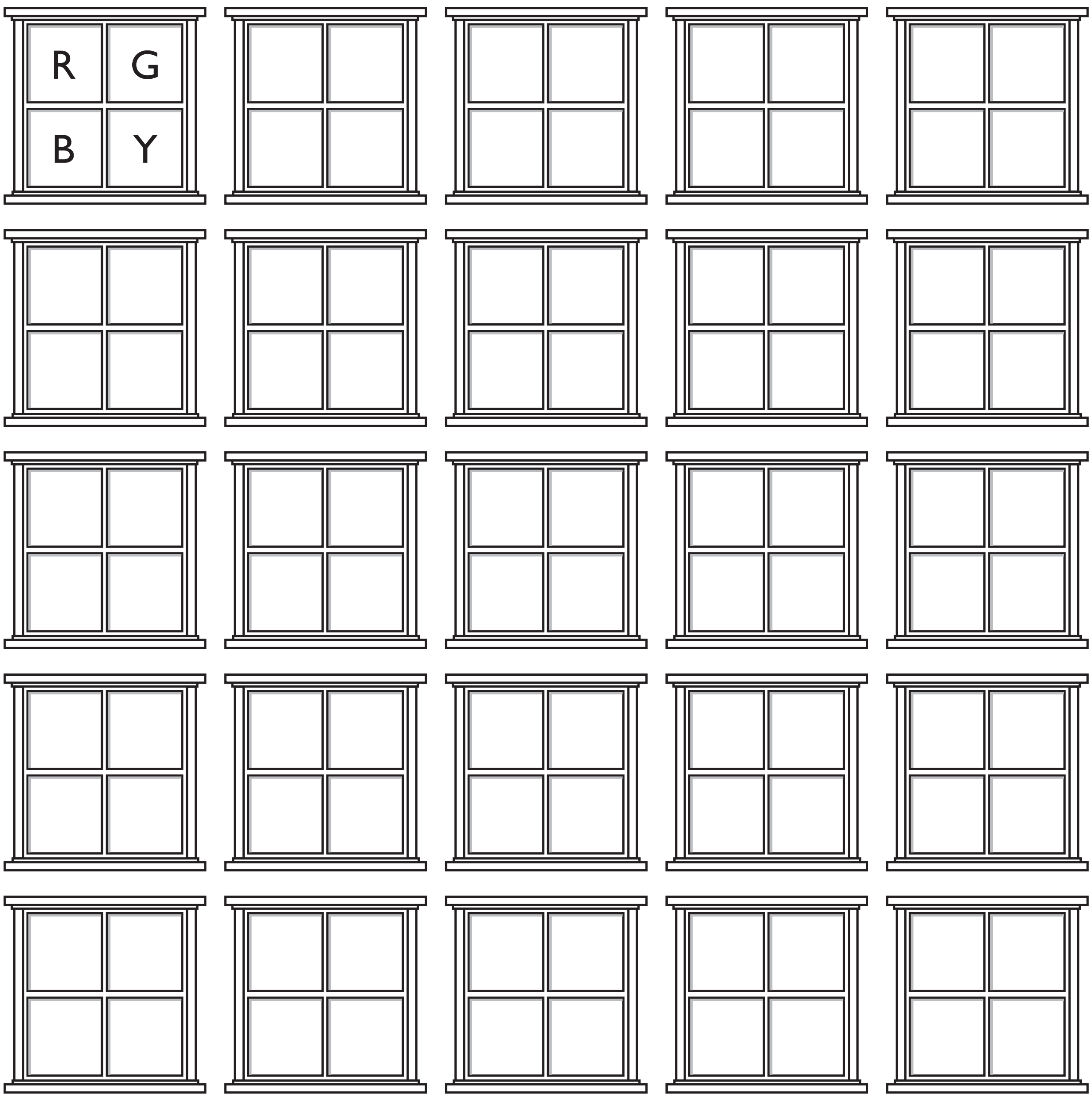
- For the "Windows" activity, discuss the methods that can be used to systematically find all of the possible designs.
- For the "Egg Cartons" and "Spider Webs" activities, discuss the methods students used to systematically find all of the possibilities without omitting or repeating any. Contrast the different ways of thinking without necessarily promoting one over another.
- Use grid paper to investigate the possibilities of having six panes of glass with two different colors.
- Investigate the egg carton problem further by using two brown eggs and two white eggs.
- Extend the spider web problem by adding another outside web line.

12.1 WINDOW PANES

Tanya is fitting colored glass into her window.

She has 1 red pane, 1 green pane, 1 blue pane, and 1 yellow pane. Each time she makes a window, she uses one of each color pane.

1. Color the different designs she can make.

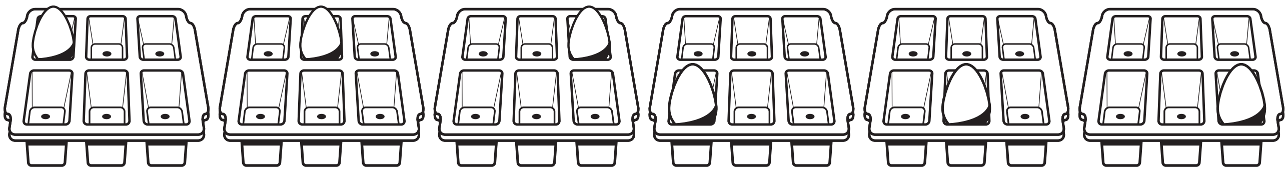


2. How many different designs can she make? ____________

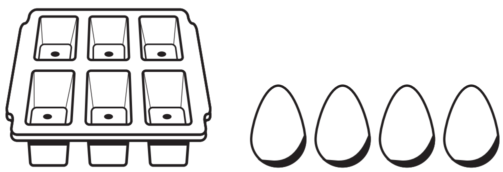
12.2 EGG CARTONS

The egg carton has room for 6 eggs.

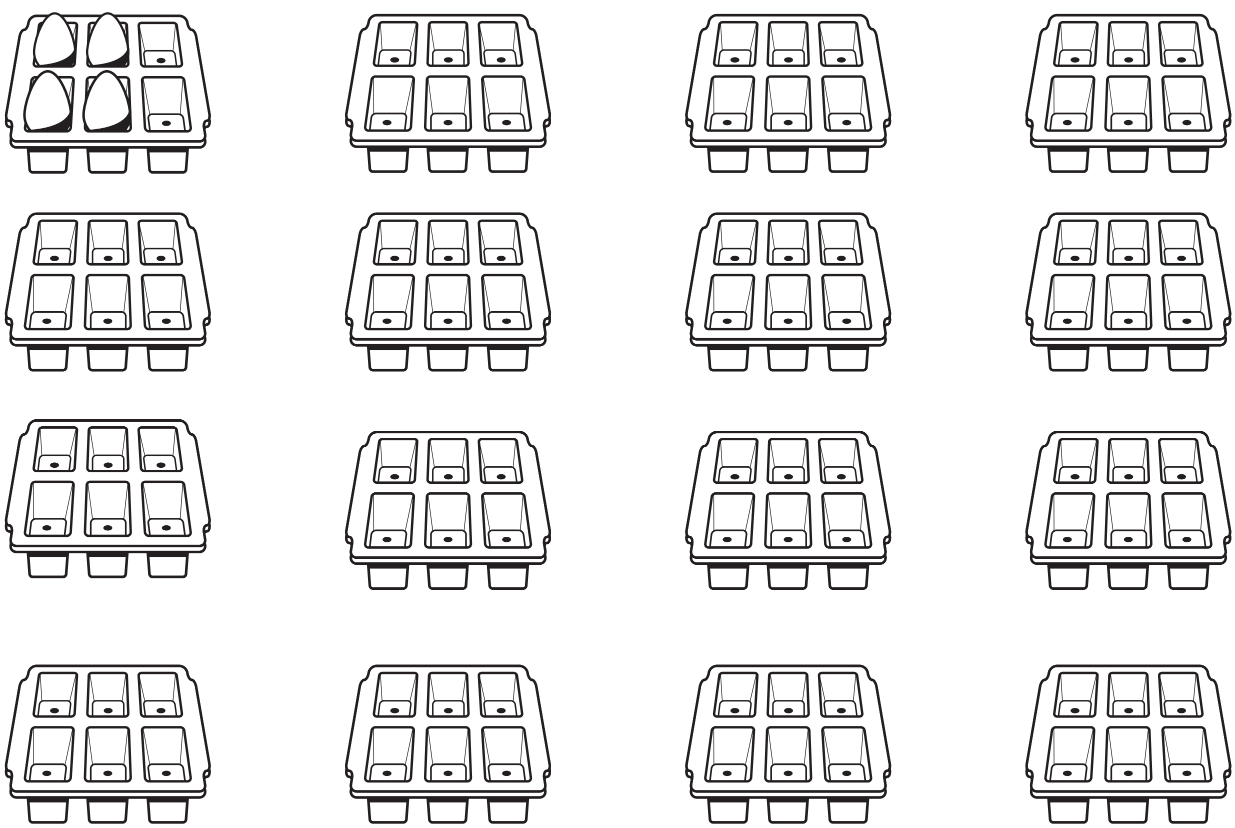
There are 6 ways you can put 1 egg into the tray.



Amanda has an egg carton and 4 eggs.



1. Draw the different ways she can put the eggs into the egg carton. The first arrangement has been done for you.



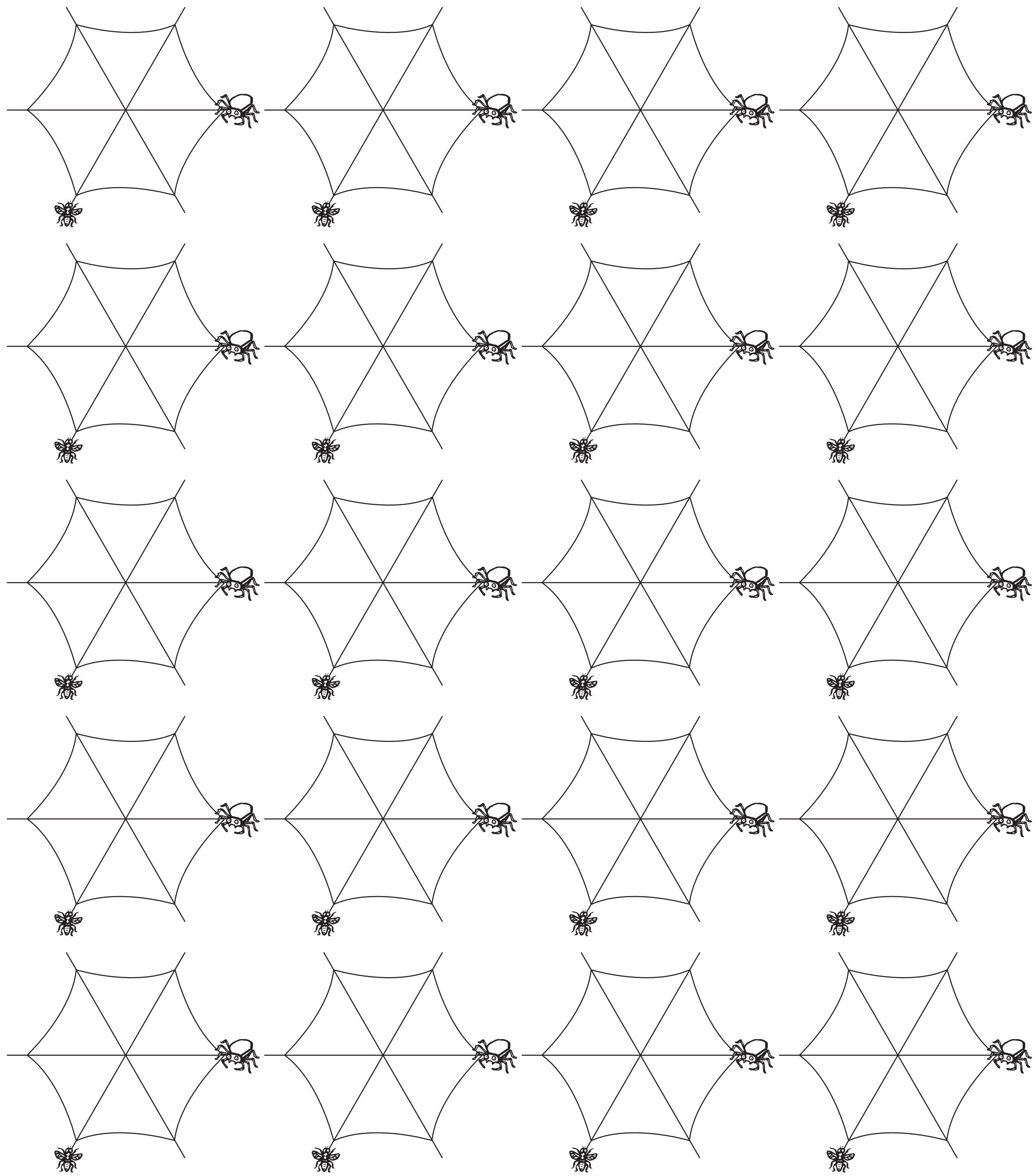
2. Did you use all of the egg cartons? ____________

3. How many did you use? ____________

12.3 SPIDER WEBS

Help the spider catch the fly.

1. On each web, draw a different path from the spider to the fly. The spider can run along each line only once.



2. Did you use all of the webs? ____________

Problem Solving

To visualize relationships shared by two-dimensional shapes

Materials

- Tangram puzzles or cutouts made from the template on page 114

  (*Note:* Explain to students that the shapes drawn on pages 69–70 are not to scale.)
- Pattern blocks or triangular grid paper

NCTM Standards

- Geometry 3.1, 3.3, 3.4
- Problem Solving

Focus

These pages explore using tangrams and pattern blocks to make shapes. Spatial thinking as well as logical thinking and organization are involved, as students investigate manipulating the pieces to form the various shapes. Being able to visualize patterning in this way will help students solve many other problems, including those involving number, measurement, and data analysis and probability, as well as other spatial problems.

Discussion

Page 69 – Tangrams 1

The first activities are designed for students to explore the relative sizes of the individual pieces and the ways in which the sides match to allow them to fit together. Practicing this will help them with the more open-ended fourth task.

Page 70 – Tangrams 2

These activities build on the comprehension developed on page 69 and help students to understand that many problems can be solved in more than one way.

Page 71 – Pattern Block Shapes

This activity requires the use of pattern block shapes or triangular grid paper. A number of different pieces can be used to cover the hexagon. When using pattern block shapes, there are eight possible arrangements if you include the hexagon piece as one of the ways to cover it. Encourage students to explore rotating and flipping the pieces when trying to find combinations of shapes that will fit together to cover the hexagon. Some student will need to try different pieces and then discard them, while others may be able to visualize various possibilities.

Possible Difficulties

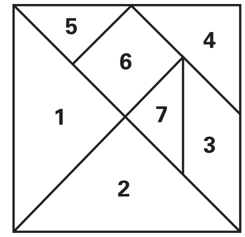
- Thinking there is only one possible solution
- Difficulty rotating or flipping the pieces as needed
- Unable to visualize possible pieces to make the pattern block shapes

Extension

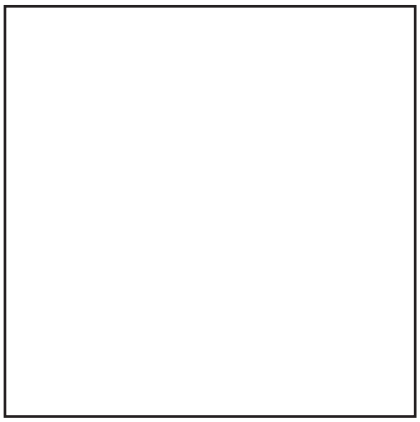
- Make the two shapes at the top of page 71 using only four tangram pieces.
- Investigate other shapes that can be made and/or covered with the pattern block shapes.

13.1 TANGRAMS 1

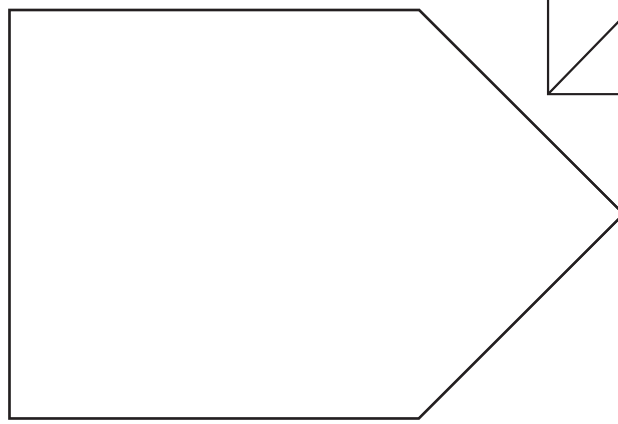
1. Cut out the tangram pieces from the sheet your teacher gives you, and use them to make each shape.



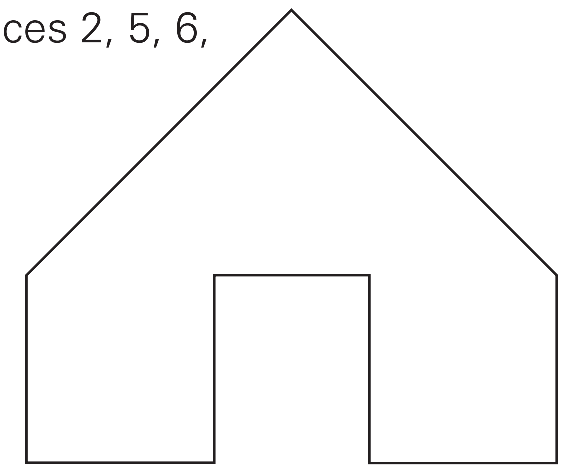
(a) Use pieces 4, 5, and 7.



(b) Use pieces 4, 5, and 6.

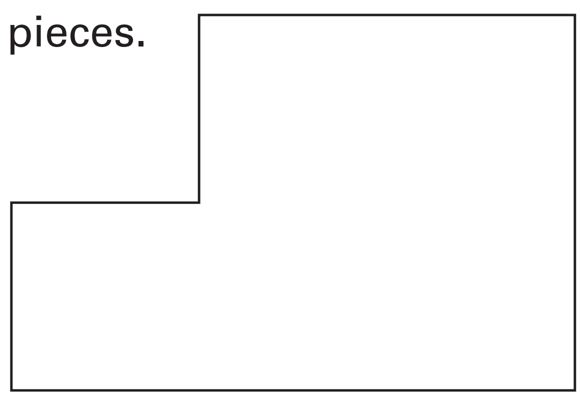


(c) Use pieces 2, 5, 6, and 7.



2. Draw lines to show the pieces you used.

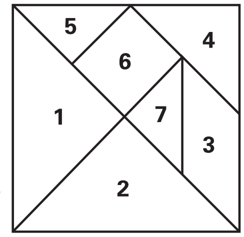
3. Make this shape. Use only three pieces.



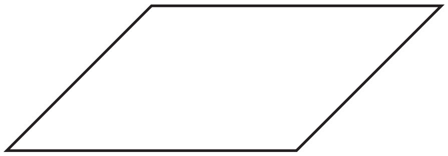
4. Can you make this shape in any other ways?

13.2 TANGRAMS 2

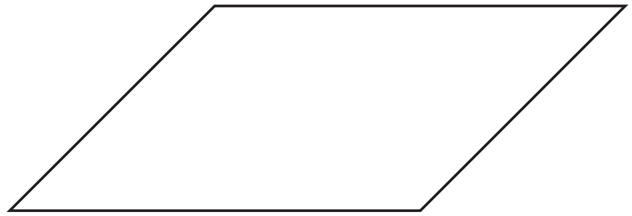
1. Cut out the tangram pieces from the sheet your teacher gives you, and use them to make each shape.



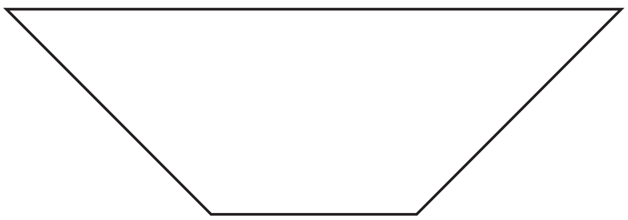
(a) Use 3 pieces.
Draw the pieces you used.



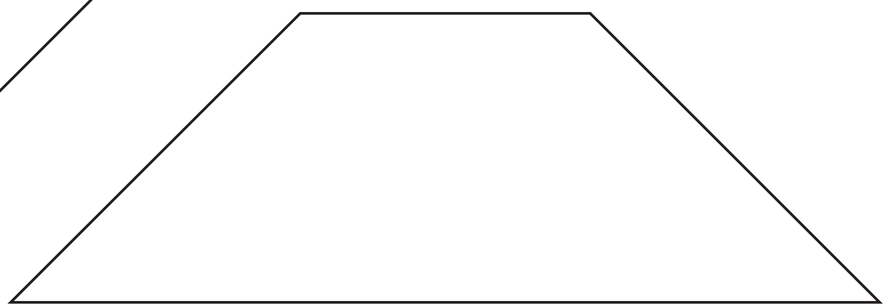
(b) Use 3 pieces.
Draw the pieces you used.



(c) Use 3 pieces.
Draw the pieces you used.

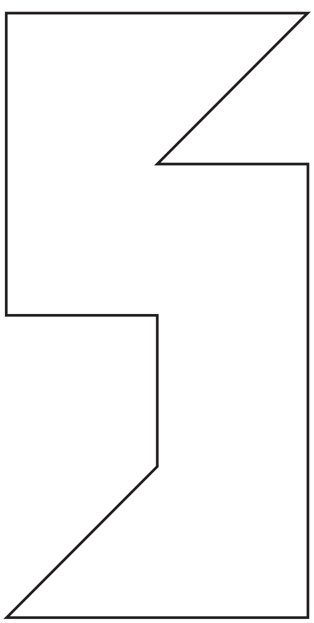


(d) Use 4 pieces.
Draw the pieces you used.

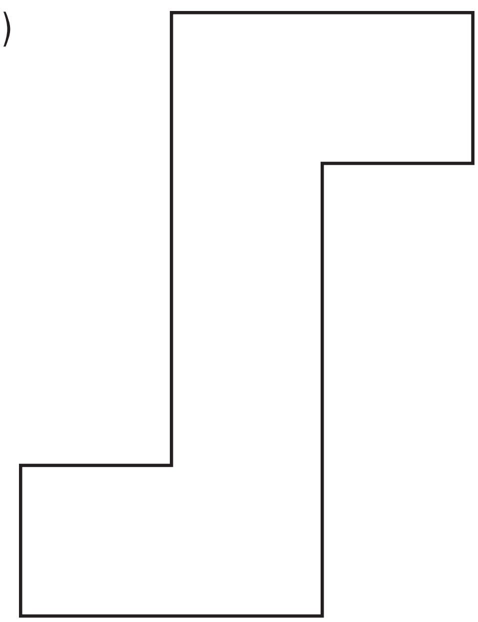


2. Use any of the pieces to make the shapes below.

(a)

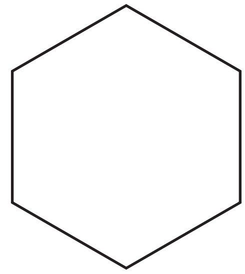


(b)

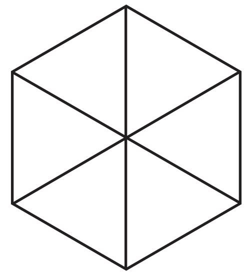


13.3 PATTERN BLOCK SHAPES

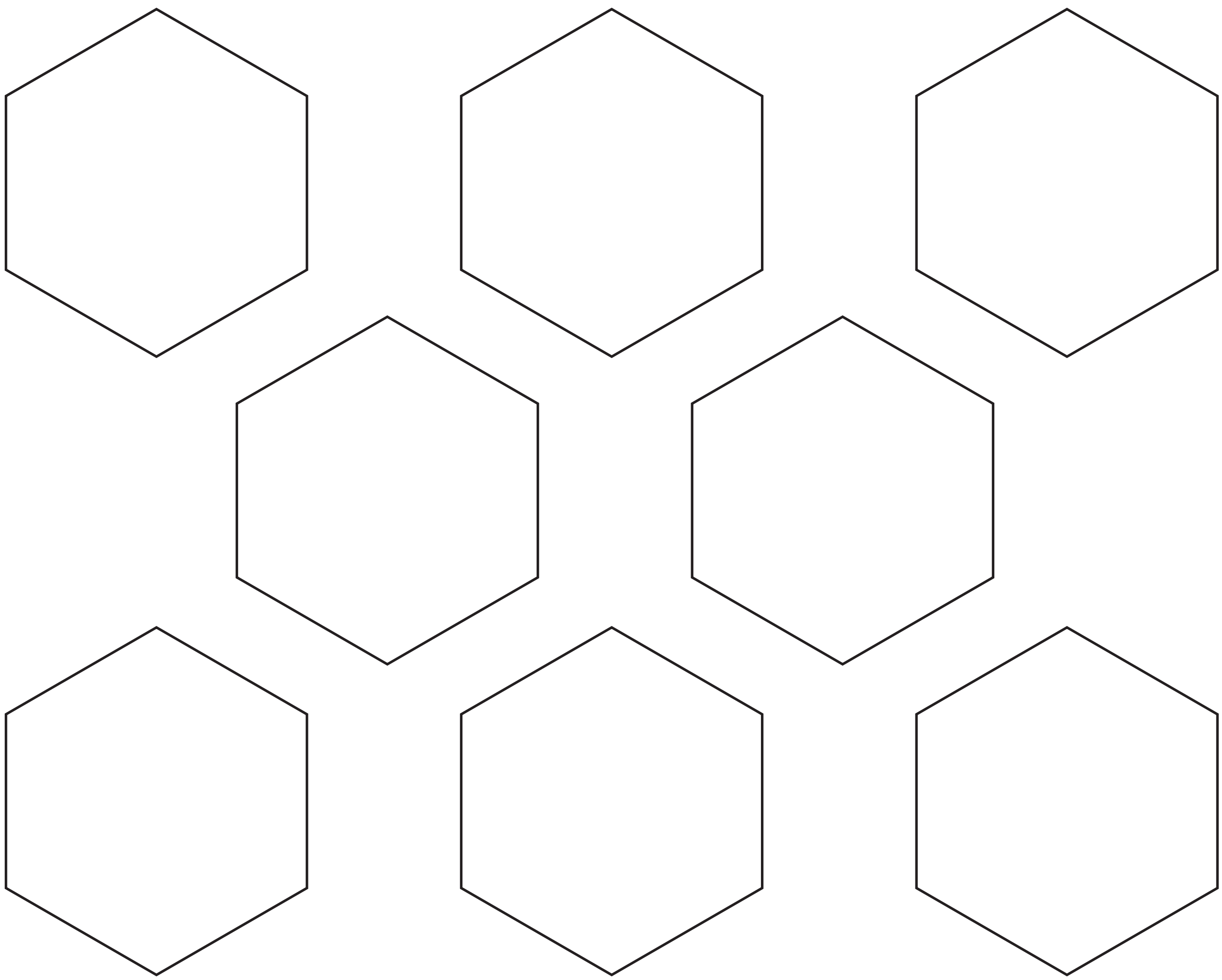
Find this pattern block shape.



Cover it with six triangles.



1. What other shapes or pattern blocks can you use to cover it?



2. Did you find 8 different ways? ___________

2-D Shapes

Problem Solving

To visualize relationships among two-dimensional shapes and use spatial visualization and reasoning to solve problems

Materials

- Square tiles
- Pattern blocks
- Grid paper or cutout squares

NCTM Standards

- Geometry 3.1, 3.3, 3.4
- Problem Solving

Focus

These pages explore arrangements of squares and other shapes. Spatial thinking as well as logical thinking and organization are involved, as students investigate all possible arrangements and extensions. Being able to visualize patterning of this form will assist students in solving many other problems, including number, measurement, and data analysis and probability, as well as other spatial situations.

Discussion

Page 73 – Using 4 Squares

A number of shapes can be made using the four squares, all of which are commonly called tetrominoes. Some students may need to physically manipulate the squares to find the combinations and analyze whether the shapes are the same or different. This might involve rotating or flipping.

Page 74 – Using 5 Squares

This activity extends from the previous problem. Again, some students may need to physically manipulate the squares to find the combinations and analyze whether the shapes are the same or different. This might involve rotating or flipping. Shapes made with five squares are called pentominoes.

Page 75 – Using Rectangles

Some students may need to physically manipulate the shapes to find the combinations that make the rectangle. This may involve rotating and/or flipping some of the pieces. Other students may be able to visualize the shapes and realize which two pieces combine to make the rectangle. Encourage students to explore rotating and flipping the pieces to come up with other shapes.

Page 76 – Growing Shapes 1

Students may need to physically manipulate squares or draw squares on grid paper to see how the pattern "grows." Encourage students to make predictions and give a verbal description of their findings.

Page 77 – Growing Shapes 2

Students will need to physically manipulate the shapes to see the pattern involved in extending each shape. The first shape replicates the manner in which the larger square was constructed. The triangles require one of the pieces to be flipped to make the larger triangle. Encourage students to continue the patterns to make the next-largest shape.

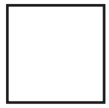
Possible Difficulties

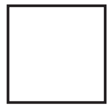
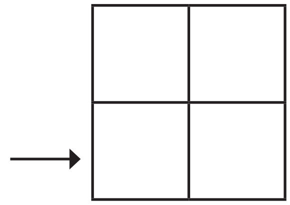
- Not joining squares along a side
- Making only one or two possible shapes
- Unable to visualize the pattern needed to grow a shape
- Not rotating or flipping the pieces as needed

Extension

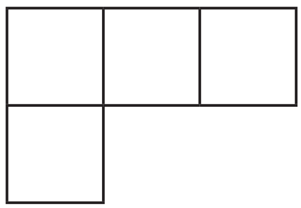
- Investigate growing other shapes using different numbers of squares or other geometric shapes.

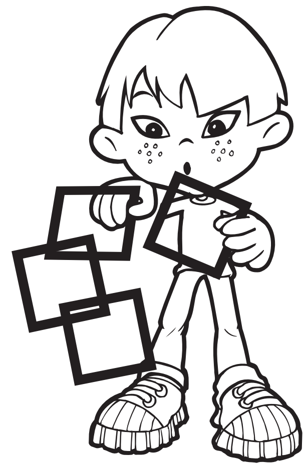
14.1 USING 4 SQUARES

This shape is a square . Find 4 squares.

Make a larger square using 4   .

What other shapes can you make using 4 squares?
Here is one.



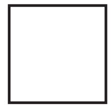


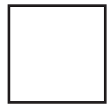
How many other shapes can you make?

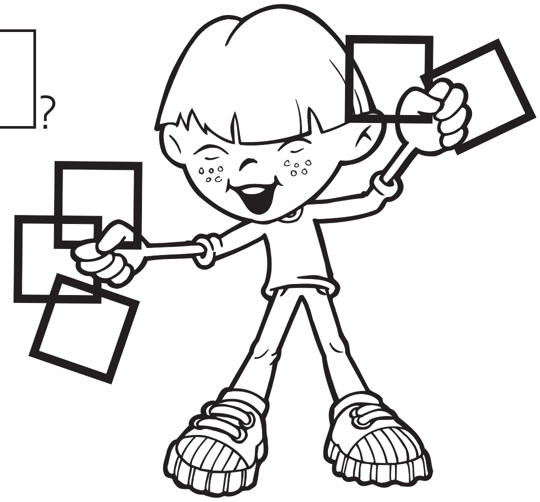
Draw each one.

How many shapes did you make? ___________

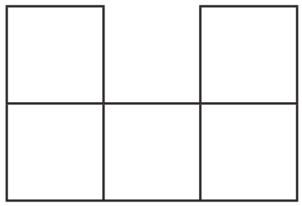
14.2 USING 5 SQUARES

This shape is a square.  Find 5 squares.

What shapes can you make using 5 ?



Here is one.

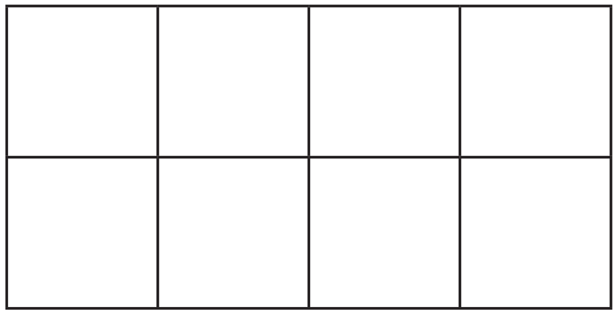


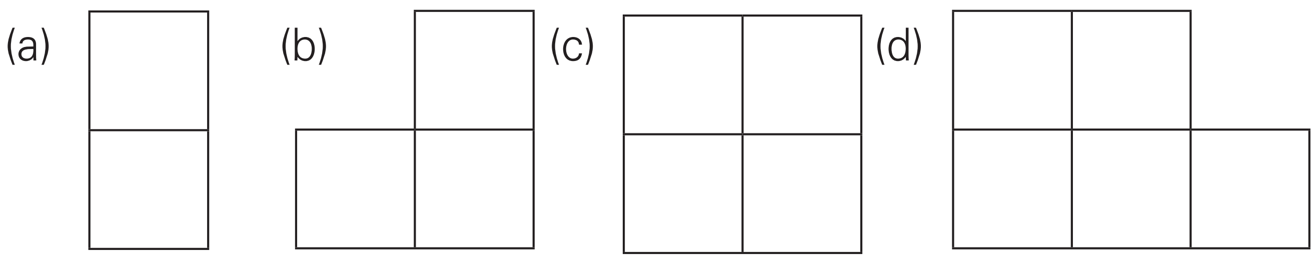
Draw each shape.

How many shapes did you make? ______________________

14.3 USING RECTANGLES

Look at the rectangle and the pieces below it.





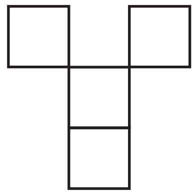
1. Which pieces fit together to make the rectangle? ______________

2. Cut out the pieces and check. What other shapes can you make?

14.4 GROWING SHAPES 1

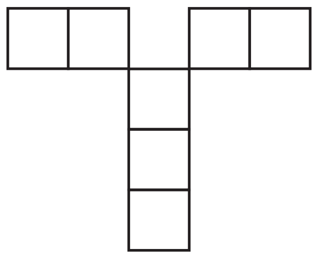
1. Make each shape. How many squares are in each shape?

Shape 1



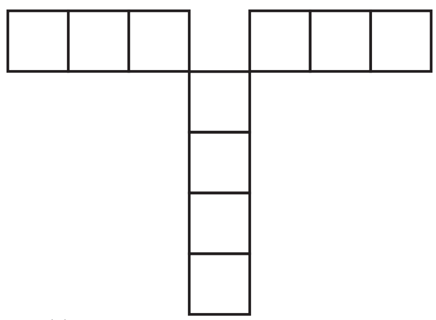
____

Shape 2



____

Shape 3



____

Look for a pattern.

2. Make Shape 4. Draw it.

How many squares are in it? _______

3. Make Shape 5. Draw it.

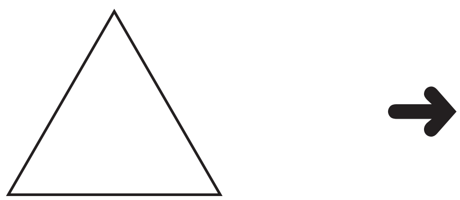
How many squares are in it? _______

14.5 GROWING SHAPES 2

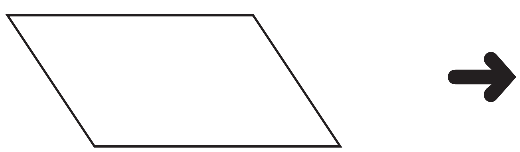
Four squares can make a larger square.

Use four of each of these shapes to make a larger copy of the shape.
Draw the new shape you made.

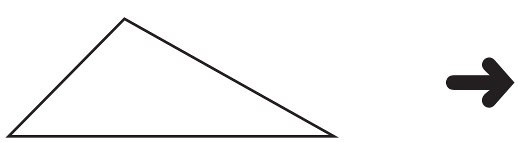
Shape 1



Shape 2



Shape 3



Problem Solving

To use spatial visualization and logical reasoning to solve problems

Materials

- Toothpicks
- Cubes, such as wooden or multilink cubes or Unifix™, in two different colors
- Isometric dot paper

NCTM Standards

- Geometry 3.1, 3.3, 3.4
- Problem Solving

Focus

These pages explore possible arrangements of two-dimensional and three-dimensional shapes to determine how particular outcomes are formed. Spatial as well as logical thinking and organization are involved as students investigate all likely arrangements to ensure the final shapes match the given criteria or visualize a given shape in terms of its component parts.

Discussion

Page 79 – Toothpicks

Students will need to manipulate the toothpicks to see how each shape can be extended using as small a number of toothpicks as possible. The original triangle is used as a building block for the larger shapes and triangles that are formed, but the number of toothpicks used for the new shapes is not simply a multiple of the three used for the first shape.

Increasing the size of the square can be considered in a similar way: using 4, then 12, then 24 toothpicks, and so on. The problem could also simply imply that the number of toothpicks on the sides of the square increases, using 4, 8, 12, and so on.

Both solutions are reasonable, based on an analysis of the problem, and should be discussed with the class.

Page 80 – Building with Cubes

Students will need to manipulate blocks to see if they can create shapes that look like those depicted on the page, building an ability to visualize three-dimensional shapes. Some may need to draw lines on the shapes in to see how the blocks can be used to form the shapes. Building a larger cube extends the thinking introduced on page 80.

Making it in two colors introduces another aspect of visualization, especially when a larger cube is constructed. The number of cubes needed increases in ways that at first might seem surprising.

Page 81 – Stacking Cubes

The drawings of the shapes on this page reverse the type of thinking needed when building shapes so that patterns can be seen. In these examples, the cubes that are partly hidden must be visualized, and careful analysis is needed to consider the whole divided into its component parts in order to determine the number of cubes used.

Possible Difficulties

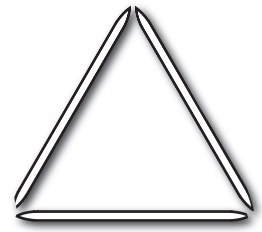
- Focusing only on building exact replicas of the triangle or square and using more toothpicks than needed
- Unable to visualize the three-dimensional cubes in their representations on the two-dimensional page
- Considering only those cubes that can be readily seen

Extension

- Have students make other shapes using five cubes, and have other students copy them.
- Make stacks of cubes for other students to replicate. The number of cubes used should be specified.
- Have students use isometric dot paper to draw the shapes they make, and have other students copy them.

15.1 TOOTHPICKS

1. Use 3 toothpicks to make a triangle.



   Now make 2 triangles using 5 toothpicks.

   Make 3 triangles.

   How many toothpicks did you need? ______

   Draw it.

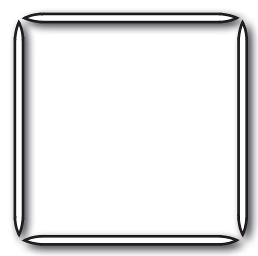
   Use 2 more toothpicks to make a larger triangle.

   Now make an even larger triangle.

   How many toothpicks did you need? ______

   Draw it.

2. Use 4 toothpicks to make a square.



   How many toothpicks do you need to make a larger square? ______

   Make an even larger square.

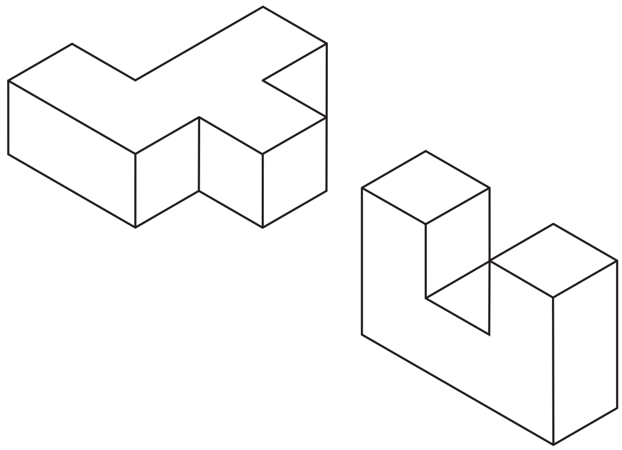
   How many toothpicks did you need? ______

   Draw it.

15.2 BUILDING WITH CUBES

Find some cubes in two different colors.

Can you make these shapes using five cubes?



Use your cubes to build a larger cube.

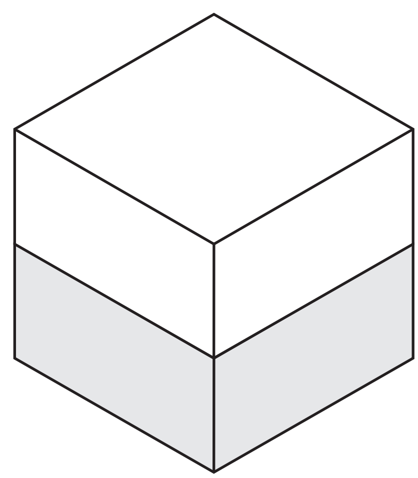
1. How many cubes did you need?

   ____

2. Can you make a cube that is half one color and half another color?

   ____

   ____



3. How many cubes of each color did you need?

   ____

4. Can you make a similar cube that is a larger size?

   ____

   ____

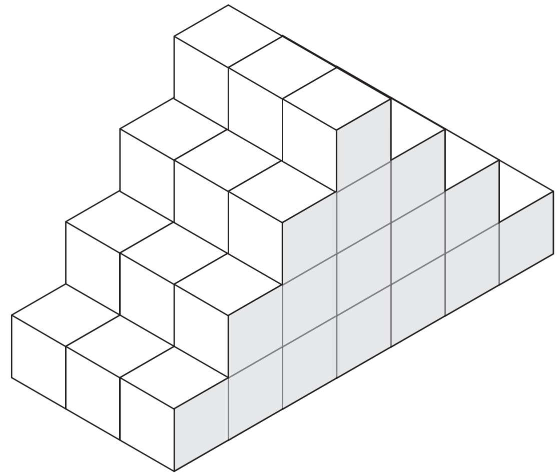
5. How many cubes of each color did you need? ____________________

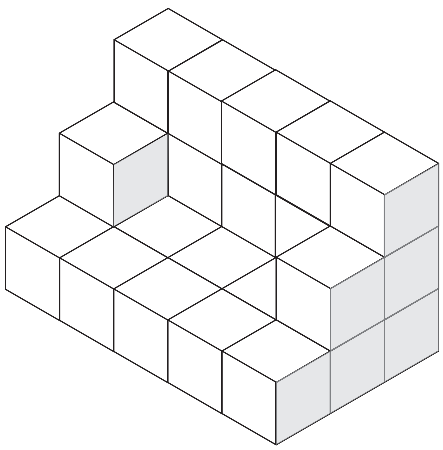
15.3 STACKING CUBES

1. This set of stairs is made by stacking cubes. How many cubes are needed to make the stairs?

   ____

   Find as many cubes as you think you need and make the staircase.





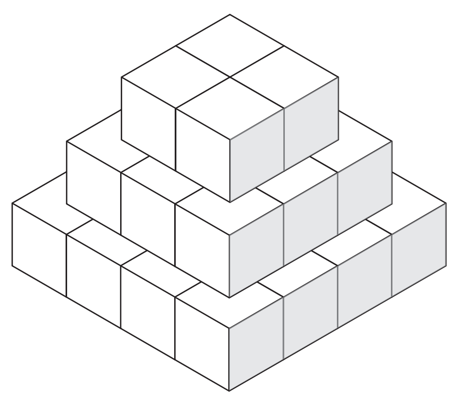
2. How many cubes are needed to make this shape?

   ____

   Take as many cubes as you think you need and make the shape.

3. How many cubes are in this pyramid?

   ____



4. How many cubes would you need in total to put one cube on top and one more layer on the bottom?

   ____

   Make the new shape to see if you were correct.

Problem Solving

To solve problems involving maps or coordinate planes, time, distance, and direction

NCTM Standards

- Geometry 3.2
- Problem Solving

Focus

These pages explore finding and interpreting information from a map or coordinate plane. Students are required to use the information given to find solutions to problems involving time, distance, and direction.

Discussion

Page 83 – Alice's Island

Analysis of the map shows the time it takes to walk various distances. Students are required to interpret this information and record it in the table. The table can then be used to help with the questions. Some destinations are not direct, and most have more than one possible route.

- Hut to lake: If it takes two hours to walk there, it will take two hours to walk back.
- Hut to cave past the forest: Involves going past the forest on the way to the cave, rather than walking the direct route past the lake, which is used on the return journey.
- Waterfall to lake: There is no direct route to the lake from the waterfall, and Alice would need to go via the cave or the hut. Discussion could focus on why one way would be better than another; for example, the route past the hut is much shorter, but the route past the cave might provide a nicer view or a better path.

Page 84 – Drive Time

This activity involves following directions and using coordinates. Students plot the path of a car, taking into account direction and distance. The first two grids are fairly simple, with just one journey in each direction. The next two grids involve moving back and forth in a number of different directions.

Page 85 – Animal Trees

Analysis of the map shows the distances between various animals' trees. Students read and interpret questions and use information from the map to find solutions. Words such as *shortest*, *longest*, *closest,* and *furthest* are used, and, in some cases, students must find all of the possible routes from one destination to another. The numbers have been kept simple to encourage mental computation and estimation.

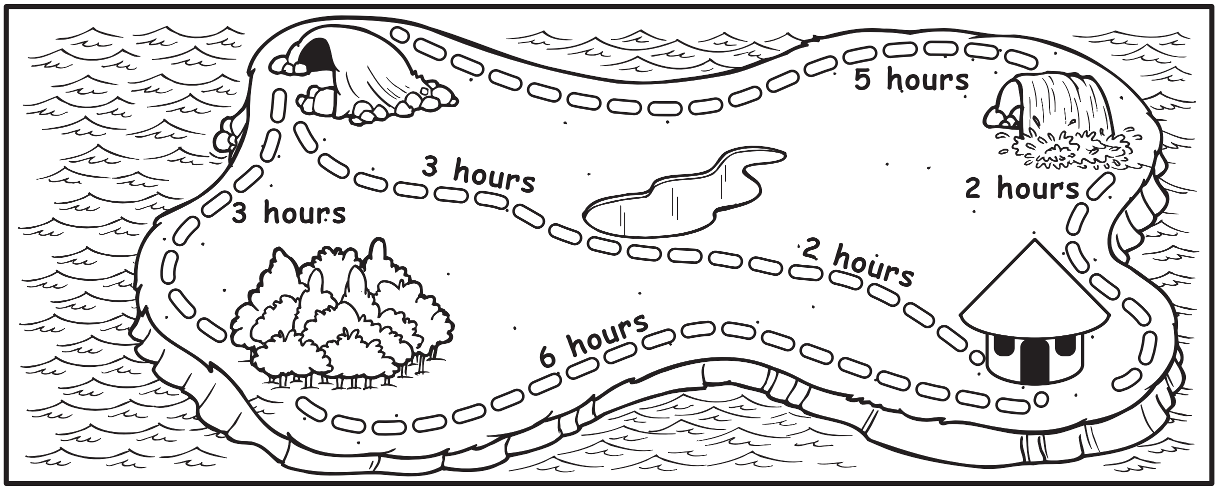
Possible Difficulties

- Unable to read the map and use the information to find solutions
- Not understanding the concept of "there and back again"
- Not knowing what to do when there is no direct route
- Confusion about the terms used to convey distance—for example, *furthest*, *closest*

Extension

- Students can use the maps and write other questions for other students to solve.

16.1 ALICE'S ISLAND



1. Fill in the walking times.

Track	Hours	Track	Hours
hut to lake		forest to cave	
hut to cave		forest to waterfall	
hut to forest		waterfall to lake	
hut to waterfall		lake to cave	
waterfall to cave			

2. How long would it take Alice to walk from her hut to the lake and back again? ______________________

3. Alice walked from her hut to the cave. She went past the forest on her way to the cave. How long did it take her?

____

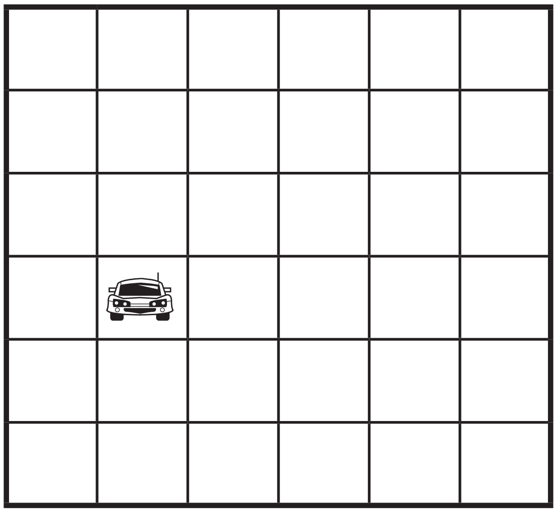
4. How long would it take her if she returned home past the lake?

____

5. How long would it take for Alice to walk from the waterfall to the lake? ______________________

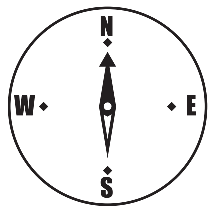
16.2 DRIVE TIME

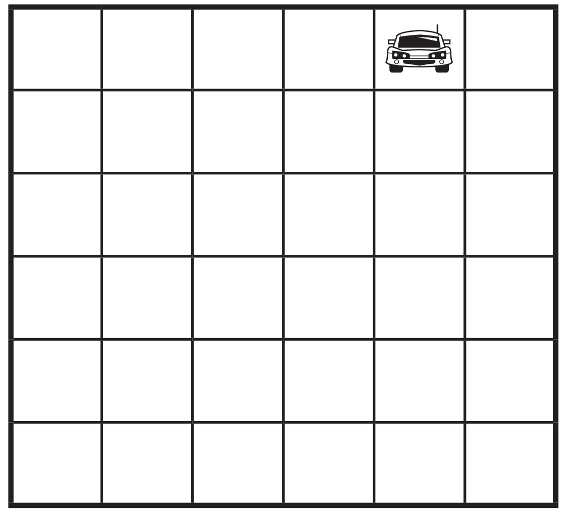
Draw the paths of the cars on the grids.

1.


Drive

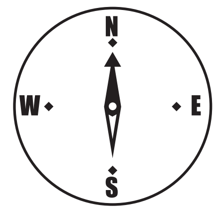
2 squares north
3 squares east
4 squares south
2 squares west

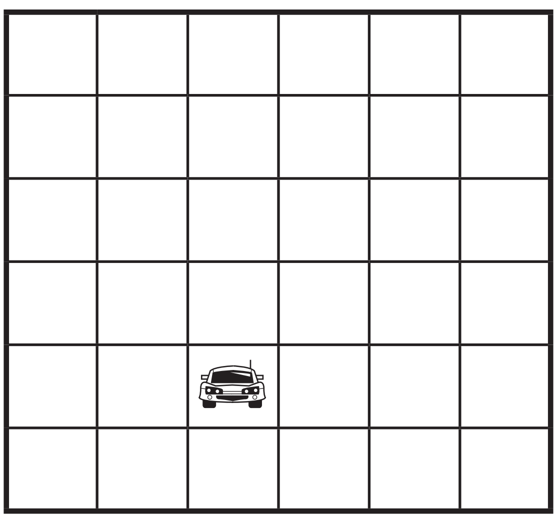


2.


Drive

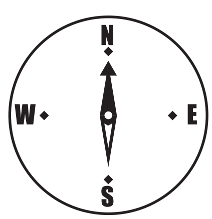
5 squares south
3 squares west
4 squares north
2 squares east

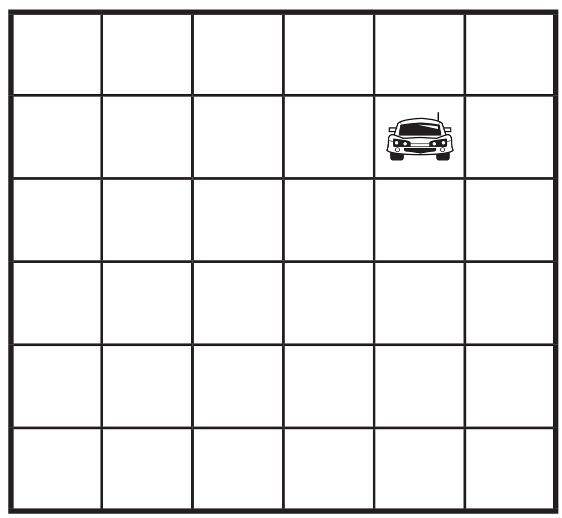


3.


Drive

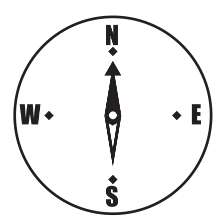
3 squares north
2 squares east
1 square north
1 square east
4 squares south
3 squares west



4.


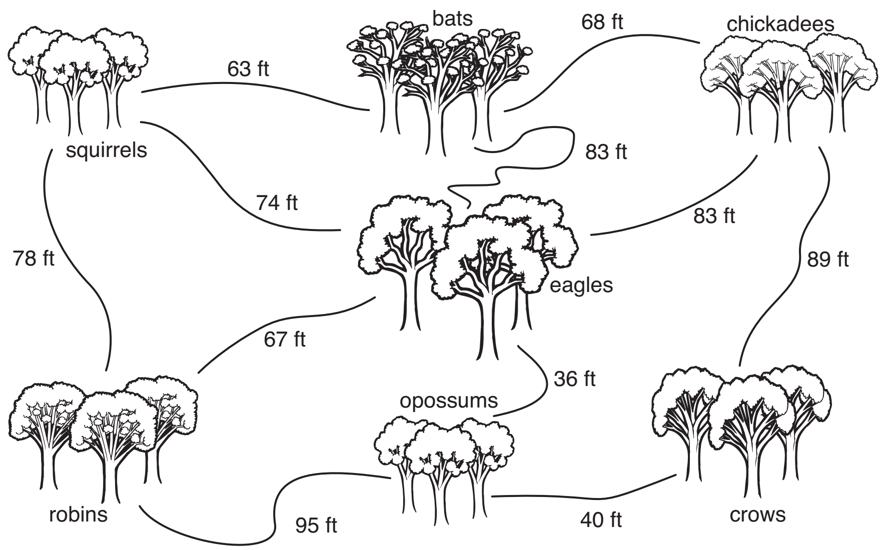
Drive

3 squares west
4 squares south
2 squares east
2 squares north
2 squares east
1 square north



16.3 ANIMAL TREES

When exploring the different ways to travel from tree to tree, each path can be walked along only once.



1. How far are the chickadees' trees from the opossums' trees?

2. Which way is the shortest way from the robins' trees to the bats' trees?

3. How many ways can you go from the squirrels' trees to the crows' trees?

4. Which way is the shortest way from the squirrels' trees to the opossums' trees?

5. Which way is the longest way from the robins' trees to the chickadees' trees?

6. What animals' trees are closest to the eagles' trees?

7. What animals' trees are the farthest from the eagles' trees?

Problem Solving

To use spatial visualization, logical reasoning, and measurement to solve problems

Materials

- Calculator
- Digital clock

NCTM Standards

- Number 1.1, 1.2
- Geometry 3.2, 3.4
- Problem Solving

Focus

These pages explore different ways of visualizing a problem and the various possibilities that may lead to a solution. Logical reasoning, as well as an understanding of metric measurement (meters and centimeters) and digital time (using both 12-hour and 24-hour time), is needed. With each problem, diagrams can be used to organize, sort, and explore the data.

Discussion

Page 87 – Animal Trails

With these problems, students must visualize the paths that the animals take as they travel around the outside of each shape. An ability to convert from centimeters to meters and vice versa is also required.

In the first problem, the snail crawls around the paddock more than once. It passes by corners A, B, and C twice before coming to rest at D after having traveled 315 cm, or 3 m 15 cm.

For the second problem, the addition required to keep track of the centipede's progress is more complex. However, some students may realize that the length of the short and long sides combined is 125 cm and use this to calculate that the distance to reach D a second time is 375 cm. When the centipede travels 85 cm further, it will have traveled 460 cm, or 4 m 60 cm, and stop at C.

The last problem is more difficult, since not all of the lengths are given and some must be calculated first from the information provided on the diagram.

Page 88 – Balance the Books

To solve these problems, students must explore the relationships among the numbers on each pan of the balance and then compare the weight of one pan to the other. Estimating or calculating the sum on each side can be used to determine which number must be subtracted to make the pans balance.

Page 89 – Taking Time

This page explores students' understanding of digital time as they investigate the ways the digits can be placed to show different times and read the times to compare which is earliest and latest. The way in which zero is used on a digital clock must also be considered. In Problem 2, there are only four possibilities (since there cannot be 90 or 95 minutes), but 0 can be used to show the hour after midnight (if using 24-hour time). In Problem 3, there are more possibilities when 24-hour time is considered, and 0 can be used in all possible positions. The final question requires interpretation of the possible times. Discuss 24-hour time—for example, Why does a new day begin in the middle of the night?

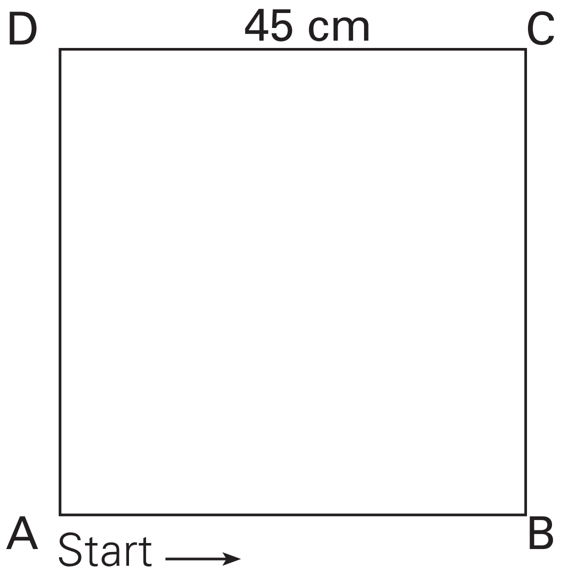
Possible Difficulties

- Unable to convert centimeters to meters and centimeters
- Confusion about moving along a side more than once
- Not calculating the sides whose length is not given
- Not understanding 24-hour time

Extension

- Students could write their own problems involving distance around a shape (perimeter), numbers on a balance, or time, and give them to other students to solve.

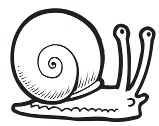
17.1 ANIMAL TRAILS



1. A snail crawls 3 m 15 cm around a square garden.

   At which corner will it stop?

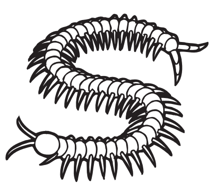
   ____

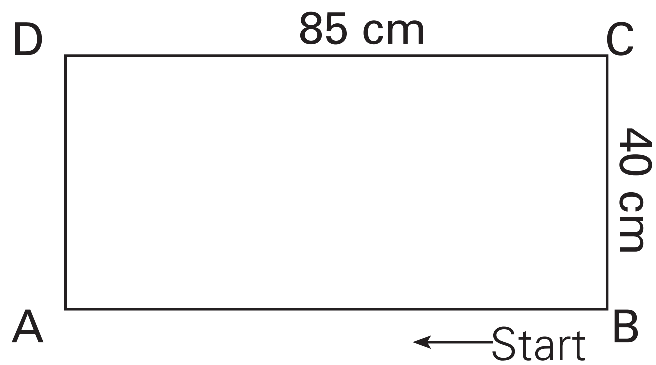


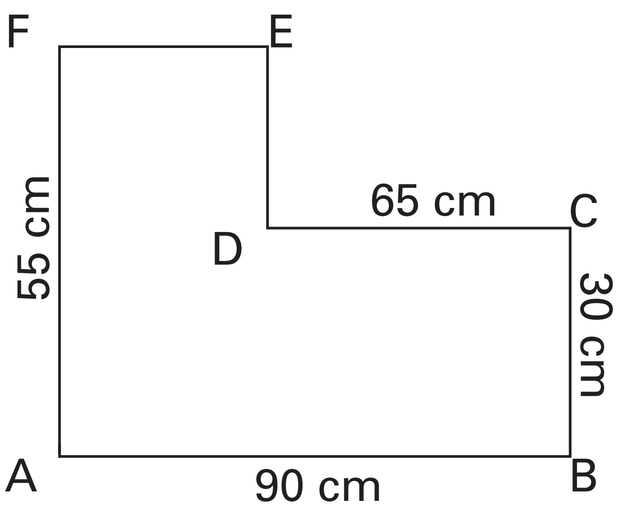
2. A centipede shuffles 4 m 60 cm around a rectangular garden.

   At which corner will it stop?

   ____



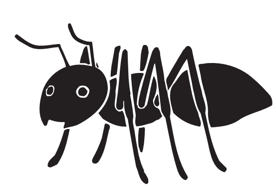




3. An ant walked clockwise around this garden. Starting at A, it stopped the third time it reached D.

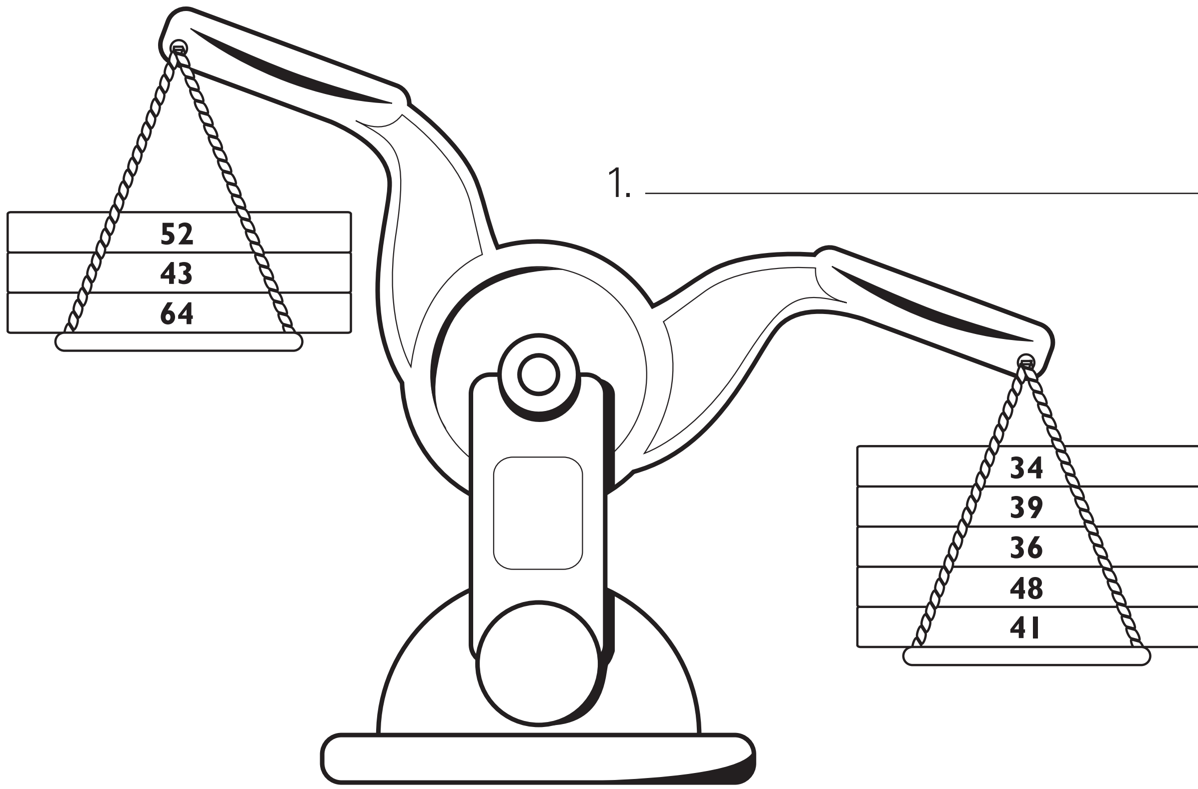
   How far did it walk?

   ____



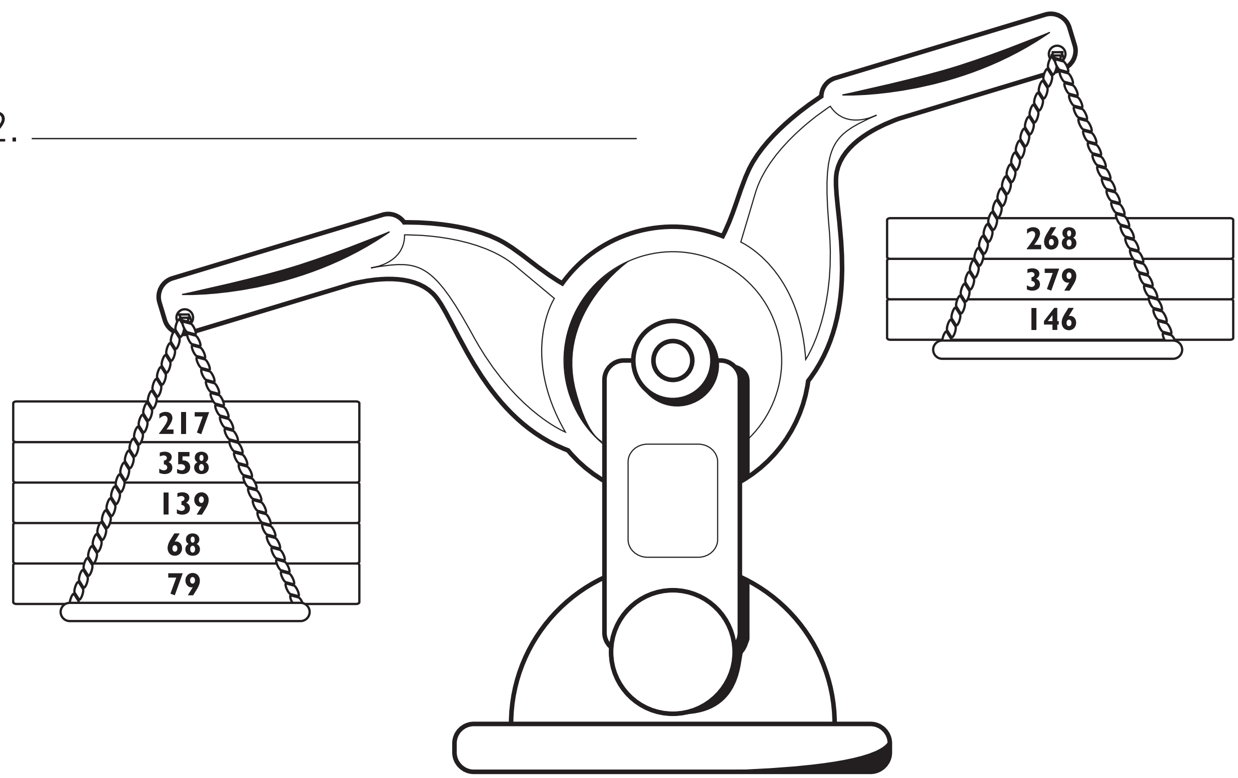
17.2 BALANCE THE BOOKS

Which book would you take off the scale to make it balance?



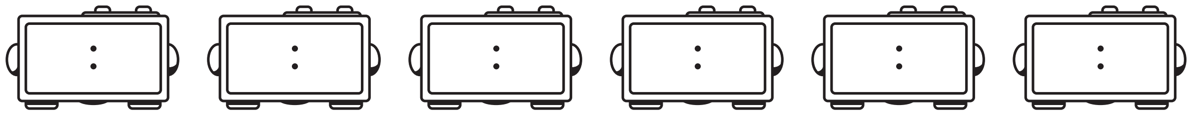
1. ______________________

2. ______________________



17.3 TAKING TIME

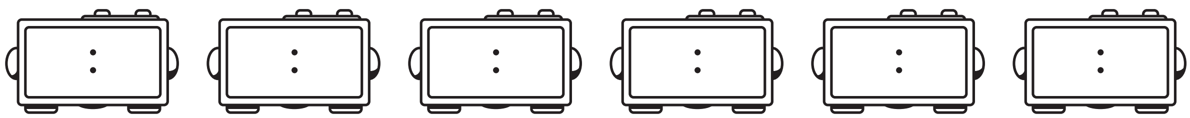
1. (a) A digital clock shows hours and minutes. What times can be shown using only the digits 3, 4, and 5?



(b) What is the latest time? ____________

(c) What is the earliest time? ____________

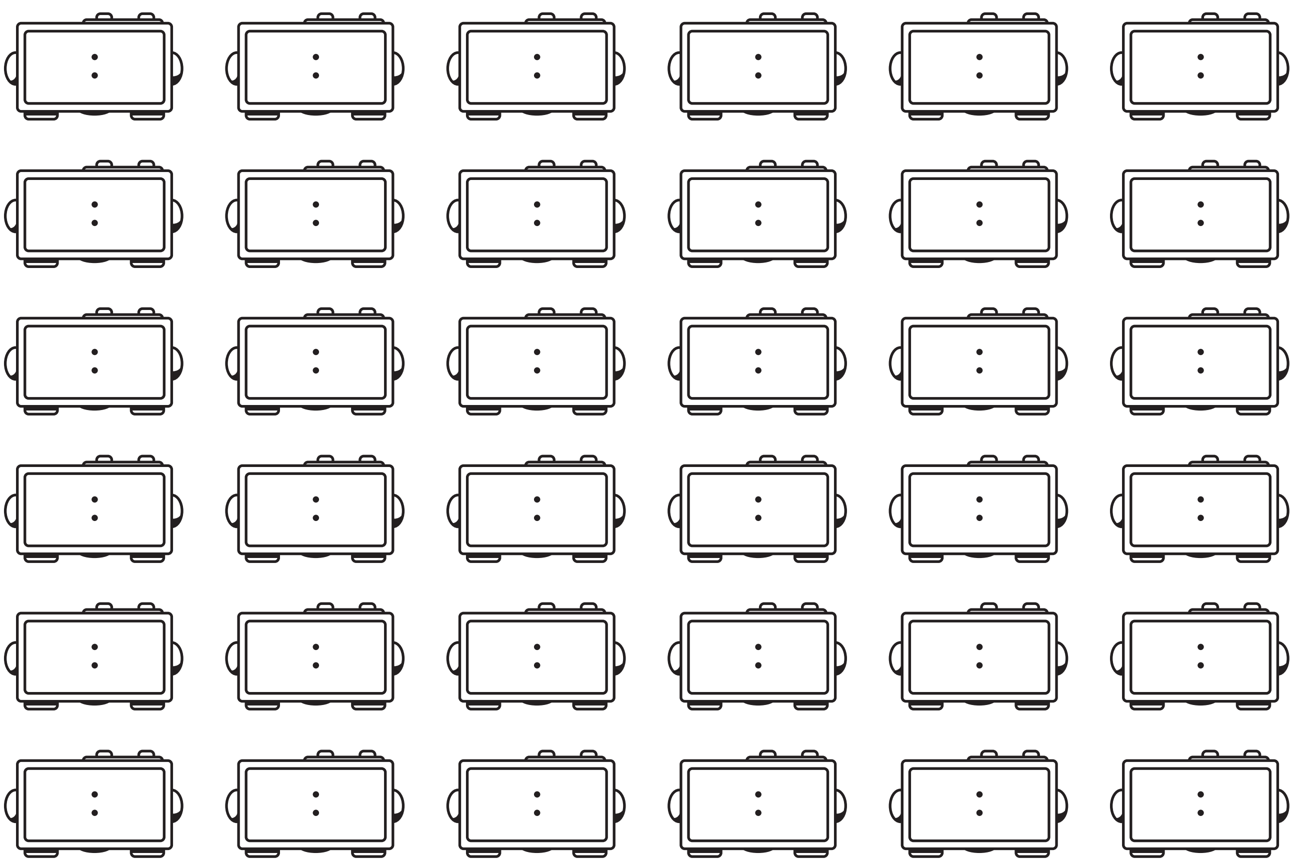
2. (a) What times can be shown using only the digits 0, 5, and 9?



(b) What is the latest time? ____________

(c) What is the earliest time? ____________

3. (a) What would the times be if you used 0, 1, 2, and 3?



(b) What are the earliest and latest times? ____________ ____________

Problem Solving

To identify and use information in a problem

Materials

- Scissors
- Glue

NCTM Standards

- Number 1.1
- Data Analysis and Probability 4.1
- Problem Solving

Focus

This page explores the reading and interpretation of information to solve problems involving numeration. No addition or subtraction is needed. Students analyze the problem to locate the required information, decide what information is not needed, and then use comparison, rather than addition or subtraction, to obtain solutions.

Discussion

Page 91 – Animal Pets

The information in the graph is analyzed to sort out which of the entries on the graph belongs to which animal. Understanding comparison enables students to work out which label goes with the largest number (dogs) and smallest number (fish).

The statement that the number of cats added to the number of fish is the same as the number of dogs is not needed, but it does show whether or not the columns have been labeled correctly. This leaves only guinea pigs and parakeets. There must be seven guinea pigs and six parakeets to match the criteria of there being one fewer parakeet than there are guinea pigs.

Possible Difficulties

- Unable to determine which information to use first when graphing

Extension

- Write other graph problems for the students to solve.

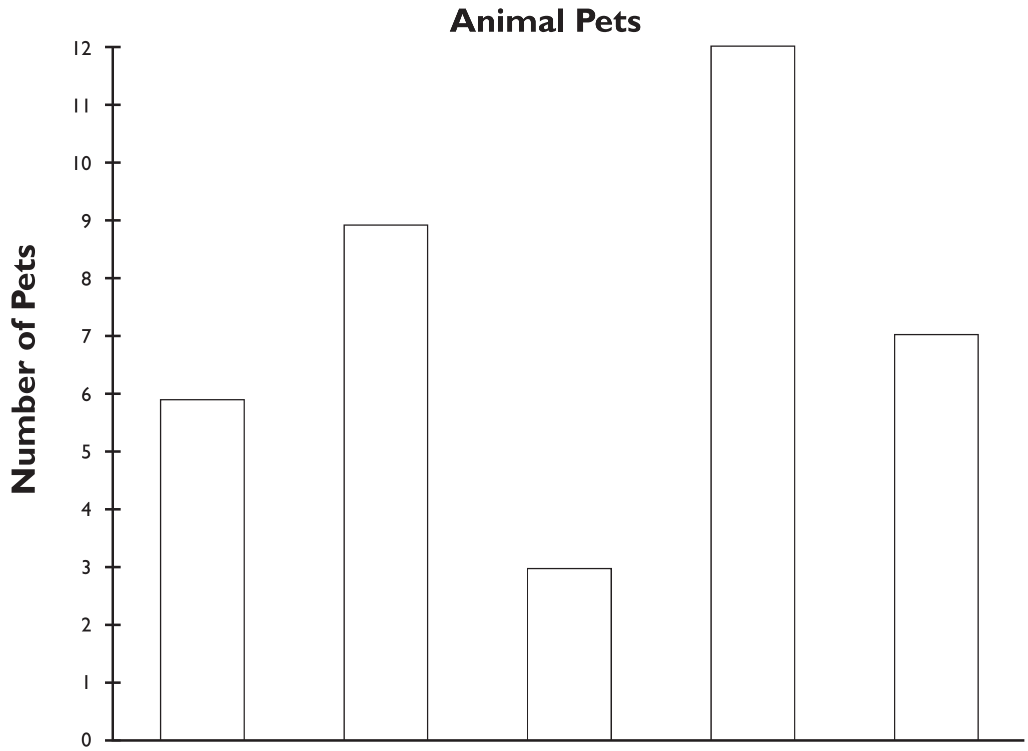
18.1 ANIMAL PETS

Sonia's class has collected information about pets.

Class members keep five different kinds of animals as pets. Below is a graph showing how many there are of each pet.

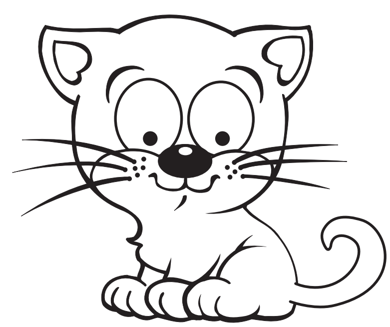
The names of the types of animals have not been put on the graph.

Cut out the animals' names and glue them onto the graph. Use the information below to help you.



Clues:

- *There are 3 fewer cats than dogs.*
- *The smallest number of pets kept are fish.*
- *The number of cats added to the number of fish is the same as the number of dogs.*
- *Most students have dogs as pets.*
- *There is one fewer parakeet than there are guinea pigs.*



cats	fish	parakeets	guinea pigs	dogs

Problem Solving

To use strategic thinking to solve problems

Materials

- 12 counters (1 one color, 11 another color)

NCTM Standards

- Number 1.1
- Data Analysis 4.1
- Problem Solving

Focus

This page introduces a more complex problem in which the most difficult step is to try to find a way of understanding what the question is asking. Using materials to explore the problem is one way this can be done. Another is to use a diagram to assist in "thinking backwards" by trying and adjusting possible answers until a solution that matches all of the conditions is found.

Discussion

Page 93 – The Big Race

The problems can be solved in several ways. One is to work backwards from the final position, reasoning that the reverse of each condition must be performed—for example, that Gina finishes in fourth position and must pass three cars and then be passed by seven cars to get back to her original position.

Some students may need counters to model the process of cars passing and being passed. Students could use the 12 counters, with one counter representing Gina's car and the other counters (of a second color) representing the other 11 cars. Other students may prefer to base their solution on a diagram that shows what has happened.

Also, rather than working backwards, some students may prefer to work forwards, choosing a position for Gina and working through each event in the race. If Gina does not end up in fourth place, an adjustment will need to be made to the original position chosen. In this way, a process of "try and adjust" can lead to the correct starting position. Note that the positive expression "try and adjust" is much more helpful than the often used "guess and check."

The second problem extends the concept; however, this time the initial position is given and the question is reversed so that students are asked for the number of cars that Jordan must pass to take the winning position.

Possible Difficulties

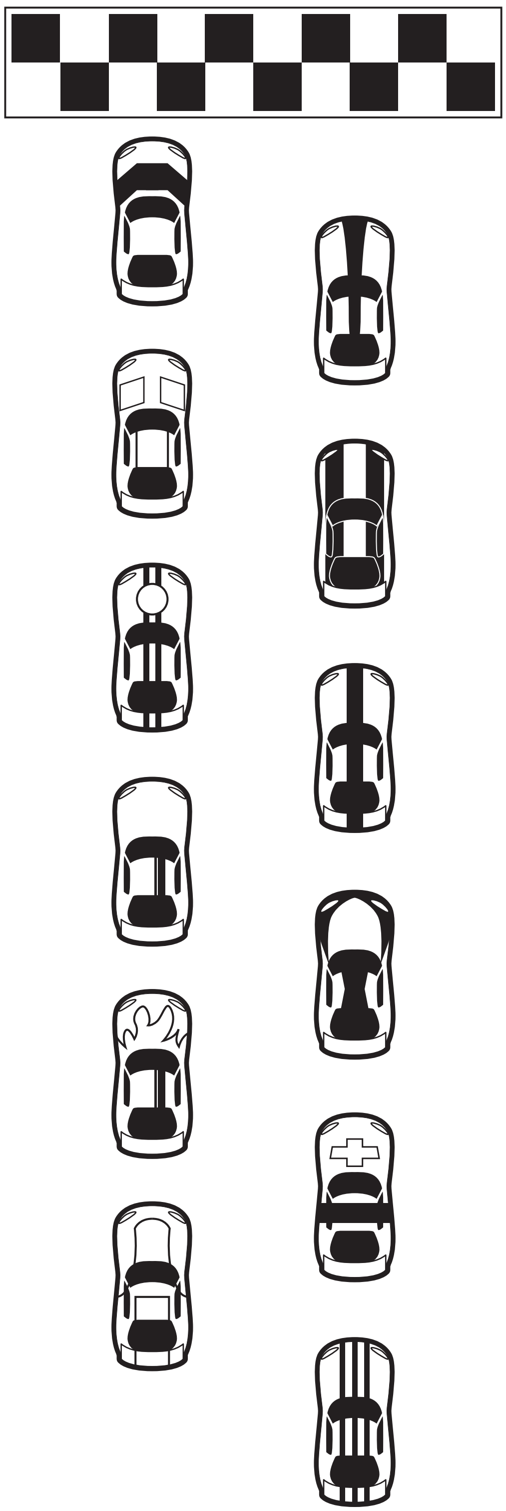
- Using only the seven cars that Gina passed to determine her starting position
- Using only the three cars that passed Gina to determine her starting position
- Thinking only of the three cars between Jordan's starting position and first place
- Thinking that Jordan must pass 10 cars, when being in tenth position means he has to pass nine cars to win

Extension

- Students write their own car race problems based on these questions.
- Use a different context rather than car racing for the stories.

19.1 THE BIG RACE

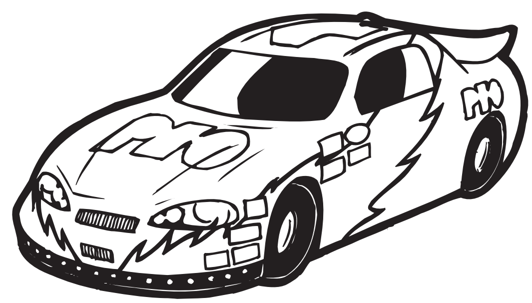
At the start of a car race, the cars line up in their starting positions.



1. Gina is driving one of the 12 cars in the race. During the race, she passes 7 cars before being passed by 3 cars. She finishes the race in fourth place.

   In what position did Gina start the race?

   ____



2. Jordan started in fourth place. During the race, he was passed by 6 cars.

   How many cars does he need to pass to win the race?

   ____

Problem Solving

To use strategic thinking to solve problems

Materials

- 0–99 number board

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

NCTM Standards

- Number and Operations 1.1
- Algebra 2.1
- Problem Solving

Focus

This page explores students' understanding of the number system and their ability to solve questions about numbers. Students must coordinate the reading and writing of numerals with the symbols involved in writing the numbers 0–99.

Discussion

Page 95 – How Many Digits?

These investigations explore students' ability to solve questions about the number system and to keep track of the possible answers they find. In coming to terms with the question, they must discuss what it means to *say* a digit as opposed to *writing* it. As the problems progress to exploring other numbers, there will be further aspects to consider, such as how "3" or "5" is read in some numbers and how the "four" in "fourteen" and "forty" sound the same even though written differently.

For the first question, students must realize that "one" is said nine times from 0–99 in the ones place for 1, 21, 31… but not for "eleven." There will be another nine of the same form from 100 to 199, together with the 100 times "one" is said with each "one hundred and …." "One" is said a total of 118 times. If students need help in organizing their solution, they can write or be given a 0–99 board that shows how place value uses tens and ones for two-digit numbers. However, they should be left to explore their own ways of coming to terms with the problem and determining solutions, rather than simply replacing it with an exercise in counting every "1" on the 0–99 board. The digit "1" is written 140 times. Careful consideration must be given to 11 and 111 to see this.

For the second question, "two" is also said nine times from 0–99, with another nine times from 100–199 and another for "200." From 1 to 200, the digit "2" is written 41 times.

If students take the problem further and try other one-digit numbers, as suggested, they will find that "three" has a similar pattern as "two." However, there is a different pattern altogether when saying "four." For this to occur, you have to include the pronunciation of the word part "four" within "fourteen," "forty," "forty-one," and so on. The pattern repeats for the digits 6 to 9 (with the pronunciation, for example, of "six" in "sixty"). When students notice this, they will have truly come to terms with the strategic thinking needed to organize and solve problems with several interacting conditions.

Possible Difficulties

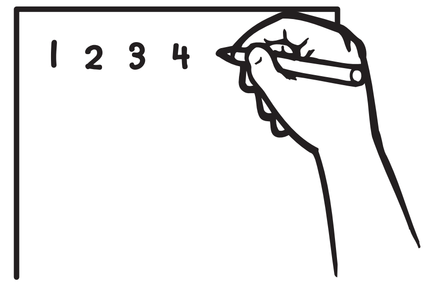
- Unable to keep track of the number of times a digit or word occurs
- Confusion between saying and writing the digits
- Confusion about 11 and 22, and seeing only the digits "1" and "2" once when they actually occur in both the ones and tens places

Extension

- Make a table for the students to display their results and present a description of the problems and their solutions to another class or group.

20.1 HOW MANY DIGITS?

1. (a) When you count from one to two hundred, how many times do you say “one”?

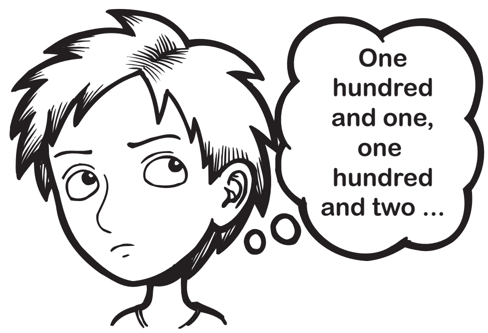


____

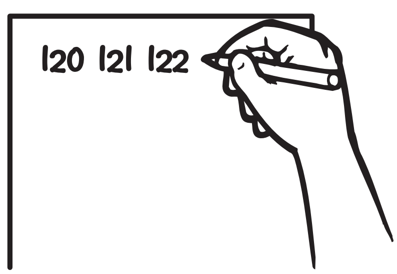
(b) How many times would you write “1” if you were to write all of the numbers from 1 to 200?

____

2. (a) When counting from one to two hundred, do you think you would say “two” more, the same, or less times than you would say “one”?



____



(b) How many times would you say “two”?

____

(c) How many times would you write “2” when writing the numbers from 1 to 200?

____

3. Try other one-digit numbers. Can you see any patterns?

____

____

____

Problem Solving

To organize data and make predictions

Materials

- Colored pencils
- Counting materials, if needed

NCTM Standards

- Data Analysis and Probability 4.3
- Problem Solving

Focus

These pages explore recording different ways of organizing scoops of ice cream and apples according to various criteria. In each investigation there are more cones or boxes than needed. This requires the students to carefully analyze their solutions and to begin to be able to justify their responses.

Discussion

Page 97 – Ice Cream Cones 1

The scoops can be placed on the cones in six different ways. In this activity eight cones are drawn on the page, and some students may simply repeat a previous combination in order to fill all the cones. The question states that students choose one of each flavor; therefore, each cone needs one chocolate scoop, one vanilla scoop, and one strawberry scoop.

Page 98 – Ice Cream Cones 2

Students may choose two scoops of ice cream from three possibilities. This problem does not state that they must choose different flavors, so there are nine possibilities, since they can have two scoops of the same flavor if they wish. Again, there are more cones on the page than needed, and some students may simply repeat a previous combination in order to fill all the cones.

Page 99 – Apples

This problem is based on a similar idea, exploring the various positions in which an item can be placed in a box. There are six possible ways to position the two apples in the box. Some students might think that because there are two apples, the solution will be the same as the two scoops of ice cream. As with the previous problems, there are more boxes than required.

Possible Difficulties

- Coloring all of the cones or boxes, whether or not they are all needed
- Randomly positioning the combinations of ice cream and apples instead of organizing the data
- Using the same combination more than once

Extension

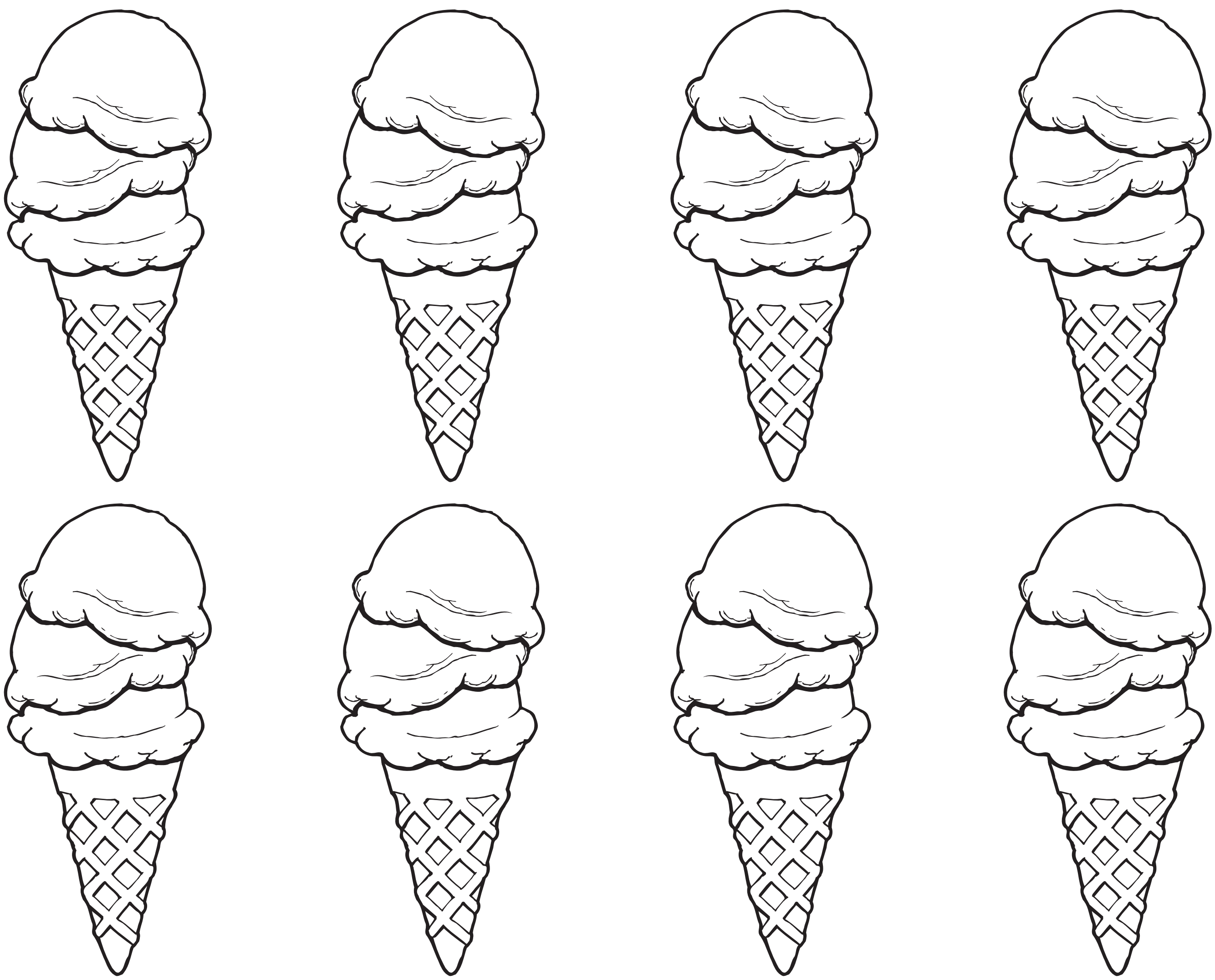
- Revisit the problem on page 97 and explore the possibilities of using any combination of flavors, rather than only one scoop of each flavor—for example, two chocolate and one vanilla.

21.1 ICE CREAM CONES 1

You are allowed 3 scoops of ice cream:

1 chocolate, 1 vanilla, and 1 strawberry.

Color the scoops to show the different ways each ice cream flavor could be placed on the cone.



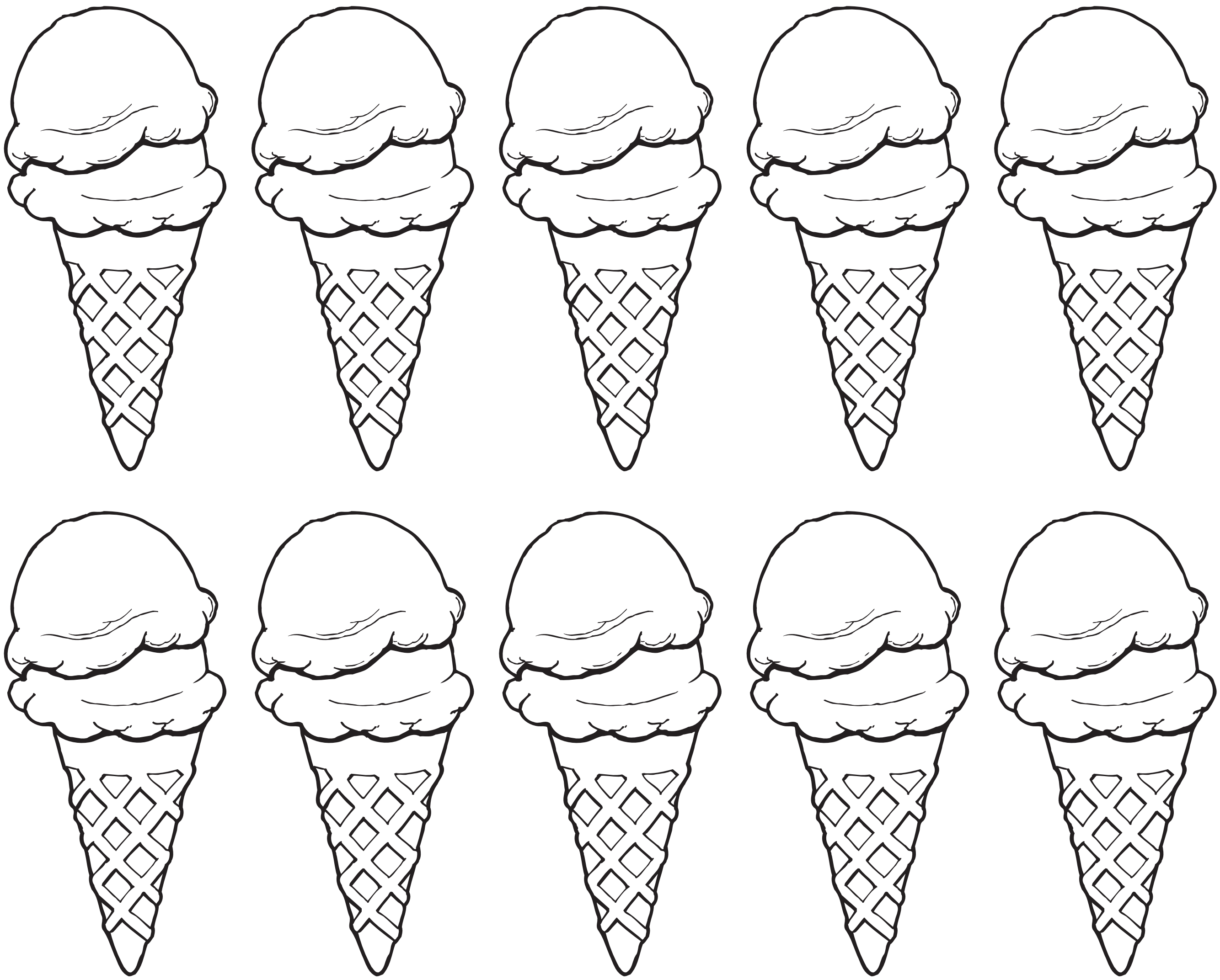
Did you need all of the cones? ____________________

21.2 ICE CREAM CONES 2

You are allowed 2 scoops of ice cream.

You can choose from vanilla, chocolate, and strawberry.

Color the scoops to show the different ways the ice cream flavors could be placed on the cone.

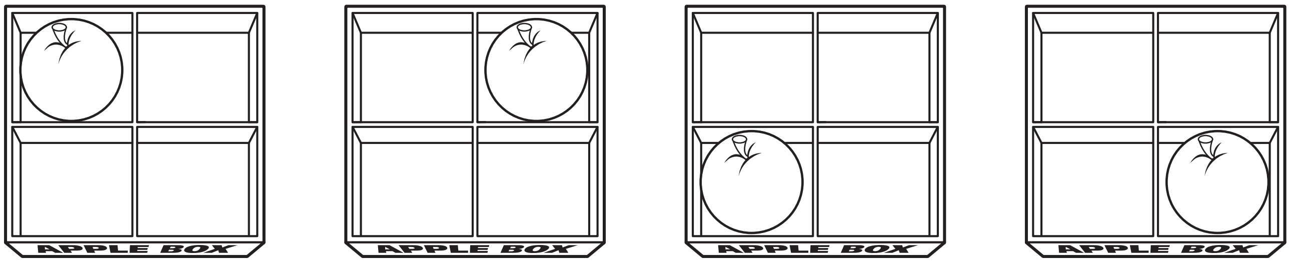


Did you need all of the cones? ____________________

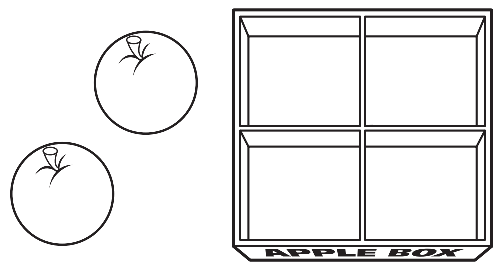
21.3 APPLES

Cathy's box has spaces for 4 apples.

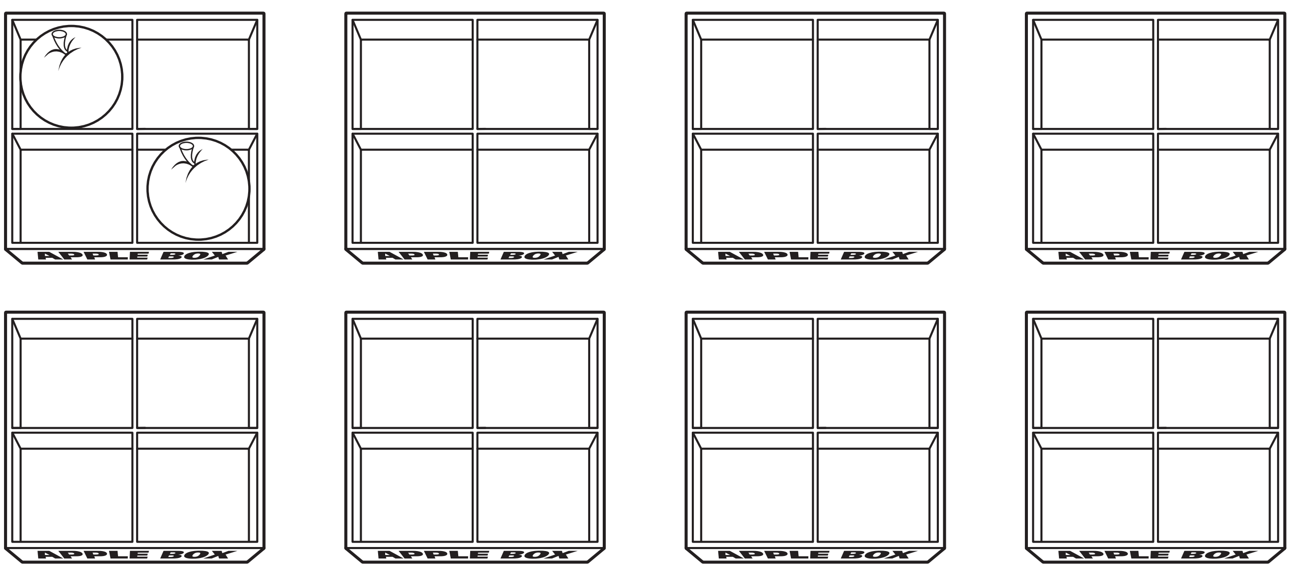
There are 4 ways she can put 1 apple into the box:



Cathy has 2 apples and 1 box.



Draw the different ways she can place the apples in the box. The first one has been done for you.



Did you use all the boxes? ________________

What if there were 3 apples? There are ____________ ways.

Problem Solving

To organize data and make predictions

Materials

- Calculator or counting blocks, if needed

NCTM Standards

- Number and Operations 1.1, 1.2, 1.3
- Algebra 2.1
- Data Analysis and Probability 4.1, 4.3
- Problem Solving

Focus

These pages explore different ways data can be analyzed and recorded. In each situation, diagrams, tables, and lists can be used to organize and sort the data in order to make predictions for further analysis and exploration. Visualization, as well as logical reasoning, are involved in these investigations.

Discussion

Page 101 – How Long?
Analysis of the problems reveals that the distance traveled entails movement back and forth. The first problem involves climbing four feet up and then slipping two feet back, so each day the distance traveled is two feet. As the pipe is 12 feet high, it will take exactly six days to reach the top.

The next problem is similar, but in this case the time will not be exact. As the well is 23 feet deep and the travel distance is three feet per hour, it will take eight hours to reach the top, with not all of the last hour needed. During the last hour, a distance of two feet must to be traveled, not three feet. The last problem entails a distance traveled of 30 meters every one and a half hours on the way up, and 30 meters every hour and 10 minutes on the way down. Again, the time will not be exact.

Page 102 – Pizza Party
Students need to organize the data to calculate the possibilities. This could be done by using a list or a table or even with counting materials. There are three possible thin-base pizzas and three possible thick-base pizzas. The extension to three types of pizza base should enable some students to immediately see that there will again be three thin-base, three thick-base, and three whole-wheat-base pizzas.

Other students may be able to see a pattern emerge. Some students will be able to look at the last question regarding three bases and two toppings and be able to answer it based solely on the previous question, since, again, it will be the same.

Page 103 – Class Work
In this investigation, students look at the diagram of the stories and visualize how many pins are needed if there are 10 stories in two separate rows. Some students may be able to look at the diagram and use that to solve it, while others may need to complete the drawing. The problem is extended in the next activity, in which the rows are now joined together, reducing the number of pins needed.

Possible Difficulties

- Not using a table or list to manage the data
- Not working out how far was really traveled when moving forward and backward
- Not realizing that a ham and cheese pizza is the same as a cheese and ham pizza
- Not remembering that the students' stories can be overlapped to save pins

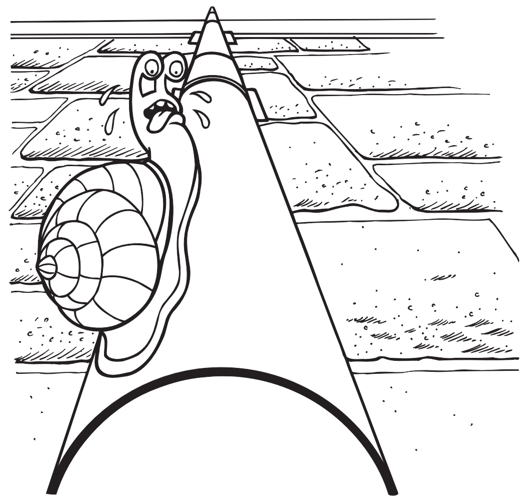
Extension

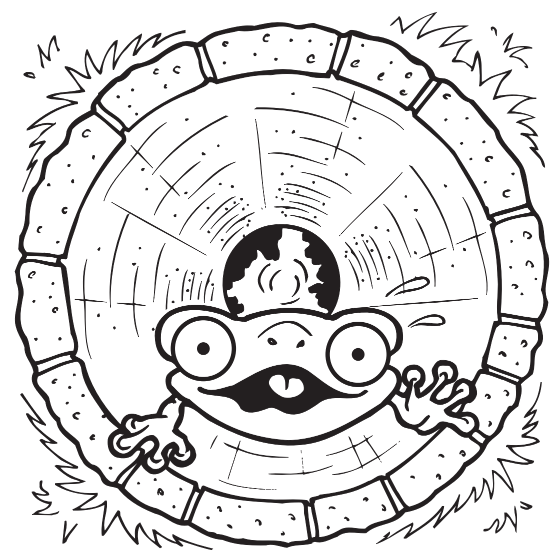
- Make a table showing how far each animal traveled per day/hour.
- Explore what would happen if you used three/four types of bases and three/four toppings.
- Explore different arrangements of artwork—two rows with six paintings each and four rows with three paintings each.

22.1 HOW LONG?

1. A drainpipe is 12 feet long. Each day, a snail climbs 4 feet up the drainpipe but then slips back 2 feet overnight. How long will it take for the snail to reach the top of the drainpipe?

____





2. A green frog is at the bottom of a well. Each hour, he climbs 5 feet but then slips back 2 feet. If the well is 23 feet deep, how long will it take the frog to get out of the well?

____

3. A tree snake climbs up and down a tree to its nest. Each hour it climbs up a distance of 30 feet and then rests for 30 minutes. If the tree is 140 feet high, how long will it take the snake to reach its nest?

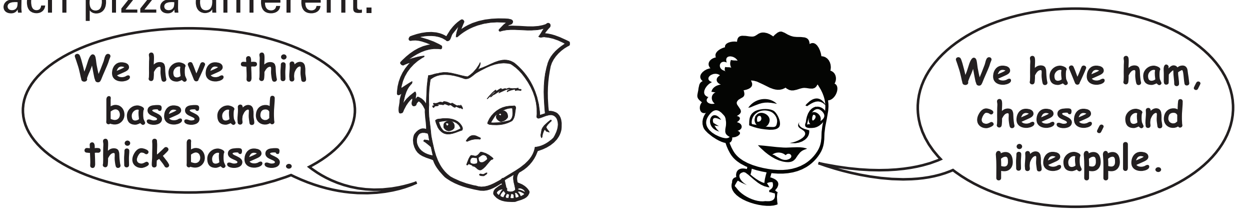
____

When climbing down the tree from the nest, it slides 30 feet every hour and then rests for 10 minutes. How long will it take for the snake to reach the ground?

____

22.2 PIZZA PARTY

Alex and Nicky are making pizzas for a party. They decide to make each pizza different.



1. Write all of the different pizzas they can make if they only use 1 topping per pizza.

THIN BASE	THICK BASE
____________	____________
____________	____________
____________	____________
____________	____________

2. How many pizzas can they make? ____________
3. What happens if they use 2 toppings per pizza?

more	less	same

4. Write all of the different ways they can make pizzas using 2 toppings.

THIN BASE	THICK BASE
____________	____________
____________	____________
____________	____________
____________	____________

5. What happens if they also have whole-wheat-pizza bases?

____

6. How many pizzas can they now make using only 1 topping?

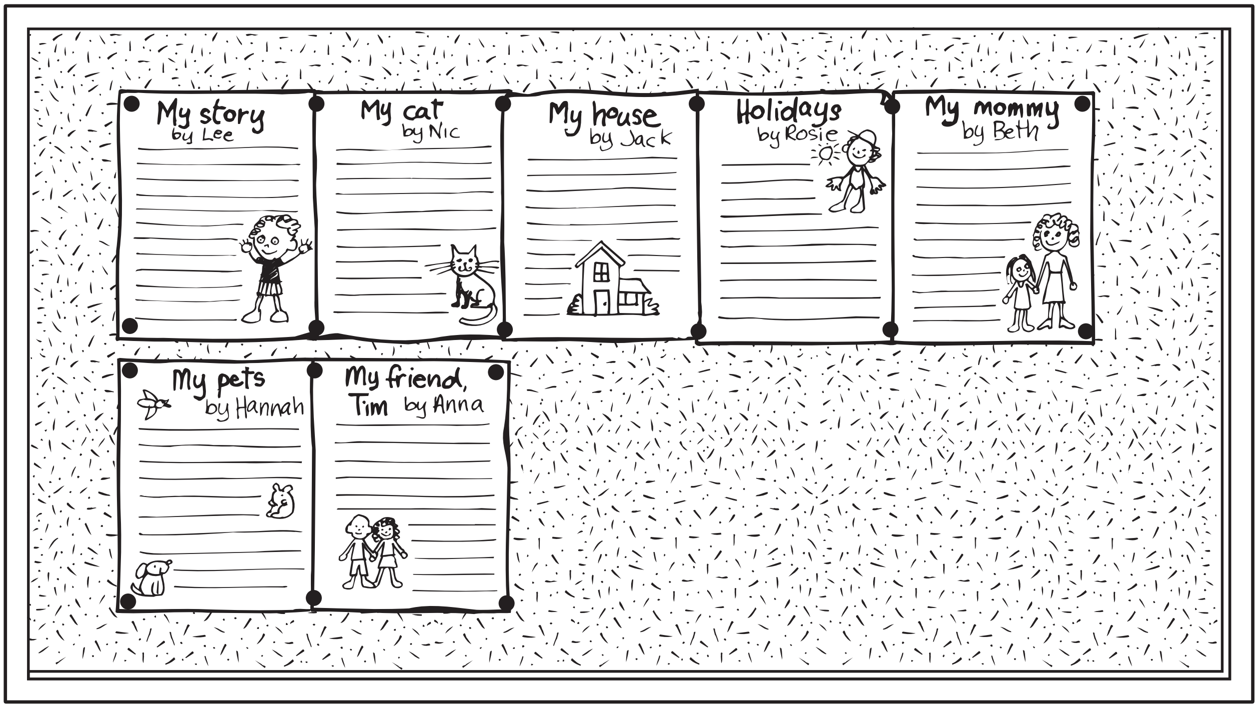
____

7. How many pizzas can they now make using all 3 toppings?

____

22.3 CLASS WORK

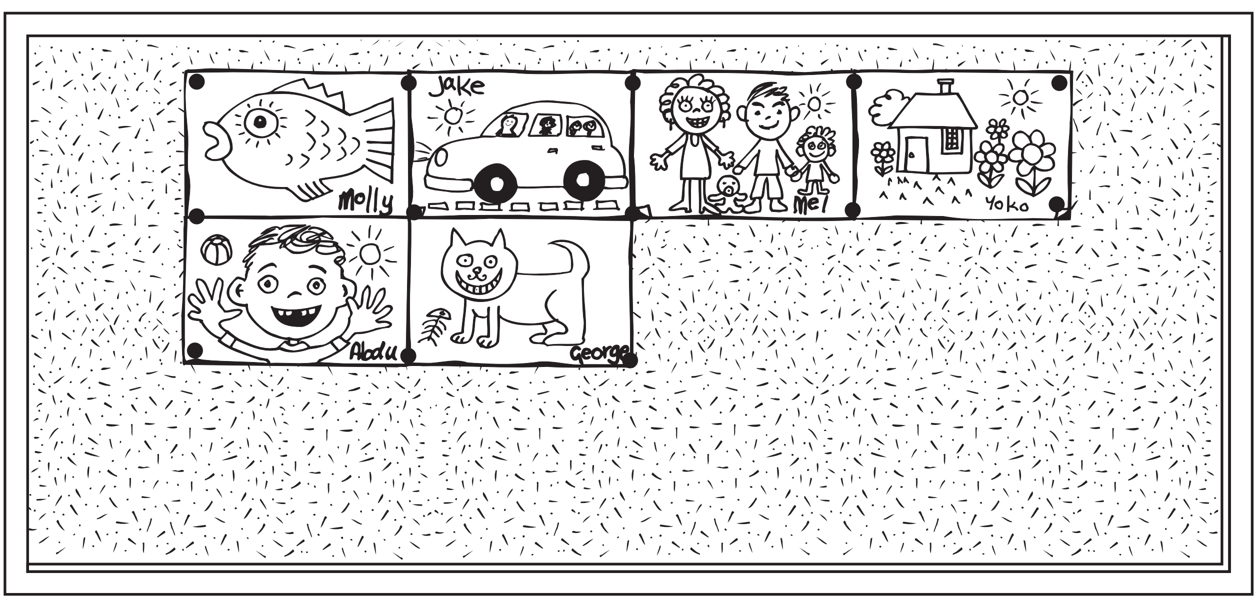
Rick is helping his teacher pin up some stories.



1. He is putting up 10 stories. How many pins does he need? ________

Melinda is helping her teacher pin up the class's artwork. She has 12 paintings that she is going to pin up in 3 rows, with 4 paintings in each row.

2. How many pins does she need? ________



Problem Solving
To organize data and make predictions

Materials
Counters or a calculator, if needed

NCTM Standards
- Data Analysis and Probability 4.2, 4.3, 4.4
- Problem Solving

Focus
This page explores different ways data can be analyzed and recorded. In each situation, diagrams, tables, and lists can be used to organize and sort the data to make predictions for further analysis and exploration. Visualization and logical reasoning are involved in these investigations.

Discussion
Page 105 – At the Mall
Using a table and listing possibilities while using the "try and adjust" strategy is a popular way of finding solutions to these types of problems. This involves trying a combination and putting it in a table and then adjusting accordingly. The first example has two possible combinations, since it involves multiples of six and 10. This results in 10 bags of six and three bags of 10, or five bags of six and six bags of 10, to make a total of 90 oranges.

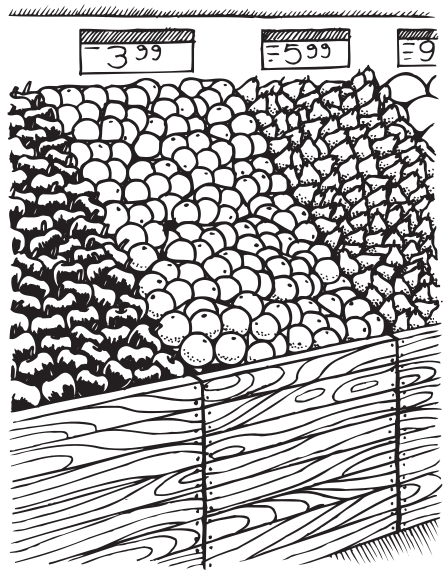
Since Question 2 involves multiples of two and four, there are many different combinations that work—for example, two puppies and 10 parakeets or three puppies and eight parakeets. At first glance the last question has two possible answers. It involves multiples of five and 12, which leads to two boxes of 12 and 15 boxes of five, or seven boxes of 12 and three boxes of five. However, when the information of "more than 10 boxes" is taken into consideration, there is only one possibility, since the second example uses only 10 boxes.

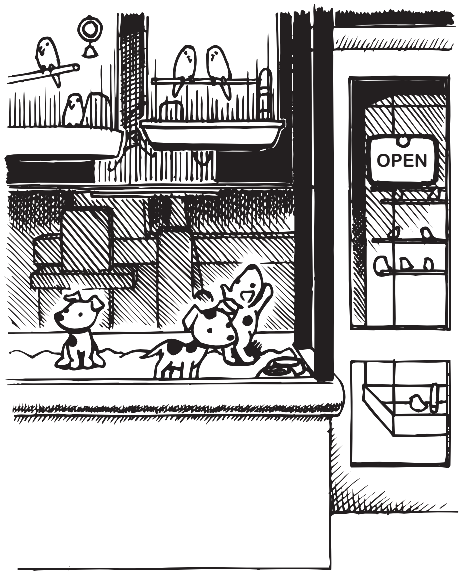
Possible Difficulties
- Not using a table or list to manage the data
- Not considering that more than 10 boxes were used

Extension
- Draw and label a diagram to show the combinations of how many oranges, and so forth.

23.1 AT THE MALL

1. A fruit store has 90 oranges available to sell. Some are packed in bags of 6 and others in bags of 10. How many are there of each size bag?





2. The pet shop has parakeets and puppies on display in the window. Combined, the owner counted a total of 28 legs on the pets. How many puppies and how many parakeets could there be?

3. The chocolate shop packages chocolates in both a small box that holds 5 chocolates and a larger box that holds 12 chocolates. The shop assistant packed 99 chocolates and used more than 10 boxes. How many of each size box did he use?

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

PATTERNS .... page 21

1. (a) blue (b) red
2. Teacher check

ORDINAL PATTERNS .... page 22

Answers will vary

CAR RACE .... page 23

1. 1st – red 2nd – yellow 3rd – green
   4th – blue 5th – orange 6th – black
2. Teacher check

GUESS MY NUMBER .... page 25

1. 73
2. 34

WHAT'S MY NUMBER? .... page 26

1. 739
2. 536
3. Teacher check

WHAT'S MY AGE? .... page 27

1. 7
2. 16
3. 5
4. 6
5. 6
6. 12
7. 15

MISSING NUMBERS .... page 29

1. (a) 3
   (b) 12
   (c) 7
2. Answers will vary.
3. Answers will vary.

MAKE YOUR OWN STORY .... page 30

1. Answers will vary.
2. Answers will vary.
3. Teacher check

MORE MISSING NUMBERS .... page 31

1. (a) 8 (b) 4
   (c) 5 (d) 43
   (e) 10
2.

	Joseph	Jacob	Jonathan
Age	12	6	9
Grade Level	7	1	4

3. Teacher check

HOW MANY? 1 .... page 33

1. Anne
2. 42
3. 58
4. Sunday
5. Jane

TRADING CARDS .... page 34

1. Julie
2.

Mark	John	Carla
53	43	72

3. Carla
4. John

MAGIC SQUARES 1 .... page 35

1. (a)

6	**1**	8
7	5	3
2	9	**4**

Magic number: 15

(b)

8	1	6
3	**5**	7
4	**9**	2

Magic number: 15

IN THE GARDEN .... page 37

1. 29 leaves
2. 24 kookaburras
3. 65 cows
4. 86 mangoes

HOW MANY? 2 .... page 38

1. 7 books
2. 10 shirts
3. 48 people
4. 14 feathers

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

HOW MANY? 3 .... page 39

1. 32 ducks
2. 23 black goats
3. 14 guinea pigs
4. 23 kangaroos

WORD PROBLEMS 1 .... page 41

1. 9 jelly beans
2. 16 cows
3. 19 kangaroos
4. 6 people
5. 6 goats

WORD PROBLEMS 2 .... page 42

1. 7 children
2. 24 stickers
3. 17 fish
4. 18 bikes

ROLAND'S ROSES/ANGELA'S PLANTS .... page 43

1. (a) 4 roses
   (b) 7 roses
   (c) 5 days (but can only pick 1 rose on day 5)
   (d) 7 roses
   (e) 1 rose
2. (a) 6 pots
   (b)

Inside	Outside
1	7
2	6
3	5
4	4
5	3
6	2
7	1

AT THE FRUIT SHOP .... page 45

1. 181 boxes
2. 36 bags of oranges
3. 10 trays
4. 55 bags

KOALA CORNER .... page 46

1. 11 koalas are not sleeping
2. addition (do not answer)
3. 26 koalas
4. 29 koalas do not have a baby on their back
5. addition (do not answer)

BIRD AVIARY .... page 47

1. 75 birds
2. 24 birds
3. 34 birds
4. 28 birds
5. 41 birds

AT THE BEACH .... page 49

1. 34 people
2. 27 people
3. 51 surfers
4. 19 people

AT THE PARTY .... page 50

1. 82 sausage rolls
2. 70 party pies
3. 25 more meatballs
4. (a) 52 soft drinks
   (b) 82 drinks

KANGAROOS .... page 51

1. 128 kangaroos
2. addition (no need to answer)
3. 136 kangaroos
4. 134 kangaroos

TOY ANIMALS .... page 53

1. Yes, they cost $3
2. Yes, they cost $5
3. $8
4. Answers will vary; for example, chicken, cat, sheep.
5. Answers will vary.

AT THE TOY STORE .... page 54

1. Yes. They cost $15.
2. No. They cost $16.
3. $4
4. Answers will vary; for example, truck, car, water pistol.
5. Answers will vary; for example, cricket bat and ball.

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

AT THE DELI .... page 55

1. Yes. They cost $5
2. No. They cost $10
3. $4
4. $4
5. Answers will vary – e.g., 3 egg sandwiches and 3 apple juices.
6. Answers will vary – e.g., 1 chicken roll, 1 chocolate milk.

MAGIC SQUARES 2.. page 57

1. 12
2. (a)

8	1	6
3	5	7
4	9	2

15

(b)

11	4	9
6	8	10
7	12	5

24

(c)

32	4	24
12	20	28
16	36	8

60

(d)

4	9	8
11	7	3
6	5	10

21

MAGIC SQUARES 3 .... page 58

1. (a)

4	3	8
9	5	1
2	7	6

15

(b)

15	1	11
5	9	13
7	17	3

27

(c)

10	3	8
5	7	9
6	11	4

21

2. (a)

5	4	9
10	6	2
3	8	7

18

(b)

16	2	12
6	10	14
8	18	4

30

(c)

11	4	9
6	8	10
7	12	5

24

(d) Yes

3. (a)

7	6	11
12	8	4
5	10	9

24

(b)

18	4	14
8	12	16
10	20	6

36

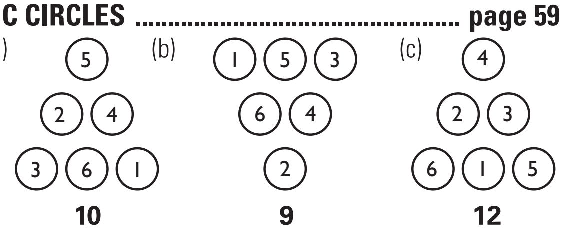
(c)

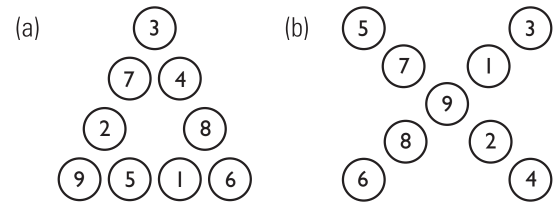
13	6	11
8	10	12
9	14	7

30

(d) Yes

MAGIC CIRCLES .... page 59

1.


2.


(c) Yes

BLOCKS .... page 61

1.

RB	RG	RY
BR	BG	BY
GR	GB	GY
YR	YB	YG

2. 12
3.

RBG	RYB	BRG	GBR	GYR	YBG
RBY	RYG	BRY	GBY	GYB	YBR
RGY	BGR	BYG	GRB	YRB	YGR
RGB	BGY	BYR	GRY	YRG	YGB

4. 24

GRID FUN .... page 62

1. Answers will vary; for example:

R	B	G
B	G	R
G	R	B

2. One diagonal is all the same color and the other is one of each color.
3. Answers will vary; for example:

R	B	G
G	R	B
B	G	R

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

MORE GRID FUN .... page 63

1. Answers will vary; for example:

R	B	G	Y
B	G	Y	R
G	Y	R	B
Y	R	B	G

2. (a) Answers may vary and include: one diagonal is all the same color and the other has only two colors.
   (b) Center square is of two repeated colors.

3. Answers will vary; for example:

R	B	G	Y
B	R	Y	G
G	Y	R	B
Y	G	B	R

WINDOW PANES 1 .... page 65

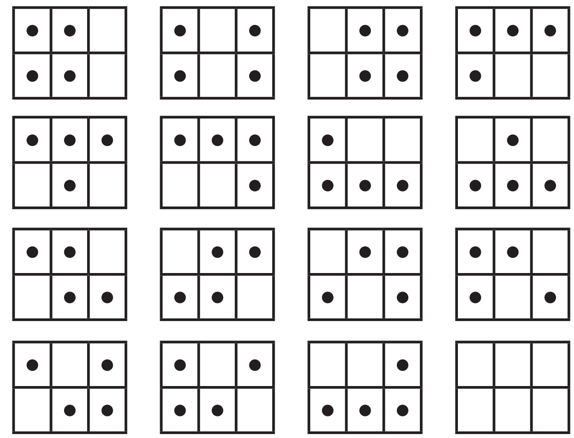
1.

R	G	R	G	R	Y	R	Y	R	B
B	Y	Y	B	B	G	G	B	Y	G
R	B	B	G	B	G	B	Y	B	Y
G	Y	R	Y	Y	R	R	G	G	R
B	R	B	R	G	R	G	B	G	B
G	R	Y	G	Y	B	Y	R	R	Y
G	Y	G	Y	Y	R	Y	B	Y	G
R	B	B	R	G	B	R	G	B	R
Y	R	Y	B	Y	G	G	R		
B	G	G	R	R	B	B	Y		

2. 24

EGG CARTONS 1 .... page 66

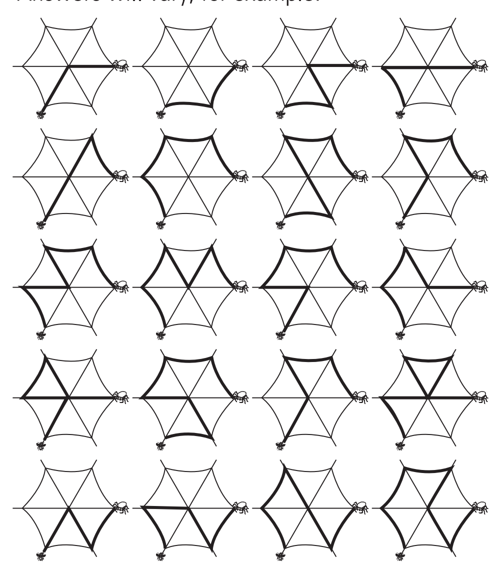
1.



2. No
3. 15

SPIDER WEBS .... page 67

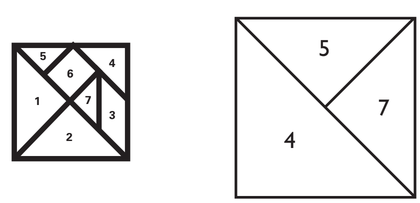
1. Answers will vary; for example:



2. Answers will vary.

TANGRAMS 1 .... page 69

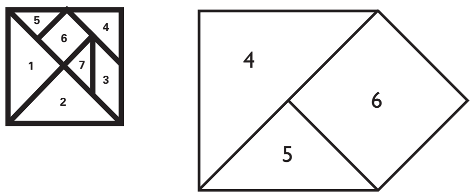
1. (a)



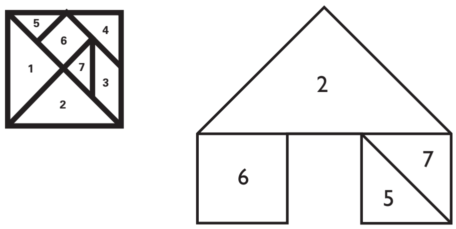
SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

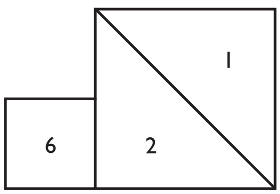
(b)



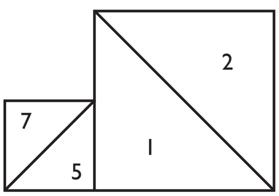
(c)

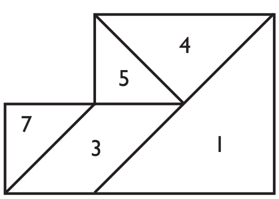


2. As above
3.



4.

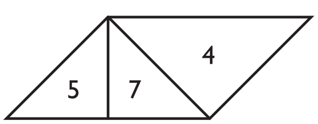




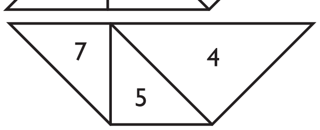
TANGRAMS 2 .... page 70

1. (a)

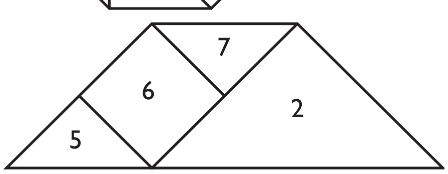
(b)



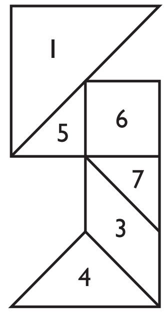
(c)



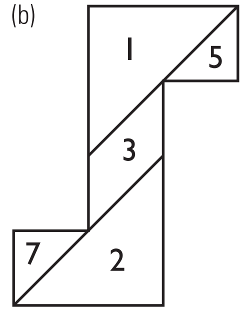
(d)



2. (a)

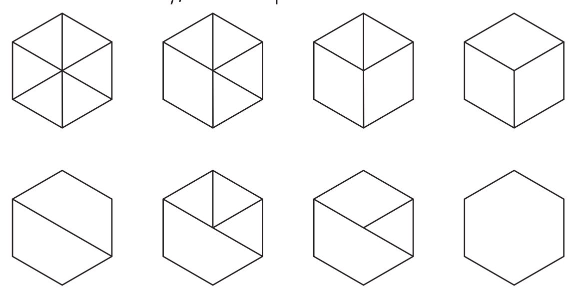


(b)



PATTERN BLOCK SHAPES .... page 71

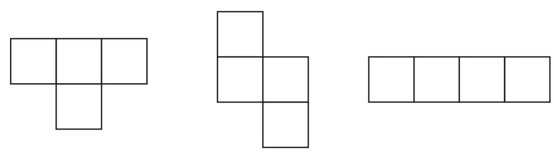
1. Answers will vary; for example:



2. Answers will vary.

USING 4 SQUARES .... page 73

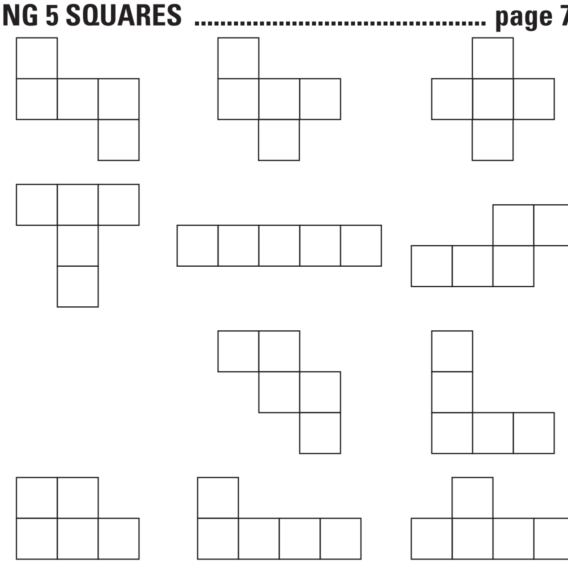
1.



2. 4 (Any other answers would be only one of the original shapes reflected or rotated.)

USING 5 SQUARES .... page 74

1.



SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

2. 11 are possible (Others are only the same shapes rotated or reflected.)

USING RECTANGLES .... page 75

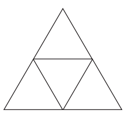
1. b and d
2. Teacher check

GROWING SHAPES 1 .... page 76

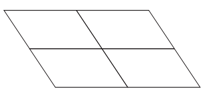
1. Shape 1 – 4
2. Shape 2 – 7
3. Shape 3 – 10
4. 13
5. 16

GROWING SHAPES 2 .... page 77

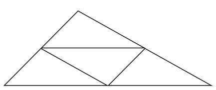
Shape 1



Shape 2

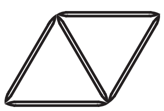


Shape 3



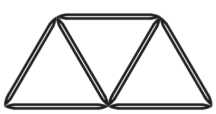
TOOTHPICKS .... page 79

1. (a)

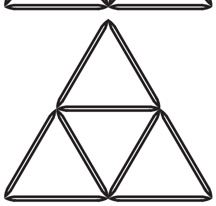


(b) 7

(c)

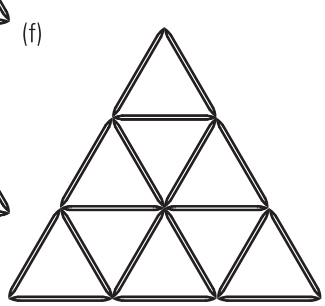


(d)



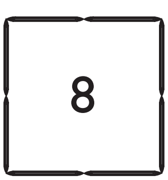
(e) 18

(f)

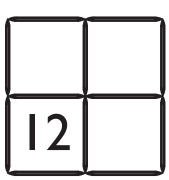


2. (a) 8 or 12

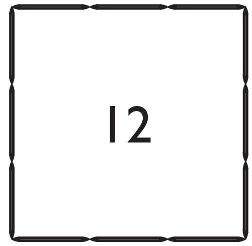
(b)



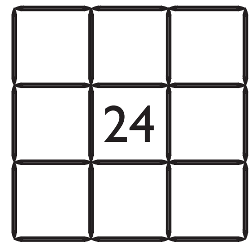
or



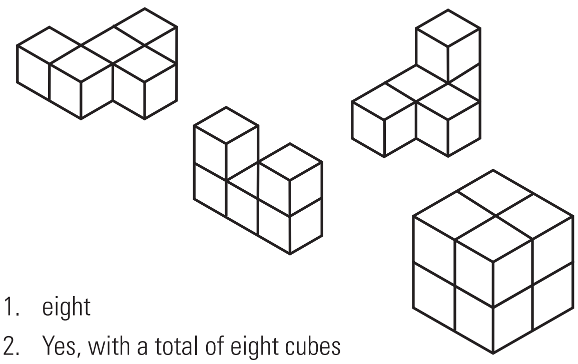
(c)



or



BUILDING WITH CUBES .... page 80



1. eight
2. Yes, with a total of eight cubes
3. four of each color
4. Yes, with a total of 64 cubes
5. 32

STACKING CUBES .... page 81

1. 48
2. 27
3. 29
4. 55

ALICE'S ISLAND .... page 83

1.

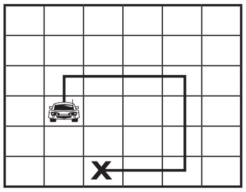
Track	Hrs	Track	Hrs
hut to lake	2	forest to cave	3
hut to cave	5	forest to waterfall	8
hut to forest	6	waterfall to lake	4
hut to waterfall	2	lake to cave	3
waterfall to cave	5		

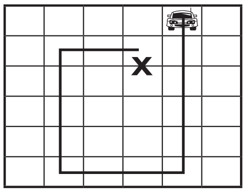
2. 4 hours
3. 9 hours
4. 5 hours
5. 4 hours (shorter route)

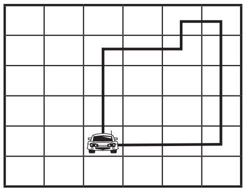
SOLUTIONS

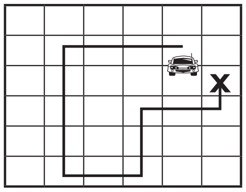
Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

DRIVE TIME .... page 84

1.


2.


3.


4.


ANIMAL TREES .... page 85

1. 119 m (shortest route)
2. robins ➜ squirrels ➜ bats (141 m)
3. At least 10 different ways
4. squirrels ➜ eagles ➜ opossums (110 m)
5. robins ➜ squirrels ➜ bats ➜ eagles ➜ opossums ➜ crows ➜ chickadees (389 m)
6. opossums
7. bats and chickadees

ANIMAL TRAILS .... page 87

1. D   2. C   3. 6 m 85 cm (68 5 cm)

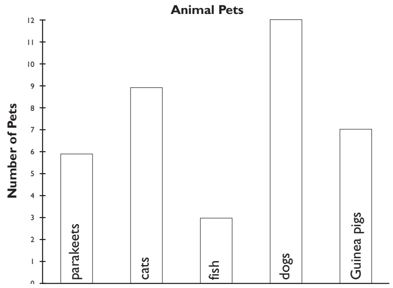
BALANCE THE BOOKS .... page 88

1. 39   2. 68

TAKING TIME .... page 89

1. (a) 3 : 45 3 : 54 4 : 35 4 : 53 5 : 34 5 : 43
   (b) 5.43 (12-hr)
   (c) 3.45 (12-hr)
2. (a) 0 : 59 5 : 09 9 : 05 9 : 50
   (b) 9.50 (12-hr)
   (c) 0.59 (24-hr)
3. (a)
   0 : 12 0 : 13 0 : 21 0 : 23 0 : 31 0 : 32
   1 : 02 1 : 03 1 : 20 1 : 30 1 : 23 1 : 32
   2 : 01 2 : 03 2 : 10 2 : 30 2 : 13 2 : 31
   3 : 01 3 : 02 3 : 10 3 : 20 3 : 12 3 : 21
   10 : 23 10 : 32 12 : 03 12 : 30 13 : 02 13 : 20
   21 : 03 21 : 30 23 : 01 23 : 10 23 : 13 20 : 31
   (b) 0:12 and 23:13.

ANIMAL PETS .... page 91



THE BIG RACE .... page 93

1. 8th position
2. 9 cars

HOW MANY DIGITS? .... page 95

1. (a) 118   (b) 140
2. (a) less   (b) 19   (c) 41
3. 3 and 5 – say 18 times, write 40 times
   4, 6, 7, 8 and 9 – say 40 times, write 40 times

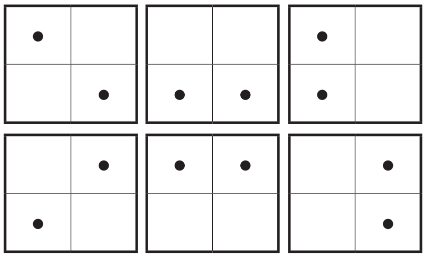
ICE CREAM CONES 1... page 97

1. There are 6 variations
2. No

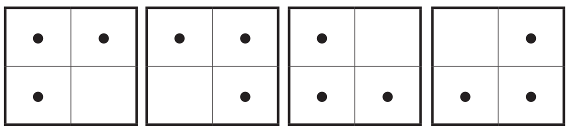
ICE CREAM CONES 2... page 98

1. There are 9 variations
2. No

APPLES .... page 99

1.


2. No, there are only 6 ways.
3. There are 4 ways:



SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

HOW LONG? .... page 101

1. 6 days
2. 8 hours (or 7 and a bit hours)
3. (a) 7 hours (or 6 hours and a bit more)
   (b) 6 hours (or 5 hours and a bit more)

PIZZA PARTY .... page 102

1.

THIN BASE	THICK BASE
ham	ham
cheese	cheese
pineapple	pineapple

2. 6 pizzas
3. same
4.

THIN BASE	THICK BASE
ham and cheese	ham and cheese
ham and pineapple	ham and pineapple
cheese and pineapple	cheese and pineapple

5. It allows for 3 more pizza options.
6. 9 pizzas
7. 3 pizzas

CLASS WORK .... page 103

1. 24 pins
2. 20 pins

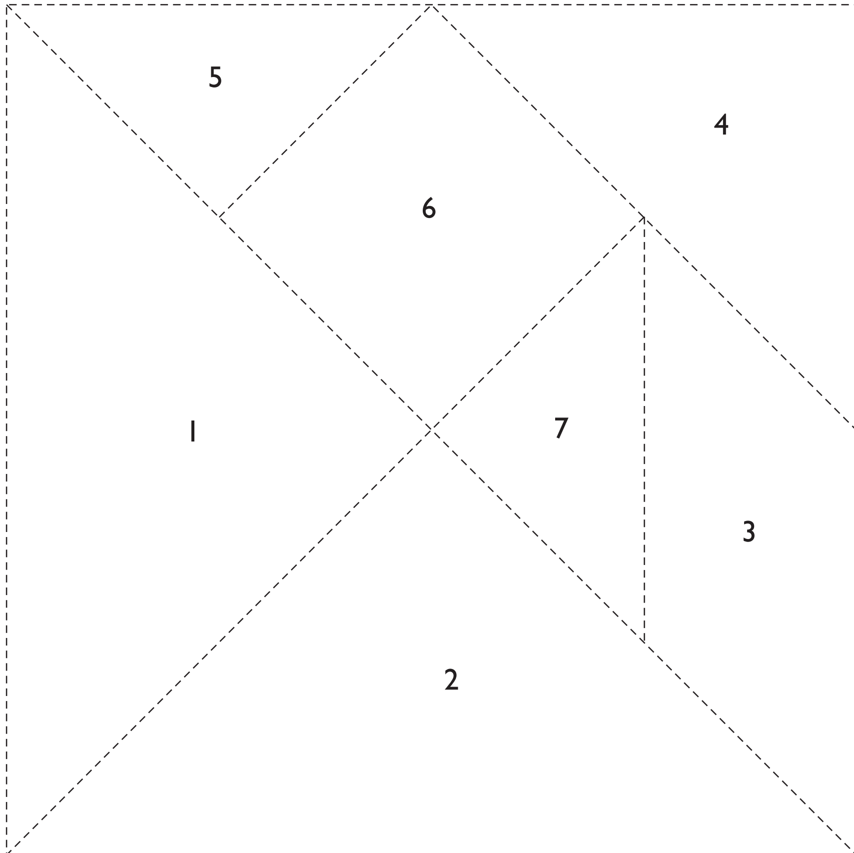
AT THE MALL .... page 105

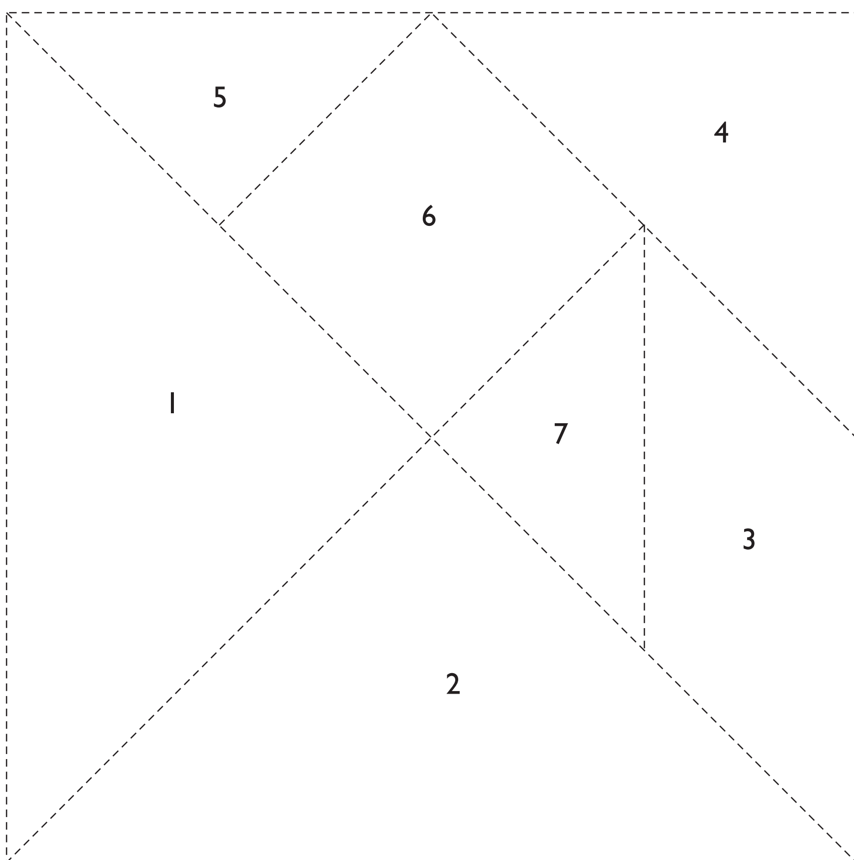
1. 10 bags of six and three bags of 10; or
   five bags of six and six bags of 10
2. Answers will vary. Combinations include:

Puppies–4 legs	2	3	4	5	6
Parakeets–2 legs	10	8	6	4	2

3. two boxes of 12, 15 boxes of five

TANGRAM

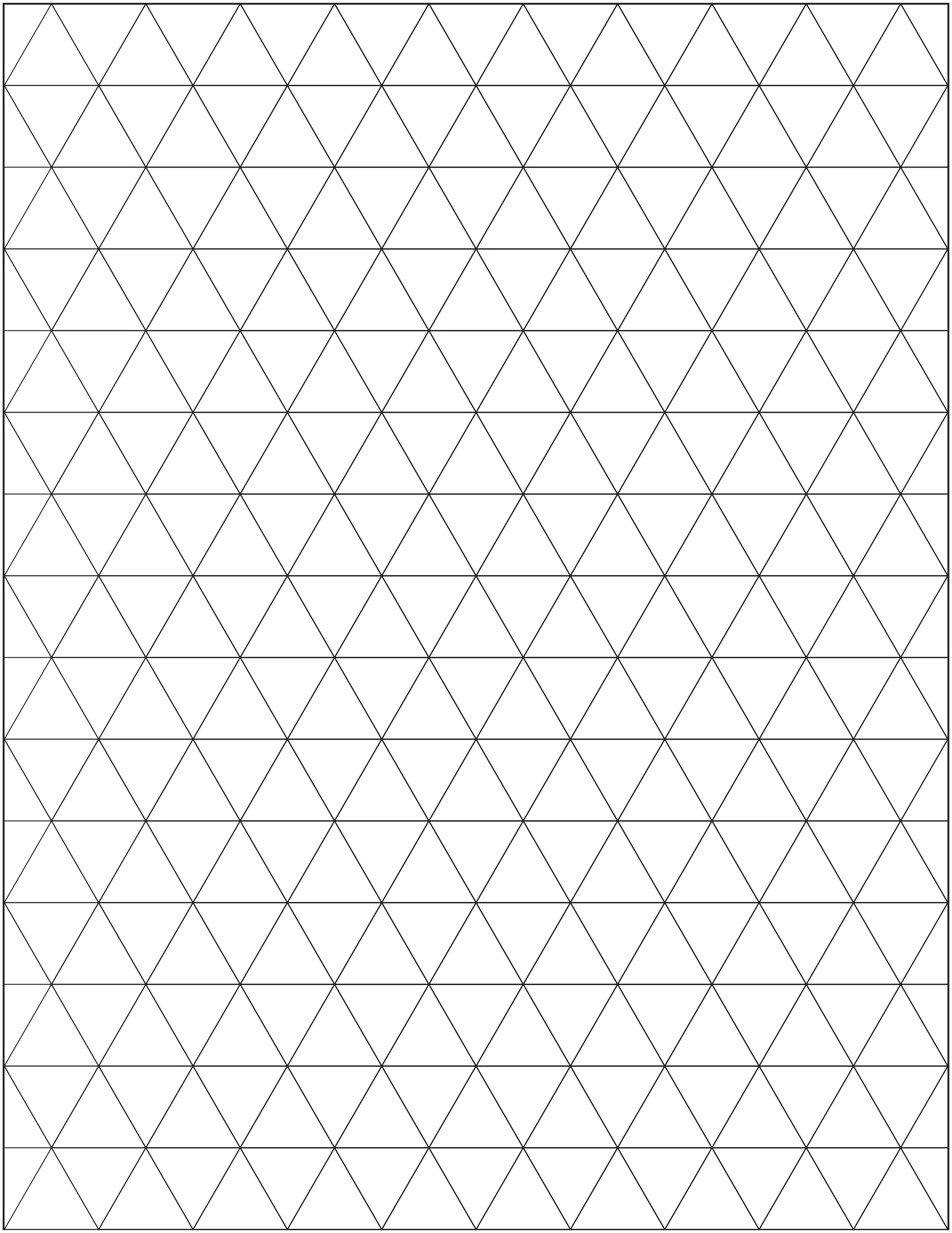




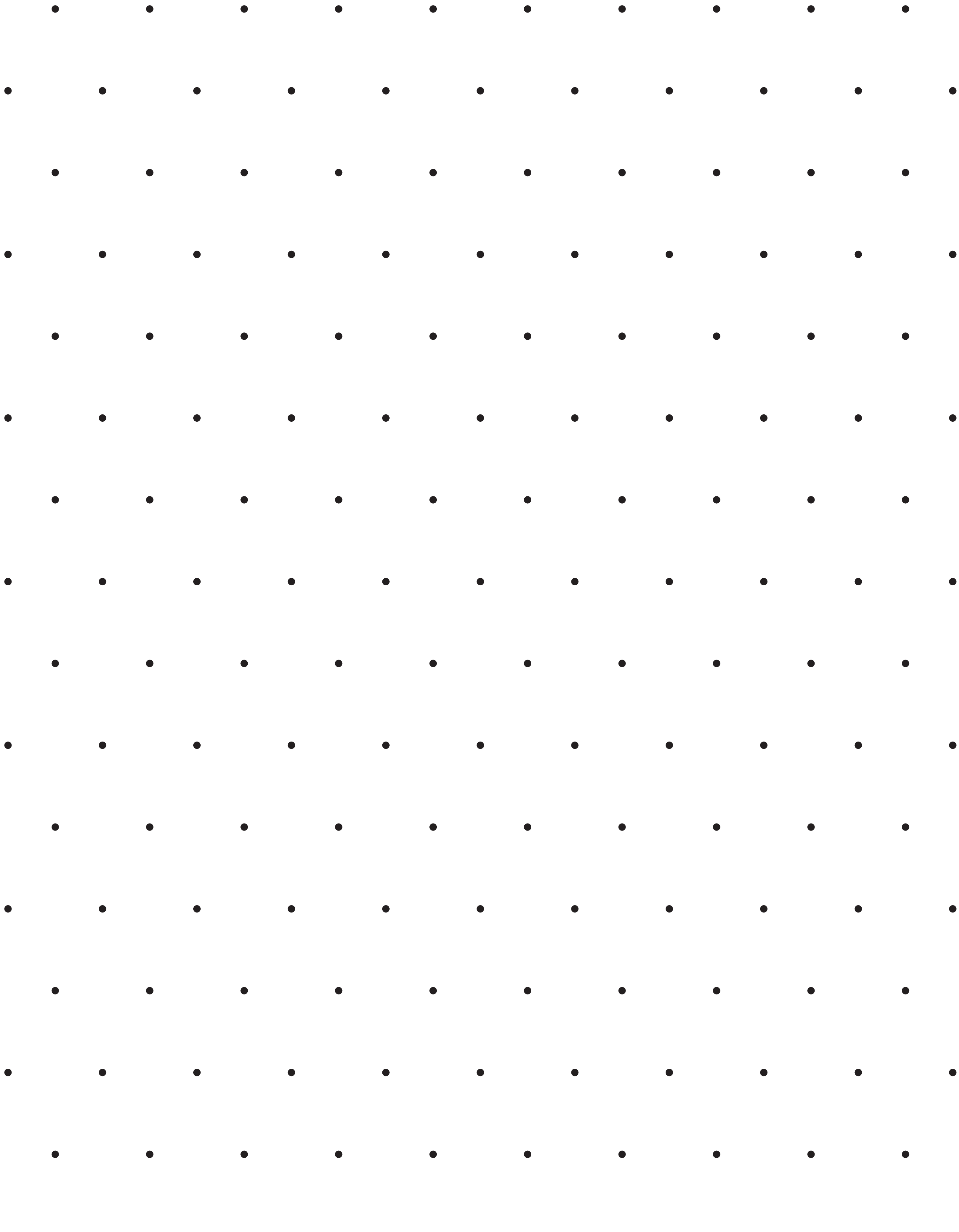
10 mm x 10 mm GRID

15 mm x 15 mm GRID

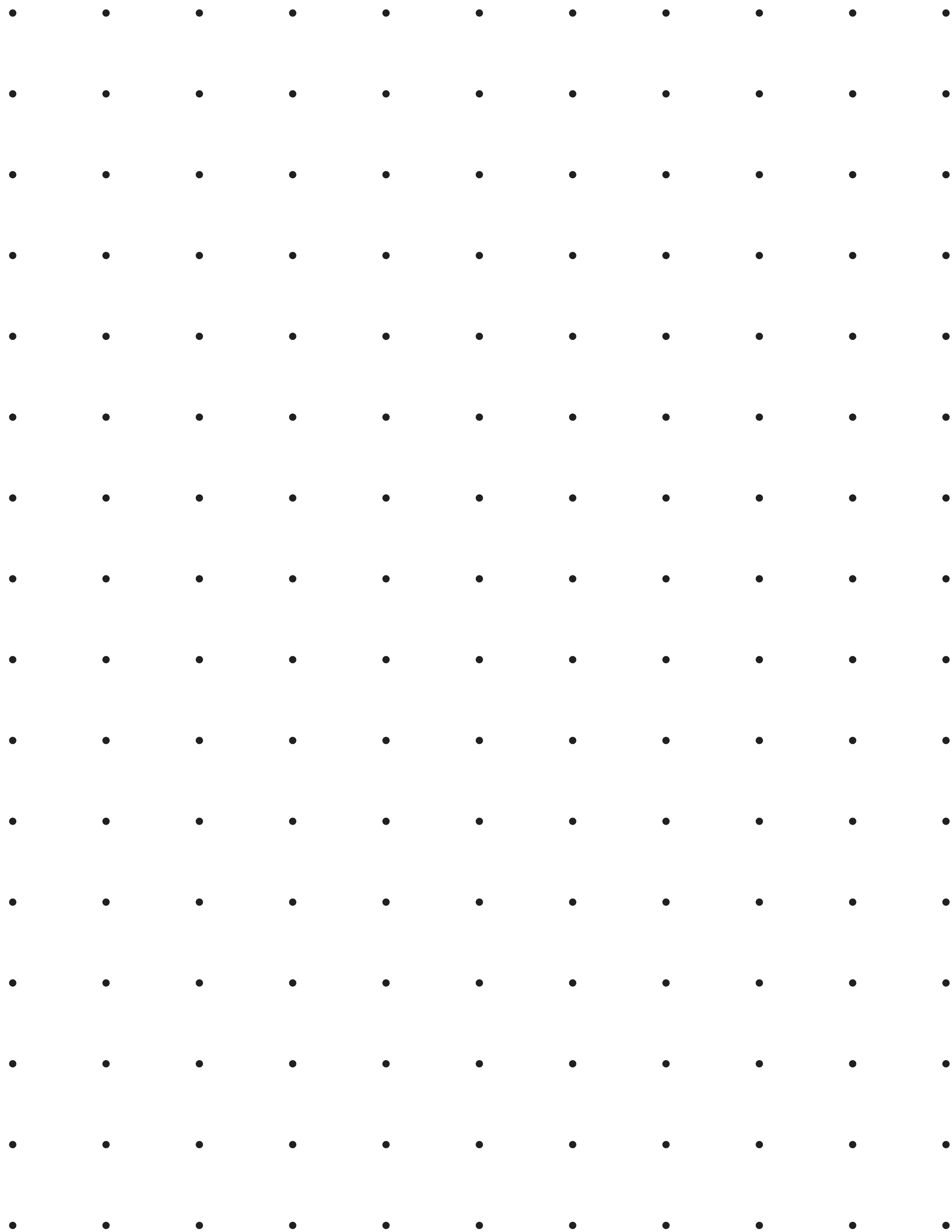
TRIANGULAR GRID



TRIANGULAR ISOMETRIC DOTS



SQUARE ISOMETRIC DOTS



4-DIGIT NUMBER EXPANDER (x 5)

ones						ones
tens						tens
hundreds						hundreds
thousands						thousands