Grades 3–4

Math PROBLEM-SOLVING Skills

Developing Successful Strategies

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CONTENTS

FOREWORD

The ***Math Problem-Solving Skills*** series has been developed to provide a rich resource for teachers of students from the elementary grades through middle school. The series of problems, discussions of ways to understand what is being asked, and means of obtaining solutions presented in these books aim to improve the problem-solving performance and persistence of all students. The authors believe it is critical that students and teachers engage with a few complex problems over an extended period rather than spend a short time on many straightforward problems or exercises. In particular, it is essential to allow students time to review and discuss what is required in the problem-solving process before moving to another and different problem. This series includes ideas for extending problems and solution strategies to help teachers implement this vital aspect of mathematics in their classrooms. The problems have been constructed and selected over many years of experience with students at all levels of mathematical talent and persistence, as well as in discussions with teachers in classrooms and professional learning and university settings.

Problem solving does not come easily to most people, so learners need many experiences engaging with problems if they are to develop this crucial ability. As they grapple with problem meaning and find solutions, students will learn a great deal about mathematics and mathematical reasoning. This leads to a focus on organizing what needs to be done rather than simply looking to apply one or more strategies.

Student and Teacher Pages

The student pages present problems chosen with a particular problem-solving focus and draw on a range of mathematical understandings and processes. For each set of related problems, teacher notes and discussion are provided. Answers to the more straightforward problems and detailed solutions to the more complex problems ensure appropriate explanations and suggest ways in which problems can be extended.

At the top of each teacher page, a statement highlights the particular thinking that the problems will demand, together with an indication of the mathematics that might be needed, a list of materials that can be used in seeking a solution, and the NCTM standards addressed. Each book is organized so that when a problem requires complicated strategic thinking, two or three problems occur on one page (supported by a teacher page with detailed discussion) to encourage students to find a solution together with a range of means that can be followed. More often, problems are grouped as a series of three interrelated pages where the level of complexity gradually increases, while the associated teacher page examines one or two of the problems in depth and highlights how the other problems might be solved in a similar manner.

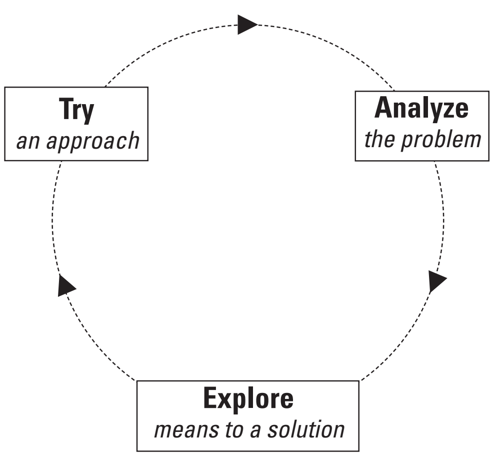
Each teacher page concludes with two further aspects critical to the successful teaching of problem solving. A section on likely difficulties points to reasoning and content inadequacies that experience has shown may well impede students' success. In this way, teachers can be on the lookout for difficulties and be prepared to guide students past these potential pitfalls. The final section suggests extensions to the problems that can build a rich array of experiences with particular solution methods.

Mathematics and Language

The difficulty of the mathematics gradually increases over the series, largely in line with what is taught at the various grade levels, although problem solving both challenges at the point of the mathematics that is being learned and provides insights and motivation for what might be learned next.

The language in which the problems are expressed is relatively straightforward, although this too increases in complexity across the series in terms of both the context in which the problems are set and the mathematical content that is required. It will always be a challenge for some students to "unpack" the meaning from a worded problem, particularly as the problems' context, information, and meanings expand. This ability is fundamental to the nature of mathematical problem solving and must be built up with time and experiences rather than diminished or left out of problem situations. It is suggested that students work in groups so that they can help one another tackle the ideas in complex problems through discussion, rather than simply leaping into the first ideas that come to mind (leaving the full extent of the problem unrealized).

An Approach to Solving Problems



The careful, gradual development of an ability to analyze problems for meaning, organize the information to make it meaningful, and make connections among problems to suggest a way forward to a solution, is fundamental to the approach taken with this series, from the first book to the last. At first, materials are used explicitly to aid these meanings and connections; however, in time they give way to diagrams, tables, and symbols as students' understanding of and experience with solving complex, engaging problems increases.

Not only is this model for the problem-solving process helpful in solving problems, but it also provides a basis for students to discuss their progress and solutions and determine whether or not they have fully answered a question. At the same time, it guides teachers' questions of students and provides a means of seeing underlying mathematical difficulties and ways in which problems can be adapted to suit particular needs and extensions. Above all, it provides a common framework for discussions between a teacher and group or whole class that focus on the problem-solving process rather than simply on the solution of particular problems. Indeed, as Alan Schoenfeld, in Steen, L. (Ed.) *Mathematics and Democracy* (2001), states so well, in problem solving:

Getting the answer is only the beginning rather than the end. … An ability to communicate thinking is equally important.

We wish all teachers and students who use these books success in fostering engagement with problem solving and building a greater capacity to come to terms with and solve mathematical problems at all levels.

George Booker and Denise Bond

Problem Solving and Mathematical Thinking

> *By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.*
>
> **— NCTM Principles and Standards for School Mathematics (2000, p. 52)**

Problem solving lies at the heart of mathematics. New mathematical concepts and processes have always grown out of problem situations and students' problem-solving capabilities develop from the very beginning of mathematics learning. A need to solve a problem can motivate students to acquire new ways of thinking as well as come to terms with concepts and processes that might not have been adequately learned when first introduced. Even those who can calculate efficiently and accurately are ill-prepared for a world where new and adaptable ways of thinking are essential, if they are unable to identify which information or processes are needed.

On the other hand, students who can analyze the meaning of problems, explore means to a solution, and carry out a plan to solve mathematical problems have acquired deeper and more useful knowledge than simply being able to complete calculations, name shapes, use formulas to make measurements, or determine measures of chance and data. It is critical that mathematics teaching focuses on enabling all students to become both able and willing to engage with and solve mathematical problems.

Well-chosen problems encourage deeper exploration of mathematical ideas, build persistence, and highlight the need to understand thinking strategies, properties, and relationships. They also reveal the central role of *sense making* in mathematical thinking—not only to evaluate the need for assessing the reasonableness of an answer or solution, but also the need to consider the interrelationships among the information provided with a problem situation. This may take the form of number sense, allowing numbers to be represented in various ways and operations to be interconnected; through spatial sense that allows the visualization of a problem in both its parts and whole; to a sense of measurement across length, area, volume, and probability and data analysis.

Problem Solving

A problem is a task or situation for which there is no immediate or obvious solution, so *problem solving* refers to the processes used when engaging with this task. When problem solving, students engage with situations for which a solution strategy is not immediately obvious, drawing on their understanding of concepts and processes they have already met, and will often develop new understandings and ways of thinking as they move towards a solution. It follows that a task that is a problem for one student may not be a problem for another and that a situation that is a problem at one level will only be an exercise or routine application of a known means to a solution at a later time.

A large number of tourists visited Canyonlands National Park during 2007. There were twice as many visitors in 2007 as in 2003 and 6,530 more visitors in 2007 than in 2006. If there were 298,460 visitors in 2003, how many were there in 2006?

For a student in grades 3 or 4, sorting out the information to see how the numbers of visitors each year are linked is a considerable task. Multiplication and subtraction with large numbers are required.

For students in the upper elementary grades, an ability to see how the problem is structured and familiarity with computation could lead them to use a calculator, key in the numbers and operations in an appropriate order, and readily obtain the answer:

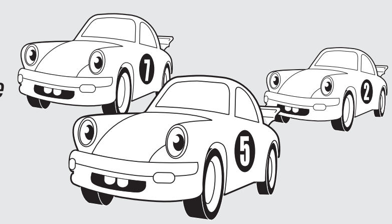
298,460 × 2 – 6,530 = 590,390

590,390 tourists visited Canyonlands in 2006

As the world in which we live becomes ever more complex, the level of mathematical thinking and problem solving needed in life and in the workplace has increased considerably. Those who understand and can use the mathematics they have learned will have opportunities opened to them that those who have not developed these ways of thinking will not. To enable students to thrive in this changing world, attitudes and ways of knowing that enable them to deal with new or unfamiliar tasks are now as essential as the procedures that have always been used to handle familiar operations readily and efficiently.

Such an attitude needs to develop from the beginning of mathematics learning as students form beliefs about meaning, the notion of taking control over the activities they engage with, and the results they obtain, and as they build an inclination to try different approaches. In other words, students need to see mathematics as a way of thinking rather than a means of providing answers to be judged right or wrong by a teacher, textbook, or some other external authority. They must be led to focus on ways of solving problems rather than on particular answers so that they understand the need to determine the meaning of a problem before beginning to work on a solution.

In a car race, Jordan started in fourth place. During the race, he was passed by six cars. How many cars does he need to pass to win the race?



To solve this problem it is not enough to simply use the numbers that are given. Rather, an analysis of the race situation is needed first to see that when Jordan started, there were 3 cars ahead of him. When another 6 cars passed him, there were now 9 ahead of him. If he is to win, he needs to pass all 9 cars. The 4 and 6 implied in the problem were not used at all! Rather, a diagram or the use of materials is needed first to interpret the situation and then see how a solution can be obtained.

However, many students feel inadequate when they encounter problem-solving questions. They seem to have no idea of how to go about finding a solution and are unable to draw on the competencies they have learned in number, geometry, and measurement. Often these difficulties stem from underdeveloped concepts for the operations, spatial thinking, and measurement processes. They may also involve an underdeveloped capacity to read problems for meaning and a tendency to be led astray by the wording or numbers in a problem situation.

Their approach may then be simply to try a series of guesses or calculations rather than consider using a diagram or materials to come to terms with what the problem is asking and using a systematic approach to organize the information given and required in the task. It is this ability to analyze problems that is the key to problem solving, enabling decisions to be made about which mathematical processes to use, which information is needed, and which ways of proceeding are likely to lead to a solution.

Making Sense in Mathematics

Making sense of the mathematics being developed and used must be seen as the central concern of learning. This is important, not only in coming to terms with problems and means to solutions, but also in terms of bringing meaning, representation, and relationships among mathematical ideas to the forefront of thinking about and with mathematics. Making sensible interpretations of any results and determining which of several possibilities is more or equally likely is critical in problem solving.

Number sense, which involves being able to work with numbers comfortably and competently, is important in many aspects of problem solving: in making judgments, interpreting information, and communicating ways of thinking. It is based on a full understanding of numeration concepts such as zero, place value, and the renaming of numbers in equivalent forms so that 207 can be seen as 20 tens and 7 ones as well as 2 hundreds and 7 ones (or that $\frac{5}{2}$, 2.5, and $2\frac{1}{2}$ are all names for the same fraction amount). Automatic, accurate access to basic facts also underpins number sense, not as an end in itself, but rather as a means of combining with numeration concepts to allow manageable mental strategies and fluent processes for larger numbers. Well-understood concepts for the operations are essential in allowing relationships within a problem to be revealed and taken into account when framing a solution.

Number sense requires:

- understanding relationships among numbers
- appreciating the relative size of numbers
- a capacity to calculate and estimate mentally
- fluent processes for larger numbers and adaptive use of calculators
- an inclination to use understanding and facility with numeration and computation in flexible ways

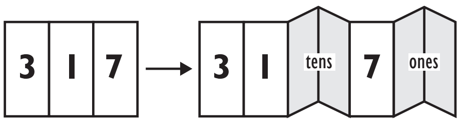
The following problem highlights the importance of these understandings.

There were 317 people at the New Year's Eve party on December 31. If each table could seat 5 couples, how many tables were needed?

Reading the problem carefully shows that each table seats five couples, or 10 people. At first glance, this problem might be solved using division; however, this would result in a decimal fraction, which is not useful in dealing with people seated at tables:

$10\overline{)317}$ is 31.7

In contrast, a full understanding of numbers allows 317 to be renamed as 31 tens and 7 ones:



This provides for all the people at the party and analysis of the number 317 shows that there have to be at least 32 tables for everyone to have a seat and allow party goers to move around and sit with others during the evening. Understanding how to *rename* a number has provided a direct solution without any need for computation. It highlights how coming to terms with a problem and integrating this with number sense provides a means of solving the problem more directly and allows an appreciation of what the solution might mean.

Spatial sense is equally important, as information is frequently presented in visual formats that must be interpreted and processed, while the use of diagrams is often essential in developing conceptual understanding across all aspects of mathematics. Using diagrams, placing information in tables or depicting a systematic way of dealing with the various possibilities in a problem assist in visualizing what is happening. It can be a very powerful tool in coming to terms with the information in a problem, and it provides insight into ways to proceed to a solution.

Spatial sense involves:

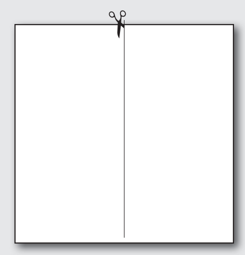
- a capacity to visualize shapes and their properties
- determining relationships among shapes and their properties
- linking two-dimensional and three-dimensional representations
- presenting and interpreting information in tables and lists
- an inclination to use diagrams and models to visualize problem situations and applications in flexible ways

The following problem shows how these understandings can be used.

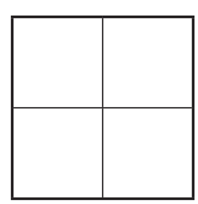
A small sheet of paper has been folded in half and then cut along the fold to make two rectangles.

The perimeter of each rectangle is 18 cm.

What was the perimeter of the original square sheet of paper?

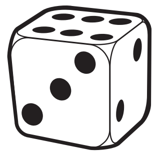


Reading the problem carefully and analyzing the diagram shows that the length of the longer side of the rectangle is the same as the one side of the square, while the other side of the rectangle is half this length. Another way to obtain this insight is to make a square, fold it in half along the cutting line, and then fold it again. This shows that the large square is made up of four smaller squares:

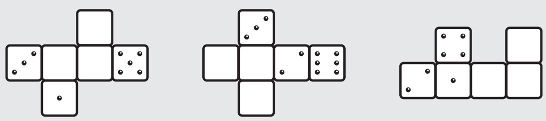


Since each rectangle contains two small squares, the side of the rectangle, 18 cm, is the same as 6 sides of the smaller square, so the side of the small square is 3 cm. The perimeter of the large square is made of 6 of these small sides, so it is 24 cm.

Similar thinking is used with arrangements of two-dimensional and three-dimensional shapes and in visualizing how they can fit together or be taken apart.



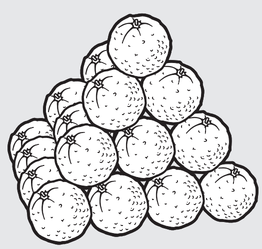
Many dice are made in the shape of a cube with arrangements of dots on each square face so that the sum of the dots on opposite faces is always 7. An arrangement of squares that can be folded to make a cube is called a net of a cube.



Which of these arrangements of squares forms a net for the dice?

Greengrocers often stack fruit as a pyramid.

How many oranges are in this stack?



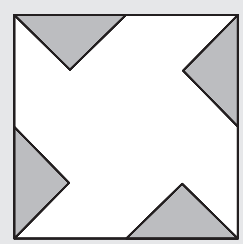
Measurement sense is dependent on both number sense and spatial sense, since attributes that are one-, two-, or three-dimensional are quantified to provide both exact and approximate measures and allow comparison. Many measurements use aspects of geometry (length, area, volume), while others use numbers on a scale (time, mass, temperature). Money can be viewed as a measure of value and uses numbers more directly, while practical activities such as map reading and determining angles require a sense of direction as well as gauging measurement. The coordination of the thinking for number and geometry, along with an understanding of how the metric system builds on place value, zero, and renaming, are critical in both building measurement understanding and using it to come to terms with and solve many practical problems and applications.

Measurement sense includes:

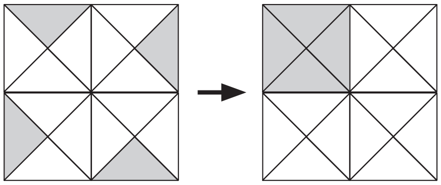
- understanding how numeration and computation underpin measurement
- extending relationships from number understanding to the metric system
- appreciating the relative size of measurements
- a capacity to use calculators and mental or written processes for exact and approximate calculations
- an inclination to use understanding and facility with measurements in flexible ways

The following problem shows how these understandings can be used.

A city square has an area of 160 m^2. Four small triangular garden beds are constructed from each corner to the midpoints of the sides of the square. What is the area of each garden bed?



Reading the problem carefully shows that there four garden beds, and each of them takes up the same proportions of the whole square. A quick look at the area of the square shows that there will not be an exact number of meters along one side. Some further thinking will be needed to determine the area of each garden bed.



If the midpoints of the four sides are connected across the square, four smaller squares are formed and each garden bed takes up $\frac{1}{4}$ of a small square. Four of the garden beds will have the same area of one small square. Since the area of the small square is $\frac{1}{4}$ the area of the large square, the area of one small square is 40 m^2 and the area of each triangular garden bed is 10 m^2.

An understanding of the problem situation given by a diagram has been integrated with spatial thinking and a capacity to calculate mentally with simple fractions to provide an appropriate solution. Both spatial sense and number sense have been used to understand the problem and suggest a means to a solution.

Data sense is an outgrowth of measurement sense and refers to an understanding of the way number sense, spatial sense, and a sense of measurement work together to deal with situations where patterns must be discerned among data or when likely outcomes must be analyzed.

Data sense involves:

- understanding how numeration and computation underpin the analysis of data
- appreciating the relative likelihood of outcomes
- a capacity to use calculators or mental and written processes for exact and approximate calculations
- presenting and interpreting data in tables and graphs
- an inclination to use understanding and facility with number combinations and arrangements in flexible ways

The following problem shows how these understandings can be used.

You are allowed 3 scoops of ice cream: 1 chocolate, 1 vanilla, and 1 strawberry. How many different ways can the scoops be placed on a cone?

There are six possibilities for placing the scoops of ice cream on a cone. Systematically treating the possible placements one at a time highlights how the use of a diagram can account for all possible arrangements.

Patterning is another critical aspect of sense-making in mathematics. Often a problem calls on discerning a pattern in the placement of materials, the numbers involved in the situation, or the possible arrangements of data or outcomes so as to determine a likely solution. Being able to see patterns is also very helpful in getting a solution more immediately or understanding whether or not a solution is complete.

A farmer has emus and alpacas in one paddock. When she counted, there were 38 heads and 100 legs. How many emus and how many alpacas are in the paddock?

There are 38 emus and alpacas. Emus have two legs. Alpacas have four legs.

Number of Alpacas	Number of Emus	Number of Legs
4	34	84 – too few
8	30	92 – too few
10	28	96 – too few
12	26	100

There are 12 alpacas and 26 emus.

As students gain more experience in solving problems, an ability to see patterns in what is occurring will also help them obtain solutions more directly and see the relationship between a new problem and one that they have solved previously. It is this ability to relate problem types, even when the context appears to be quite different, that often distinguishes a good problem solver from one who is more hesitant.

Building a Problem-Solving Process

While the teaching of problem solving has often centered on the use of particular strategies that could apply to various classes of problems, many students are unable to access and use these strategies to solve problems outside of the teaching situations in which they were introduced. Rather than acquire a process for solving problems, they may attempt to memorize a set of procedures and view mathematics as a set of learned rules where success follows the use of the right procedure, to the numbers given in the problem. Any use of strategies may be based on familiarity, personal preference, or recent exposure rather than through a consideration of the problem to be solved. A student may even feel it is sufficient to have only one strategy and that the strategy should work all of the time—and if it doesn't, then the problem can't be solved.

In contrast, observation of successful problem solvers shows that their success depends more on an analysis of the problem itself—what is being asked, what information might be used, what answer might be likely and so on—so that a particular approach is used only after the intent of the problem is determined. Establishing the meaning of the problem before any plan is drawn up or work on a solution begins is critical. Students need to see that discussion about the problem's meaning, and the ways of obtaining a solution, must take precedence over a focus on the answer. Using collaborative groups when problem solving, rather than tasks assigned individually, is an approach that helps to develop this disposition.

Looking at a problem and working through what is needed to solve it will shed light on the problem-solving process.

On Saturday, Peta went to the mall to buy a new outfit to wear at her friend's birthday party. She spent half of her money on a dress and then one-third of what she had left on a pair of sandals. After her purchases, she had $60.00 left in her purse. How much money did she have to start with?

INTRODUCTION

By reading the problem carefully, it can be determined that Peta had an original amount of money to spend. She spent some on a dress and some on shoes and then had $60.00 left. All of the information required to solve the problem is available, and no further information is needed. The question at the end asks how much money did she start with, but really the problem is how much did she spend on the dress and then on the sandals.

The discussion of this problem serves to identify the key elements in the problem-solving process. To begin, it was necessary to analyze the problem to discover what must be considered. What a problem is really asking is rarely found in the problem statement. In this phase, it is necessary to look below the surface of the problem and come to terms with its structure. Reading the problem aloud, recalling previous difficult problems and other similar problems, selecting the important information, and discussing the problem's meaning are all essential.

The next step is to explore possible ways to solve the problem. If the analysis stage has been completed, then ways in which the problem might be solved will emerge. It is here that strategies, and how they might be useful to solving a problem, can arise. However, most problems can be solved in a variety of ways, using different approaches, and students must be encouraged to select a method that makes sense and appears achievable.

Ways that may come to mind during the analysis include:

- *Materials* – Base 10 materials could be used to represent the money spent and to help the student work backwards through the problem from when Peta had $60.00 left.
- *Try and adjust* – Select an amount that Peta might have taken shopping, try it in the context of the question, examine the resulting amounts, and then adjust them, if necessary, until $60.00 is the result.
- *Backtrack using the numbers* – The sandals were one-third of what was left after the dress, so the $60.00 would be two-thirds of what was left. Together, these two amounts would match the cost of the dress.
- *Use a diagram* to represent the information in the problem.
- *Think of a similar problem* – For example, it is like the car race problem in that the relative portions (places) are known and the final result (money left, winning position) are given.

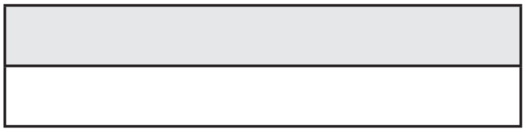
Now *one* of the possible means to a solution can be selected to try. Backtracking shows that $60 was two-thirds of what Peta had left, so the sandals (which are one-third of what she had left) must have cost $30. Together, these are half of what Peta took, which is also the cost of the dress. As the dress cost $90, Peta took $180 to spend.

Materials could also have been used with which to work backwards: 6 tens represent the $60 left, so the sandals would cost 3 tens and the dress 9 tens—she took 18 tens, or $180, shopping.

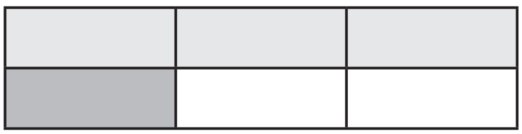
Another way to solve the problem is with a diagram. If we use a rectangle to represent how much money Peta took with her, we can show by shading how much she spent on a dress and sandals:

Total amount available to spend:

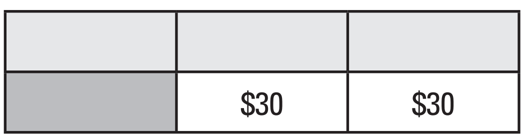
She spent half of her money on a dress.



She then spent one-third of what she had left on sandals, which has minimized and simplified the calculations.



At this point she had $60 left, so the two unshaded parts must be worth $60 or $30 per part—which has again minimized and simplified the calculations.

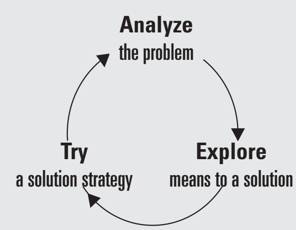


Each of the six equal parts represents $30, so Peta took $180 to spend.

Having tried an idea, an answer must be analyzed in the light of the problem in case another solution is required. It is essential to compare an answer to the original analysis of the problem to determine whether the solution obtained is reasonable and answers the problem. It will also raise the question as to whether other answers exist, and even whether there might be other solution strategies. In this way the process is cyclic, and should the answer be unreasonable, then the process would need to begin again.

We believe that Peta took $180 to shop with. She spent half (or $90) on a dress, leaving $90. She spent one-third of the $90 on sandals ($30), leaving $60. Looking again at the problem, we see that this is correct, and the diagram has provided a direct means to the solution that has minimized and simplified the calculations.

Thinking about the various ways this problem was solved highlights the key elements in the problem-solving process. When starting the process, it is necessary to ***analyze*** the problem to unfold its layers, discover its structure, and determine what the problem is really asking. Next, all possible ways to solve the problem are ***explored*** before one, or a combination of ways, are selected to ***try***. Finally, once something has been tried, it is important to check the solution to see if it is reasonable. This process highlights the cyclical nature of problem solving and brings to the fore the importance of understanding the problem (and its structure) before proceeding. This process can be summarized as:



A Plan to Manage Problem Solving

This model provides students with a way of talking about the steps they engage in whenever they have a problem to solve. Discussing how they initially analyzed the problem, explored various ways that might provide a solution, and then tried one or more possible paths to obtain a solution—which they then analyzed for completeness and sense making—reinforces the very methods that will help them solve future problems.

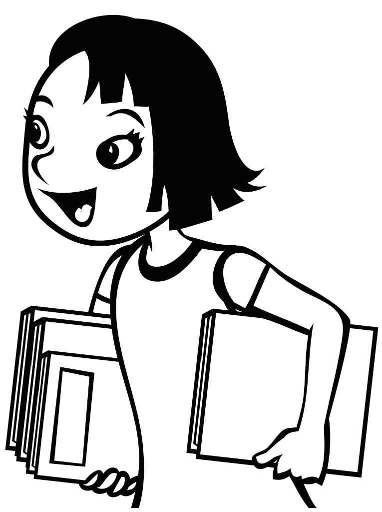
Further, returning to an analysis of any answers and solution strategies highlights the importance of reflecting on what has been done. Taking time to reflect on any plans drawn up, processes followed, and strategies used brings out the significance of coming to terms with the nature of the problem, as well as the value and applicability of particular approaches that might be used with other problems.

Thinking of how a related problem was solved is often the key to solving another problem at a later stage. It allows the thinking to be carried over to the new situation in a way that simply trying to think of the strategy used often fails to reveal. Analyzing problems in this way also highlights that a problem is not solved until the answer obtained can be justified. Learning to reflect on the *whole* process leads to the development of a deeper understanding of problem solving, and time must be allowed for reflection and discussion to fully build mathematical thinking.

Managing a Problem-Solving Program

Teaching problem solving differs from many other aspects of mathematics in that collaborative work can be more productive than individual work. Students who may be tempted to quickly give up when working on their own can be encouraged to see ways of proceeding when discussing a problem in a group, therefore building greater confidence in their capacity to solve problems and learning the value of persisting with a problem in order to tease out what is required. What is discussed with their peers is more likely to be recalled when other problems are met, while the observations made in the group increase the range of approaches that a student can access. Thus, time has to be allowed for discussion and exploration rather than insisting that students spend time on task, as for routine activities.

Correct answers that fully solve a problem are always important, but developing a capacity to use an effective problem-solving process must be the highest priority. Students who have an answer should be encouraged to discuss their solution with others who believe they have a solution, rather than tell their answer to another student or simply move on to another problem. In particular, explaining to others why they believe an answer is reasonable, as well as why it provides a solution, gets other students to focus on the entire problem-solving process rather than on just quickly getting an answer.



Expressing an answer in a sentence that relates to the question stated in the problem also encourages reflection on what was done and ensures that the focus is on solving the problem rather than providing an answer. These aspects of the teaching of problem solving should then be taken further, as particular groups discuss their solutions with the whole class and all students are able to participate in the discussion of the problem. In this way, problem solving as a way of thinking comes to the fore, rather than focusing on the answers as the main aim of their mathematical activities.

Questions must encourage students to explore possible means to a solution and try one or more of them, rather than point to a particular procedure. They can also help students to see how to progress in their thinking, rather than get stuck in a loop where the same steps are repeated over and over. While asking too many questions that focus on the way to a solution may end up undermining the problem-solving process, asking too few may cause students to become frustrated with the task and think that it is beyond them.

Students need to experience the challenge of problem solving and gain pleasure from working through the process that leads to a full solution. Taking time to listen to students as they try out their ideas, without comment and without directing them to a particular strategy, is also important. Listening provides a sense of how students' problem solving is developing, as assessing this aspect of mathematics can be difficult. After all, solving one problem will not necessarily lead to success on the next problem, nor will difficulty with a particular problem mean that the problems that follow will also be as challenging.

A teacher also may need to extend or adapt a given problem to ensure that the problem-solving process is understood and can be used in other situations, instead of moving on to a different problem in another area of mathematics learning. This can help students to understand the significance of asking questions of a problem, as well as seeing how a way of thinking can be adapted to other related problems. Having students engage in this process of posing questions is another way of both assessing them and bringing them to terms with the overall process of solving problems.

Building a Problem-Solving Process

The cyclical model *Analyze–Explore–Try* provides a very helpful means of organizing and discussing possible solutions. However, care must be taken that it is not seen simply as a procedure to be memorized and then applied in a routine manner to every new problem. Rather, it needs to be carefully developed over a range of different problems, highlighting the components that are developed with each new problem.

Analyze

- As students read a problem, the need to first read for the *meaning* of the problem can be stressed. This may require reading more than once and can be helped by asking students to state in their own words what the problem is asking them to do.
- Further reading will be needed to sort out which information is needed and whether some is not needed or if other information must be gathered from the problem's context (for example, data presented within the illustration or table accompanying the problem) or whether the students' mathematical understandings must be used to find other relationships among the information. As the form of the problems becomes more complex, this thinking will be extended to incorporate further ways of dealing with the information; for example, measurement units, fractions, and larger numbers might need to be renamed to the same mathematical form.
- Thinking about any processes that might be needed and the order in which they are used, as well as the type of answer that could result, should also be developed in the context of new levels of problem structure.

- Developing a capacity to see through the problem's expression—or context to see similarities between new problems and others that might already have been met—is a critical way of building expertise in coming to terms with and solving problems.

Expanding the Problem-Solving Process

Explore

- When a problem is being explored, some problems will require the use of materials to think through the whole of the problem's context. Others will demand the use of diagrams to show what is needed. Another will show how systematic analysis of the situation using a sequence of diagrams, on a list or table, is helpful. As these ways of thinking about the problem are understood, they can be included in the cycle of steps.

Try

- Many students often try to guess a result. This can even be encouraged by talking about "guess and check" as a way of solving problems. Changing to "try and adjust" is more helpful in building a way of thinking and can lead to a very powerful way of finding solutions.
- When materials, a diagram, or a table has been used, another means to a solution is to look for a pattern in the results. When these have revealed what is needed to try for a solution, it may also be reasonable to use pencil and paper or a calculator.

Analyze

- The point in the cycle where an answer is assessed for reasonableness (for example, whether it provides a solution, is only one of several solutions, or there may be another way to solve the problem) also needs to be brought to the fore as different problems are met.

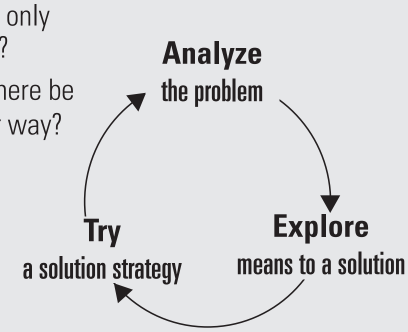
The Role of Calculators

When calculators are used, students devote less time to basic calculations, providing time that might be used to either explore a solution or find an answer to a problem. In this way, attention is shifted from computation, which the calculator can do, to thinking about the problem and its solution—work that the calculator cannot do. It also allows more realistic problems to be addressed in problem-solving sessions. In these situations, a calculator serves as a tool rather than a crutch, requiring students to think through the problem's solution in order to know how to use the calculator appropriately. It also underpins the need to make sense of the steps along the way and any answers that result, as keying incorrect numbers, operations, or order of operations quickly leads to results that are not appropriate.

A fuller model to manage problem solving can gradually emerge:

- Read carefully.
- What is the problem asking?
- What is the meaning of the information? Is it all needed? Is there too little? Too much?
- Which operations will be needed and in what order?
- What sort of answer is likely?
- Have I seen a problem like this before?

- Put the solution back into the problem.
- Does the answer make sense?
- Does it solve the problem?
- Is it the only answer?
- Could there be another way?



- Use materials or a model.
- Use a calculator.
- Use pencil and paper.
- Look for a pattern.

- Use a diagram or materials.
- Work backwards or backtrack.
- Put the information into a table.
- Try and adjust.

Choosing, Adapting, and Extending Problems

When problems are selected, they need to be examined to see if students already have an understanding of the underlying mathematics required and that the problem's expression can be meaningfully read by the group of students who will be attempting the solution—though not necessarily by ***all*** students in the group. The problem itself should be neither too easy (so that it is just an exercise, repeating something readily done before) nor too difficult (thus beyond the capabilities of most or all in the group). A problem should engage the interest of the students and also be able to be solved in more than one way.

As a problem and its solution are reviewed, posing similar questions—where the numbers, shapes, or measurements are changed—focuses attention back on what was entailed in analyzing the problem and in exploring the means to a solution. Extending these processes to more complex situations shows how the particular approach can be extended to other situations and how patterns can be analyzed to obtain more general methods or results. It also highlights the importance of a systematic approach when conceiving and discussing a solution and can lead students to ask themselves further questions about the situation and to pose problems of their own as the significance of the problem's structure is uncovered.

Problem Structure and Expression

When analyzing a problem, it is also possible to discern critical aspects of the problem's form and relate this to an appropriate level of mathematics and problem expression when choosing or extending problems. A problem of first-level complexity uses simple mathematics and simple language. A second-level problem may have simple language and more difficult mathematics or more difficult language and simple mathematics, while a third-level problem has yet more difficult language and mathematics. Within a problem, the processes that must be used may be more or less obvious, the information that is required for a solution may be too much or too little, and strategic thinking may be needed in order to come to terms with what the problem is asking.

Level	processes obvious	processes less obvious	too much information	too little information	strategic thinking
increasing difficulty with problem's expression and mathematics required	simple expression, simple mathematics				
	more complex expression, simple mathematics simple expression, more complex mathematics				
	complex expression, complex mathematics				

The Varying Levels of Problem Structure and Expression

(i) The processes to be used are relatively obvious; these problems are comparatively straightforward and contain all the information necessary to find a solution.

(ii) The processes required are not immediately obvious; these problems contain all the information necessary to find a solution but demand further analysis to sort out what is wanted, and students may need to reverse what initially seemed to be required.

(iii) The problem contains more information than is needed for a solution as these problems contain not only all the information needed to find a solution, but also additional information in the form of times, numbers, shapes, or measurements.

(iv) Further information must be gathered and applied to the problem in order to obtain a solution. These problems do not contain all the information necessary to find a solution but do contain a means to obtain the required information. The problem's setting, the student's mathematical understanding, or the problem's wording must be searched for the additional material.

(v) Strategic thinking is required to analyze the question in order to determine a solution strategy. Deeper analysis, often aided by the use of diagrams or tables, is needed to come to terms with what the problem is asking so a means to a solution can be determined.

This analysis of the nature of problems can also serve as a means of evaluating the provision of problems within a mathematics program. In particular, it can lead to the development of a full range of problems, ensuring they are included across all problem forms, with the mathematics and expression suited to the level of the students.

Assessing Problem Solving

Assessment of problem solving requires careful and close observation of students working in a problem-solving setting. These observations can reveal the range of problem forms and the level of complexity in the expression and underlying mathematics that a student is able to confidently deal with. Further analysis of these observations can show to what extent the student is able to analyze the question, explore ways to a solution, select one or more methods to try, and then analyze any results obtained.

Assessment informs:

What?
- *Problem form*
- *Problem expression*
- *Mathematics required*

How?
- *Analyze*
- *Explore*
- *Try*

It is the combination of two fundamental aspects—the types of problem that can be solved and the manner in which solutions are carried out—that will give a measure of a student's developing problem-solving abilities, rather than a one-off test in which some problems are solved and others are not.

Observations based on this analysis have led to a categorization of many of the possible difficulties

Problem	***Likely Causes***
Student is unable to make any attempt at a solution.	• not interested • feels overwhelmed • cannot think of how to start to answer question • needs to reconsider complexity of steps and information
Student has no means of linking the situation to the implicit mathematical meaning.	• needs to create diagram or use materials • needs to consider separate parts of question and then bring parts together
Students uses an inappropriate operation.	• misled by word cues or numbers • has underdeveloped concepts • uses rote procedures rather than real understanding
Student is unable to translate a problem into a more familiar process.	• cannot see interactions between operations • lack of understanding means he/she unable to reverse situations • data may need to be used in an order not evident in the problem statement or in an order contrary to that in which it is presented

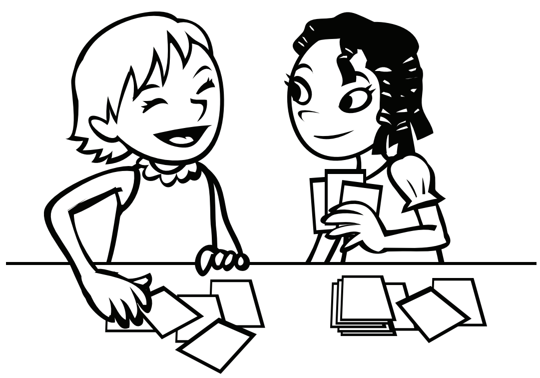
that students experience with problem solving as a whole, rather than the misconceptions they may have about particular problems. These often involve inappropriate attempts at a solution based on little understanding of the problem.

A major cause of possible difficulties is the *lack of a well-developed plan* of attack, leading students to focus on the *surface level* of problems. In such cases, students:

- locate and manipulate numbers with little or no thought about their relevance to the problem
- try a succession of different operations if the first ones attempted do not yield a (likely) result
- focus on keywords for an indication of what might be done without considering their significance within the problem as a whole
- read problems quickly and cursorily to locate the numbers to be used
- use the first available word cue to suggest the operation that might be needed.

Other possible difficulties result from a focus on being quick, which leads to:

- no attempt to assess the reasonableness of an answer
- little perseverance if an answer is not obtained using the first approach tried
- not being able to access strategies to which they have been introduced



When the approaches to problem processing developed in this series are followed and the specific suggestions for solving particular problems or types of problems are discussed with students, these difficulties can be minimized, if not entirely avoided. Analyzing the problem before starting leads to an understanding of the problem's meanings. The cycle of steps within the model means that nothing is tried before the intent of the problem is clear and the means to a solution have been considered. Focusing on a problem's meaning and discussing what needs to be done build perseverance. Making sense of the steps that must be followed and any answers that result is central to the problem-solving process. These difficulties are unlikely to occur among those who have built up an understanding of this way of thinking.

A Final Comment

If an approach to problem solving can be built up using the ideas developed here and the problems in the investigations on the pages that follow, students will develop a way of thinking about and with mathematics that will allow them to readily solve problems and generalize from what they already know to understand new mathematical ideas. They will engage with these emerging mathematical conceptions from their very beginnings, be prepared to debate and discuss their own ideas, and develop attitudes that will allow them to tackle new problems and topics. Mathematics can then be a subject that is readily engaged with and can become one in which the student feels in control, instead of one in which many rules devoid of meaning have to be memorized and applied at the right time. This early enthusiasm for learning and the ability to think mathematically will lead to a search for meaning in new situations and processes that will allow mathematical ideas to be used across a range of applications in school and everyday life.

A NOTE ON CALCULATOR USE

Many of the problems in this series require the use of a number of consecutive calculations, often requiring adding, subtracting, multiplying, or dividing the same amount to complete entries in a table or see a pattern. This demands (or will build) a certain amount of sophisticated use of the memory and constant functions of a simple calculator.

1. To add a number such as 9 repeatedly, it is sufficient on most calculators to enter an initial number (e.g., 30) and then press + 9 = = = = to add 9 over and over.
   - *30, 39, 48, 57, 66, …*
   - *To add 9 to a range of numbers, enter the first number (e.g., 30) and then press + 9 =. 30 + 9 = 39; 7 = gives 16; 3 = gives 12; 21 = gives 30; …*
   - *These are the answers when 9 is added to each number.*

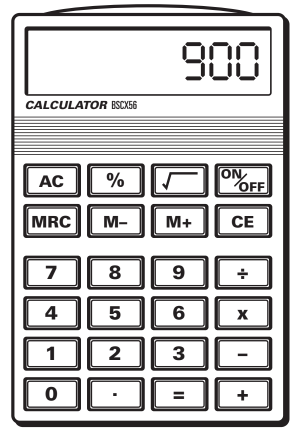
2. To subtract a number such as 5 repeatedly, it is sufficient on most calculators to enter an initial number (e.g., 92) and then press – 5 = = = = to subtract 5 over and over.
   - *92, 87, 82, 77, 72, …*
   - *To subtract 5 from a range of numbers, enter the first number (e.g., 92) and then press – 5 =. 92 – 5 = 87; 68 = gives 63; 43 = gives 38; 72 = gives 67; …*
   - *These are the answers when 5 is subtracted from each number.*

3. To multiply a number such as 10 repeatedly, most calculators now reverse the order in which the numbers are entered. Enter 10 × and then press an initial number (e.g., 15) = = = = to multiply by 10 over and over.
   - *150, 1,500, 15,000, 150,000, …*
   - *These are the answers when the given number is multiplied by 10.*
   - *This also allows squaring of numbers: 4 × = gives 16 or* 4^2.
   - *Continuing to press = gives more powers:*
   - *4 × = = gives* 4^3 *or 64; 4 × = = = gives* 4^4*; 4 × = = = = gives* 4^5*, and so on.*
   - *To multiply a range of numbers by 10, enter 10 × and then the first number (e.g., 90) and =.*
   - *10 x 90 = 900; 45 = gives 450; 21 = gives 210; 162 = gives 1,620; …*
   - *These are the answers when each number is multiplied by 10.*

4. To divide by a number such as 4 repeatedly, enter a number (e.g., 128).
   - *Then press ÷ 4 = = = = to divide each result by 4.*
   - *32, 8, 2, 0.5, …*
   - *These are the answers when the given number is divided by 4.*
   - *To divide a range of numbers by 4, enter the first number (e.g., 128) and ÷ 4 =. 128 ÷ 4 = 32; 64 = gives 16; 32 = gives 8; 12 = gives 3; …*
   - *These are the answers when each number is divided by 4.*

5. Using the memory keys M+, M–, and MR will also simplify calculations. A result can be calculated and added to memory (M+). Then a second result can be calculated and added to (M+) or subtracted from (M–) the result in the memory. Pressing MR will display the result. Often this will need to be performed for several examples as they are entered onto a table or patterns are explored directly. Clearing the memory after each completed calculation is essential!

   A number of calculations may also need to be made before addition, subtraction, multiplication, or division with a given number. That number can be placed in memory and used each time without having to rekey it.



6. The % key can be used to find percentage increases and decreases directly.
   - *To increase or decrease a number by a certain percent (e.g., 20%), simply key in the number and press = 20% or – 20% to get the answer:*
   - *80 + 20% gives 96 (not 100). 20% of 80 is 16; 80 + 16 is 96.*
   - *90 – 20% gives 72 (not 70). 20% of 90 is 18; 90 – 18 is 72.*

7. While the square root key can be used directly, finding other roots is best done by a "try and adjust" approach using the multiplication constant described above (in point 3).

MEETING THE NCTM STANDARDS

Numbers and Operations	
1.1 Understand numbers, ways of representing numbers, relationships among numbers, and number systems	pp. 30, 36, 42, 46, 48, 54, 56, 58, 68, 76, 82, 86, 94, 98
1.2 Understand meanings of operations and how they relate to one another	pp. 24, 30, 36, 42, 58, 86, 90
1.3 Compute fluently and make reasonable estimates	pp. 24, 30, 36, 40, 42, 48, 50, 58, 86, 90

Algebra	
2.1 Understand patterns, relations, and functions	pp. 54, 56, 58
2.2 Represent and analyze mathematical situations and structures using algebraic symbols	
2.3 Use mathematical models to represent and understand quantitative relationships	
2.4 Analyze change in various contexts	

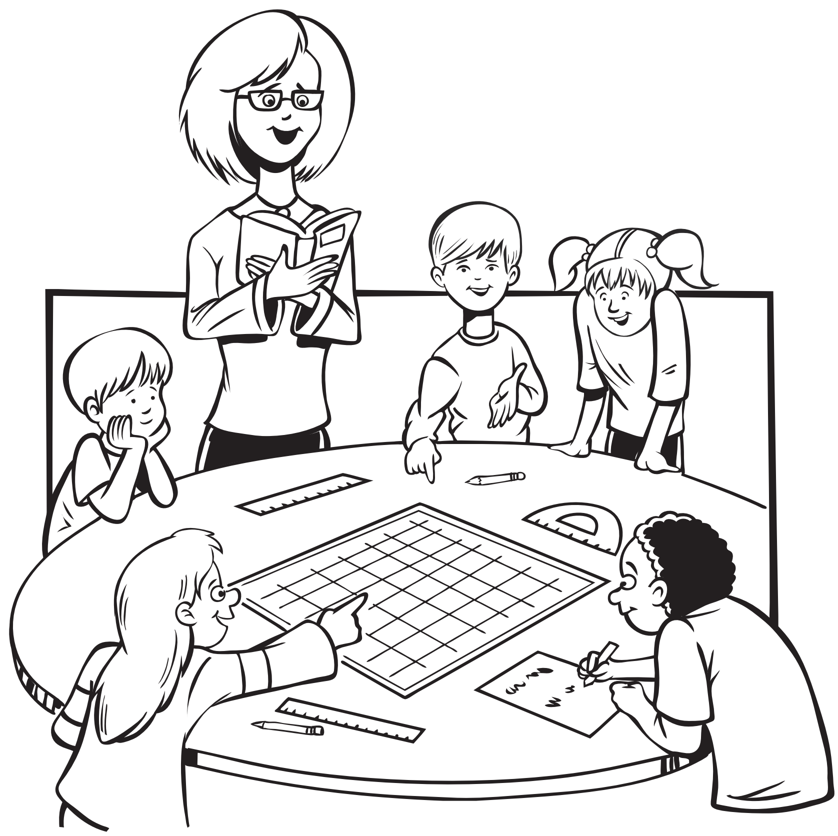
Geometry	
3.1 Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	pp. 62, 72, 76
3.2 Specify locations and describe spatial relationships using coordinate geometry and other representational systems	pp. 78, 82, 86, 94, 98
3.3 Apply transformations and use symmetry to analyze mathematical situations	p. 62
3.4 Use visualization, spatial reasoning, and geometric modeling to solve problems	pp. 62, 68, 72, 76

Data Analysis and Probability	
4.1 Formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them	pp. 82, 86, 90, 94, 98
4.2 Select and use appropriate statistical methods to analyze data	
4.3 Develop and evaluate inferences and predictions that are based on data	
4.4 Understand and apply basic concepts of probability	

Problem Solving, Reasoning and Proof, Communications, Connections, Representation	all activities

Math Problem-Solving Skills:

Teacher Notes and Student Worksheets



Problem-Solving Objective

To analyze and use information in word problems

Materials

Base 10 materials, place value chart, calculator

NCTM Content Standards

- Number and Operations 1.2, 1.3

Focus

These pages explore word problems that mostly require addition, subtraction, or multiplication. Students must determine what the problem is asking and, in many cases, calculate more than one step in order to find solutions. Analysis of the problems reveals that some questions contain additional information that is not needed.

If necessary, materials can be used to assist with the calculation, since these problems are about determining what the problem is asking rather than computation or basic facts.

Discussion

Page 25 – The Water Park

These problems involve more than one step and may involve addition as well as subtraction. The wording has been kept simple to assist with the problem-solving process. Students may choose a number of different ways to find a solution. For example, in the second problem, the number of people getting out (43) could be subtracted from the people going swimming (79) and then this number (36) could be added to 397. Alternatively, 79 could be added to 397 and then 43 subtracted to obtain a solution. Students should be encouraged to explore and try different ways of arriving at a solution.

Page 26 – Lily Pads and Frogs

This investigation relates to information about a lake with lily pads and frogs. The scenario begins with a certain number of frogs and lily pads. As new information is introduced, the numbers change to meet the new criteria; lily pads flower, grow, and die, while the frogs move from one lake to another. Students are required to keep track of the new information and use it to answer the subsequent questions.

Page 27 – Samantha's Flower Shop

These problems involve addition, subtraction, and multiplication. In most cases more than one step is needed to find a solution. The wording has been kept fairly simple to assist with the problem-solving process. The last problem requires students to work backwards to find a solution: If Samantha needs 82 lilies and only orders an extra 47, then she would have had 35 left over on Friday. When the delivery is short by seven lilies, she will have 75 lilies rather than the 82 she needs.

Page 28 – The Herb Market

These investigations involve addition, subtraction, and multiplication. Students may find it helpful to draw a diagram to work out what is happening in the story and to determine what needs to be multiplied to find a solution. The problems about selling the bunches of parsley in the morning and afternoon (questions 5 and 6) also involves information from Problem 4. These problems explore how many bunches of parsley Simon has not sold.

Page 29 – At the Bakery

The wording and the steps involved in these problems have been kept fairly simple to assist with the problem-solving process. Students may find it helpful to draw a picture in order to work out what is happening in each story. In most cases, more than one step is needed to find a solution. Problem 2 contains information about white loaves and multigrain loaves that is not needed to find a solution.

Possible Difficulties

- Inability to identify the need to add, subtract, or multiply
- Confusion over the need to carry out more than one step to arrive at a solution
- Using all the numbers given in the problem rather than just the numbers needed

Extension

- Students could write their own problems and give them to other students to solve.

1.1 THE WATER PARK

1. 361 adults and 173 children go through the gates before lunch, and 219 adults and 106 children enter after lunch. How many more adults than children are there?

   ____

2. A total of 397 people are swimming in the six pools. Another 79 people go swimming, while 43 people get out. How many people are now swimming in the pools?

   ____

3. 248 people are lying on their towels. Later, 78 people go swimming, 26 people go for a walk, and 36 people leave to get something to eat. How many people are still lying on their towels?

   ____

4. In the water, 93 people are on floats, 134 are swimming, and 83 are wading in the shallow water. Soon, another 21 people with floats arrive, but 14 also get out. How many people are now on floats?

   ____

5. At the cafeteria, 143 people are sitting eating lunch and 31 are standing in line waiting to order lunch. Two large tables of 12 finish their lunch and leave. How many people are now in the cafeteria?

   ____

6. In the wave pool, 73 surfers are waiting to catch a wave. A large wave comes, and 36 surfers catch and ride it to the beach. How many did not catch the wave?

   ____

1.2 LILY PADS AND FROGS

A lake has 279 lily pads and 372 frogs.

1. 87 of the lily pads are in flower. If each lily pad has three flowers, how many flowers are there altogether?

2. How many lily pads are not in flower?

3. Frogs like to sleep under the lily flowers. How many frogs cannot sleep under a flower?

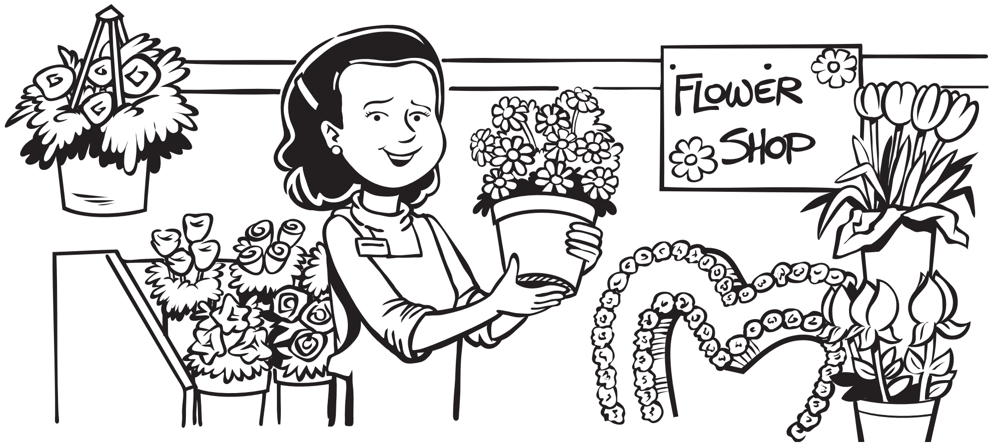
4. When it rains, 38 more lily pads burst into flower. How many lily pads are now in flower?

5. Are there now enough lily flowers for each frog to sleep under?

6. During spring, 129 new lily pads grow, 186 tadpoles turn into frogs, and 75 lily pads die. How many lily pads are now in the lake?

7. During summer, some frogs move to another lake. If 148 frogs move to another lake, how many stay behind?

1.3 SAMANTHA'S FLOWER SHOP



1. Samantha has 38 pots of chrysanthemums left over from Friday. On Saturday, she receives a delivery of another 75 pots of chrysanthemums and sells 59 pots. How many pots of chrysanthemums does she have available to sell on Sunday?

____

2. (a) The delivery truck delivers 384 roses. 186 roses are sold in the morning. In the afternoon, she receives an order for 26 bunches of 5 roses. Does Samantha have enough roses for this order?

____

   (b) How many roses does she still have available to sell after that order?

____

3. Samantha has 120 red roses and 100 yellow roses. She makes 14 bunches, using 9 roses in each bunch. Does she have enough roses to make another 8 bunches?

____

4. (a) Samantha has some lilies left at the end of the day on Friday. She has orders for 82 lilies for her customers on Saturday. She orders 47 lilies to make up the shortfall. How many lilies did she have left over on Friday?

____

   (b) When she goes to work on Saturday, she finds that only 40 lilies were delivered. How many lilies does she have for her orders?

____

1.4 THE HERB MARKET

1. Simon has an herb stall. He sold 373 bunches of fresh herbs and 218 pots of herb plants. How many more bunches of fresh herbs than pots were sold?

2. Simon has 6 trays of herb pots. How many pots does he have if each tray holds 24 pots?

3. Simon sells small bags of herb seeds. Each bag holds 8 seeds. How many seeds does he need to make 45 bags?

4. Simon has 3 boxes of parsley. Each box has 65 bunches of parsley. How many bunches of parsley can he sell?

5. Simon sells 41 bunches of parsley in the morning and 48 bunches in the afternoon. How many bunches does he have left?

6. The next day, he sells 54 bunches of parsley in the morning and 52 bunches in the afternoon. How many bunches does he have now?

7. Simon has 8 boxes of herb plants delivered. Each box contains 12 plants. He unpacks 3 boxes in the morning and 2 boxes in the afternoon. How many plants has he unpacked?

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1.5 AT THE BAKERY

1. The baker baked 624 cakes. She took orders for 287 cakes on Monday, 265 cakes on Tuesday, and 298 cakes on Wednesday. How many more cakes does she need to make?

2. On Saturday, 83 white loaves, 147 whole-wheat loaves, and 163 multigrain loaves were sold. On Sunday, 132 white loaves, 169 whole-wheat loaves, and 178 multigrain loaves were sold. How many whole-wheat loaves were sold over the weekend?

3. (a) The bakery bakes 15 trays of pies. Each tray holds 12 pies. How many pies does the bakery have available to sell?

   (b) If the bakery sells 83 pies, how many pies were not sold?

4. The bakery has 14 trays of brownies. Each tray has 16 brownies. It sold 23 brownies during the first hour, 84 brownies during the second hour, and 73 brownies during the third hour. How many brownies can it sell for the rest of the day?

Problem-Solving Objective

To analyze and calculate information in word problems

Materials

Base 10 materials, place value chart, or a calculator

NCTM Content Standards

- Number and Operations 1.1, 1.2, 1.3

Focus

These pages explore word problems that mostly require more than one step and involve a number of operations, including multiplication and division. Students must determine what the problem is asking and, in many cases, carry out more than one calculation to find a solution. Analysis reveals that some problems contain additional information that is not needed. Materials can, if necessary, be used to assist with the calculation, since these problems are about determining what the problem is asking rather than computation or basic facts.

Discussion

Page 31 – The Plant Nursery

Students must read each problem carefully to determine what the problem is asking. The third investigation involves the concept of profit. Students must have an understanding that "profit" is the money left over after all costs have been calculated. The fourth problem involves pots being fertilized and plants being watered; however, the solution involves only plants being watered, and the information about pots and fertilizer is not required.

Page 32 – The Citrus Orchard

These investigations involve information about a citrus orchard with 12 acres of orange trees. Using the information given in the beginning statement as a basis, the numbers change to meet new criteria as oranges are picked, sorted, and packed into trays. Some solutions may not necessarily be exact; for example, in Problem 1, 273 trees are watered each day, so it will take three days for 819 trees to be watered, leaving just 117 trees to be watered on the fourth day. Some students may discuss their solution in terms of "three days and a little more," while others may say "four days." Some questions require students to keep track of new information and use it to answer the subsequent questions.

Page 33 – Animal World

The wording of these problems has been kept simple to assist with the problem-solving process. Some problems give additional information, often involving weight, which is not needed, and some problems may not have an exact answer; for example, Question 1 involves 52 boxes of fish. If the dolphins eat eight boxes of fish a day, then there is enough fish for six full days, with four boxes left over. Students could discuss their solution in terms of "six days" or even "six and a half days." Students should be encouraged to try different ways of arriving at a solution.

Page 34 – Beading

The problems mostly require multiplication, with some addition and subtraction. Each problem is fairly straightforward, and no additional information is needed. Students may find it helpful to draw a diagram to figure out what is happening in the problem. Addition can be used at times rather than multiplication.

Page 35 – The Commuter Train

The problems focus on trains, cars, people, and seats. In some problems, people are getting on and off, and in others it is necessary to determine how many people are sitting or standing in a car. Each problem requires more than one step, and two or more operations are needed to find a solution. There are a number of ways to find a solution, and students should be encouraged to explore different ways of arriving at a solution.

Possible Difficulties

- Inability to identify the need to add, subtract, multiply, or divide
- Confusion about the need to carry out more than one step to arrive at a solution
- Using all of the numbers given in the problems rather than just the numbers needed

Extension

- Discuss how problems can have more than one answer, depending on different interpretations.
- Explore how many of these problems could be solved using the repeated addition function on the calculator.

2.1 THE PLANT NURSERY

1. During the week, the plant nursery planted 45 rows of flower seedlings and 18 rows of tree seedlings. There are 18 tree seedlings or 56 flower seedlings in each row. How many seedlings were planted?

2. The nursery sold 28 bags of bark mulch on Monday, 23 bags on Tuesday, 34 bags on Wednesday, and 29 on Thursday. If each bag sells for $27, how much money did the nursery make from selling bark mulch?

3. If the nursery buys each bag of bark mulch for $12, how much profit did the nursery make from selling the bark mulch?

4. During the morning, 46 pots were fertilized and 45 rows of plants were watered. During the afternoon, another 56 rows of plants were watered and 36 pots were fertilized. If there are 65 plants in each row, how many plants were watered?

5. The nursery uses large trays that hold 9 seedlings and small trays that hold 6 seedlings. How many seedlings were planted if 236 large trays were used?

2.2 THE CITRUS ORCHARD

The citrus grove has 12 acres of oranges, 8 acres of lemons, and 3 acres of limes.

1. Each acre of oranges has 78 trees and is watered on a rotational basis. Three and a half acres are watered each day before sunrise. How many trees are watered each day, and how many days will it take for all of the orange trees to be watered?

____

The orange crop is ready to harvest. Assume each tree has 80 oranges.

2. How many oranges are ready for sorting and packing into trays after 6 acres have been picked?

____

After harvesting, the oranges are sent to the packing shed for sorting and are packed into crates of 16. Some damaged and blemished oranges are rejected and are not packed into trays. About 7 oranges per 100 are rejected.

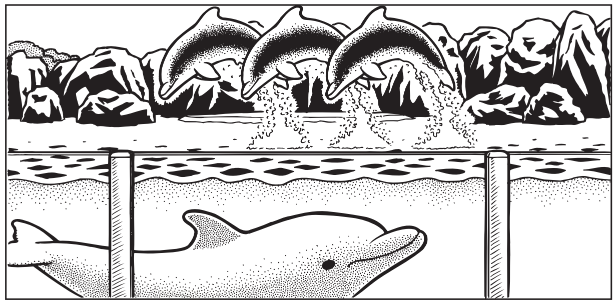
3. How many full crates of oranges were packed after the 6 acres were picked?

____

4. How many full crates of oranges were packed after all of the oranges were picked?

____

2.3 ANIMAL WORLD



1. The dolphins at Animal World eat 8 boxes of fish a day. Each box weighs 5 pounds and costs $9.50. How many days does $494 worth of fish last?

2. The parking lot has 9 sections for cars to park in. Each section holds up to 230 cars. If sections one to six are completely full and sections seven to nine are half full, how many cars are in the parking lot?

3. The giraffes eat 7 bales of hay over two days. Each bale of hay costs $12.00. How much would it cost to feed the giraffes during the month of September?

4. The tropical birds in the walk-through aviary eat 14 bags of seed a week. Each bag weighs 40 kilograms and costs $58.00. How many bags are used over one year?

5. The area to see the dolphin show has 18 rows of seats, with 26 seats in each row. If there are 247 adults and 59 children already seated, how many more people can watch the show?

6. The seals eat 2 cartons of fish a week. Each carton weighs 25 pounds, contains 6 boxes of fish, and costs $57.00. How many pounds of fish do the seals eat over a year?

2.4 BEADING

1. Judy has 6 bags of colored beads, with 42 beads in each bag. If she uses 29 beads to make a necklace, how many beads does she have left?

2. Manu has 7 bags of beads. He buys 2 more bags and uses 25 beads to make some bracelets. If each bag contains 32 beads, how many beads does he have now?

3. Clarence bought 3 bags of beads on Monday, 5 bags on Tuesday, and 4 bags on Wednesday. Each bag contains 48 beads. How many beads did he buy?

4. Marty has 25 bags of beads. She gives 6 bags to her friend Elly and 8 bags to her other friend, Sarah. If each bag holds 36 beads, how many beads did she give away?

5. Ned has 4 large bags of beads and 7 small bags of beads. The large bags hold 25 beads and the small bags hold 15 beads. He uses 24 beads to make a necklace and 18 beads per bracelet to make two bracelets. How many beads does he have now?

6. Zena has 19 bags of beads, with 28 beads in each bag. She uses 36 beads to make 2 necklaces and 54 beads to make 6 bracelets. She wants to make 5 more sets of the same amount of jewelry. Does she have enough beads?

2.5 THE COMMUTER TRAIN

1. During rush hour, a daily commuter train has 18 cars. Each car has 47 seats and can hold 65 people. How many people can travel on the train, and how many of them can sit?

   ____

2. There were 1,104 people on the train. At the first station, 208 people got on and 394 people got off. At the next station, 243 people got on and 172 people got off. How many people are now on the train?

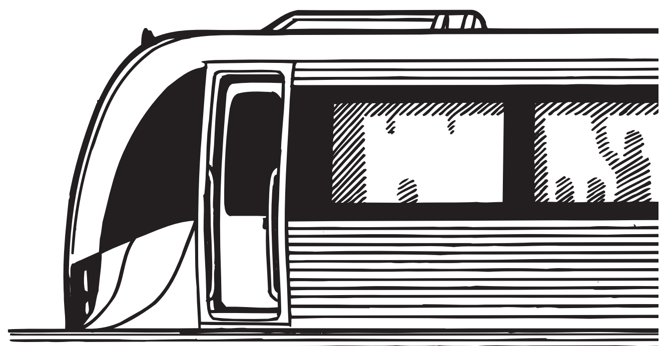
   ____

3. At the end of the line, there are 41 people in the first car, 52 in the second car, 39 in the third car, and 47 in the fourth car. How many more people are needed to fill the train to capacity?

   ____

4. On the weekend the train has only 11 cars. After the fourth stop, there are 527 people on the train. If each seat is taken, how many people are not sitting down?

   ____



5. At the first station, 143 people get on and 238 people get off. At the next station, 154 people get on and 236 people get off. There are now 491 people on the train. How many people were on the train to begin with?

   ____

6. On Tuesday, the train breaks down and all of the passengers have to be taken by bus to the next station. If there are 14 cars on the train and each car is full to capacity, how many buses are needed to take the passengers to the next station if each bus holds 72 people?

   ____

Problem-Solving Objective

To analyze and use information in word problems

Materials

Place value chart, calculator

NCTM Content Standards

- Number and Operations 1.1, 1.2, 1.3

Focus

These pages explore word problems that require a number of operations, including division. The wording has been kept fairly simple to help with the problem-solving process. Students must determine what the problem is asking and in many cases carry out more than one operation to find a solution. If necessary, materials can be used to assist with the calculation, since these problems are about determining what the problem is asking rather than computation or basic facts.

Discussion

Page 37 – The Fruit Farm

These problems require more than one step and involve a number of operations, including division. The wording has been kept fairly simple to assist with the problem-solving process. In many cases, the problem involves more than one fruit, and students must read the problem carefully to determine what is needed to find a solution. For example, in Problem 4 students need to consider apricots and pears as well as trays and boxes to find a solution.

Page 38 – At the Deli

These investigations involve more than one step and use the concepts of grams, kilograms, ounces, and pounds (weight). The concept of profit and repackaging is explored in many of the problems. Students may find it helpful to draw a diagram to visualize what is happening in order to find a solution; for example, in the first question, students could draw and label the cost of a large container followed by 10 small tubs. In this way they can visualize the actual process of repackaging and the cost involved in buying the large container and selling the smaller tubs. Problem 2 requires students to think in terms of how many containers are needed, instead of an exact amount. Four containers are needed to make 34 tubs, but not all of the tomatoes will have been used, since another six tubs could be made from the last container.

Page 39 – The Sugar Mill

These investigations involve weight—in particular, metric tons. In most cases more than one calculation is needed to find a solution. The first question results in a calculation of 763.3 bins. This is an example of a problem in which the answer does not result in a whole number and needs to be thought of as either 763 full bins and 1 bin that is not full, or 764 bins. Similar thinking is required in Problem 6. Since three trucks are used, there would be 37 trips using all three trucks and one trip using only two trucks.

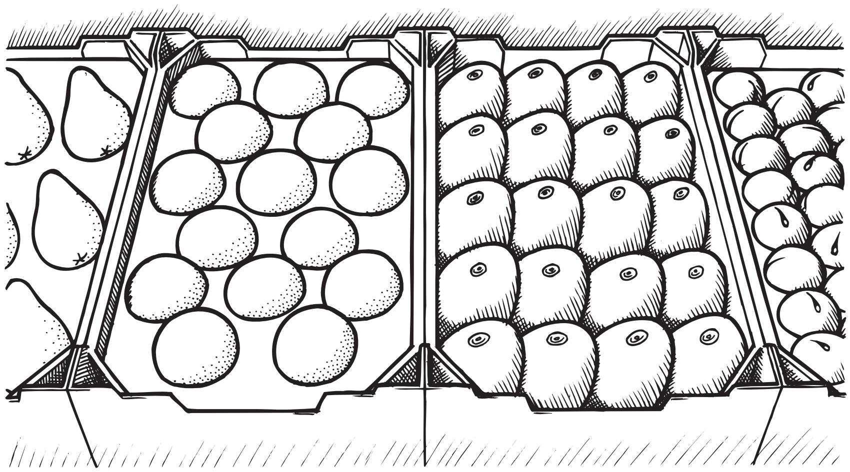
Possible Difficulties

- Inability to identify the need to add, subtract, multiply, or divide
- Confusion about the need to carry out more than one calculation to arrive at a solution
- Using all of the numbers given in the problems rather than just the numbers needed
- Not thinking in terms of the problem and writing solutions such as "763.3 bins"
- Difficulty with the concept of metric tons

Extension

- Students could write their own problems and give them to other students to solve.

3.1 THE FRUIT FARM



1. An amount of pears, peaches, and apricots have been picked and are ready for packing into trays. There are 406 pears, 739 peaches, and 615 apricots. If each tray holds 9 peaches, how many peach trays are needed?

____

2. A box of pears holds 4 trays, and each tray holds 12 large pears. If there are 1,152 pears to be packed, how many boxes and trays are needed?

____

3. (a) The fruit pickers picked 2,340 medium-sized mangoes over 6 days, which were then packed into 52 boxes. How many mangoes are in each box?

   ____

   (b) If each box holds 3 trays, how many mangoes are in each tray?

   ____

4. There are 35 boxes of apricots and 68 trays of pears ready for the market. Each box has 48 apricots, and each tray has 12 pears. How many pears and apricots are packed?

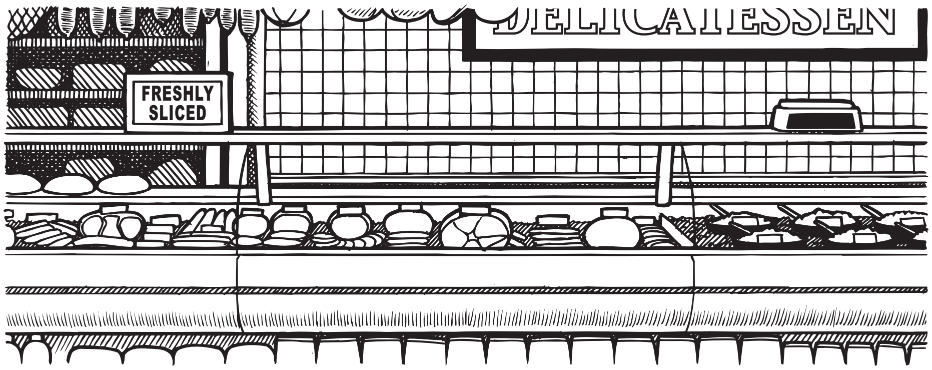
____

5. The fruit pickers picked 487 peaches in the morning and 364 peaches in the afternoon. If they worked for 6 hours, approximately how many peaches did they pick per hour?

____

3.2 AT THE DELI

OPEN SIX DAYS A WEEK (CLOSED MONDAYS)



1. The deli buys 2 kg containers of sun-dried tomatoes, which they repackage and sell in 200 gm tubs for $6.00. If each container costs them $19, how much profit do they make on each container?

   ____

2. About 34 tubs are sold each week. How many containers does the deli need to buy each week so that there are enough tubs to sell?

   ____

3. The deli sells marinated green olives in 4 oz tubs and loose by the pound. The tubs sell for $3.50 each and the loose olives sell for $11.50 per pound. Which is the better way to buy 1.6 lb of olives?

   ____

4. The deli buys its loose olives for $7.25 per pound. During the summer months, about 13 lb of loose olives are sold each week, and during the winter months about 9 lb are sold. What is the difference in weekly profit between the summer and winter months?

   ____

5. Eighteen boxes of bacon were delivered. Each box contains 12 packages of bacon. Six boxes were unpacked in the morning and 8 boxes in the afternoon. How many packages have been unpacked?

   ____

3.3 THE SUGAR MILL

1. During the crushing season, a sugarcane farm transported 4,580 metric tons of cane by train to the sugar mill. A train usually hauls between 135 and 145 bins. If each bin holds 6 metric tons of cut sugarcane, how many bins of cane were transported to the sugar mill?

____

2. If each train is able to haul a maximum of 145 bins, how many train trips to the mill were there during the season?

____

3. During one week, trains transported cane from 4 different farms. If 19,370 metric tons of cane was transported, how many bins and trains were used?

____

4. During the harvesting season, one farm transported 6,790 metric tons of sugarcane to be crushed. If the harvesting season is from June to December, approximately how much cane was harvested per month?

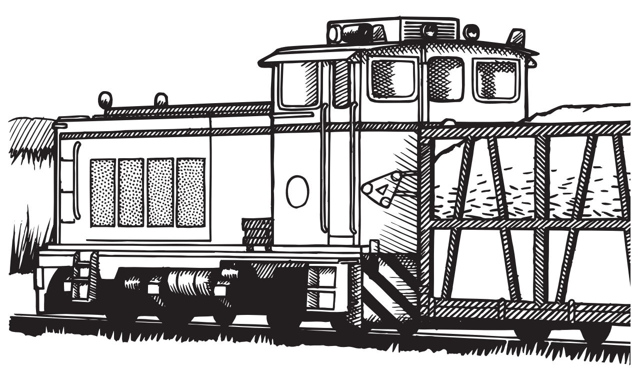
____

5. It takes approximately 8 metric tons of cut sugarcane to produce one metric ton of raw sugar. How much raw sugar would be produced from 8,976 metric tons of cut sugarcane?

____

6. During the crushing season, raw sugar from the mill is transported to the port by trucks, each of which can carry 7 metric tons. If 3 trucks were used, how many trips would be needed to transport 790 metric tons of raw sugar?

____



Problem-Solving Objective
To read and interpret information using a calculator

Materials
Calculator, number expander

NCTM Content Standards
- Number and Operations 1.3

Focus
This pages explores concepts of place value and number sense.

Discussion
Page 41 – Calculator Problems
These problems give information about numbers that students must read, interpret, and enter into a calculator to find a given number. An understanding of place value is needed to enter the information. Students may use a number expander to help them with place value, if needed. No formal addition or subtraction is needed.

With Question 1, students must start with a number 100 less than 4,086. Using an understanding of place value, we know that the number has 40 hundreds, and 1 hundred less would be 39 hundreds, so the starting number is 3,986.

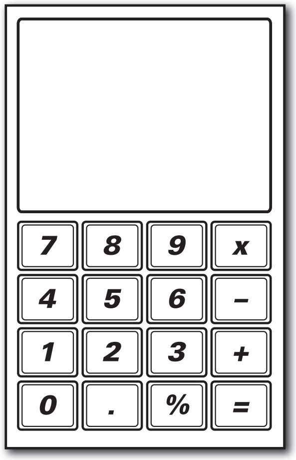
Possible Difficulties
- Poor understanding of place value
- Wanting to add, subtract, or multiply rather than using place value or number sense
- Not considering all of the criteria

Extension
- Students could think up their own calculator problems and write the criteria to match.

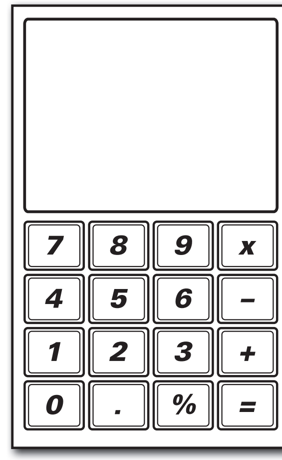
4.1 CALCULATOR PROBLEMS

Solve the problems. Use your calculator to help you.

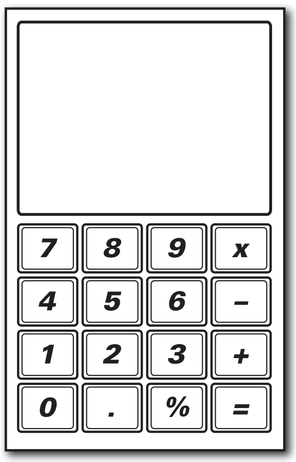
1. Enter the number that is 100 less than 4,086. Take away 317 and add 2,006. What number am I?



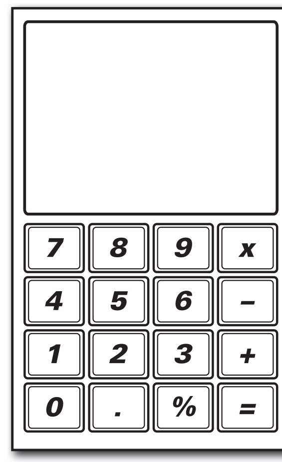
2. Enter the number with 562 tens and 9 ones. Add 23 hundreds, 4 tens, and 7 ones. Take away 10 hundreds. What number am I?



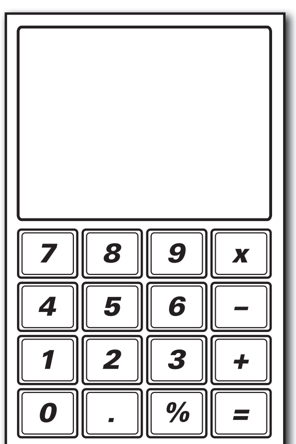
3. Enter the number 1,000 before 8,293. Take away 24 tens. Add 3 ones and 634 tens. What number am I?



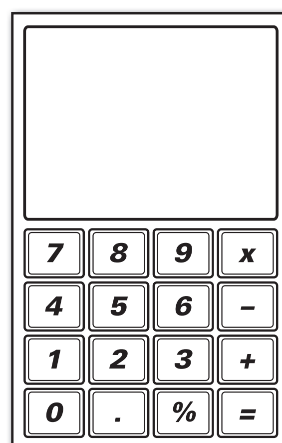
4. Enter the number 100 more than 6,958. Make it 1,000 more. Make it 100 less. Add 17 hundreds. What number am I?



5. Enter the number with 82 hundreds and 5 ones. Add 34 tens. Take away 800 and 6. What number am I?



6. Enter the number one before 9,900. Take away 89 tens. Make it 100 more. Take away 3,000 and 14. What number am I?



Problem-Solving Objective

To read, interpret, and analyze information

Materials

Calculator, number expander

NCTM Content Standards

- Number and Operations 1.1, 1.2, 1.3

Focus

These pages explore concepts of place value and number sense. The relationships among numbers and place value are analyzed, and students are encouraged to not only find numbers that are possible but also to disregard numbers that are not possible. In most cases place value and number sense are needed rather than addition or multiplication.

Discussion

Page 43 – Ice Cream Cones

These problems require students to think in terms of place value. Since 10 cones fit into one box and 10 boxes fit into one carton, an understanding of place value can be used to solve each problem. For example, Question 1 involves 9 boxes and 24 cartons. It can be solved by thinking in tens or by thinking in tens and hundreds. By understanding place value, it is known that there will be 90 cones in the boxes and 2,400 cones in the cartons, giving a total of 2,490 cones. No formal multiplication or addition is required.

Page 44 – The Library

This investigation provides information about a library, with a number of interrelated questions arising from it. The questions begin with a set number of books, magazines, and CD-ROMs. Using this information as a basis, the numbers of each item are changed to meet new criteria, with some books, magazines, and CD-ROMs being borrowed, returned, shelved, and dusted. Students are required to keep track of the new information and use it to answer the subsequent questions.

Computation is not always needed to find a solution. For example, Problem 1 states that each shelf holds 100 books, so if there are 9,472 books in the library, 95 shelves are needed. No division is necessary, since place value tells us that 9,472 has 94 hundreds, so 95 shelves must be needed. Similarly with Problem 2, 48 shelves are being dusted, which means 48 hundreds. Therefore, 4,800 books have been dusted.

Page 45 – Bookworms

This investigation requires students to read and interpret information while thinking in terms of place value. Since the book has 10 chapters of equal length, an understanding of place value can be used to solve each problem. Given that the book has 1,380 pages, we know from place value that this number has 138 tens, so each chapter must be 138 pages long. This information can be used to calculate the various problems.

Before starting, students may find it helpful to draw up a table, noting the starting page of each chapter. Each chapter has 138 pages and Chapter 1 starts on page 1, so Chapter 2 must start on page 139, and so on. The page number that Wanda has read to is given in the beginning, and this information is needed to answer some of the problems. For example, to determine how many pages Wanda has read past the middle of the book, it is necessary to keep in mind that there are 1,380 pages in the book and she has read up to page 760. Similarly, this information is needed again to determine how many pages she must read to finish the book.

Possible Difficulties

- Not using place value to solve the problems
- Not considering all of the criteria
- Confusion over the need to carry out more than one step to arrive at a solution
- Not taking into consideration the starting page of each chapter

Extension

- Students could write other problems involving the ice cream cones.
- Change the criteria involving the number of pages in a book and the page number read to and explore the problems again based on the new criteria.

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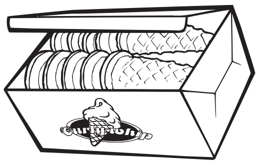
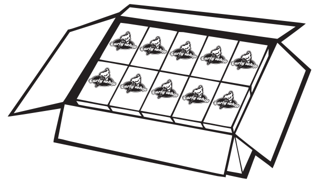
5.1 ICE-CREAM CONES

There are 10 ice-cream cones in a box.



There are 10 boxes in a carton.



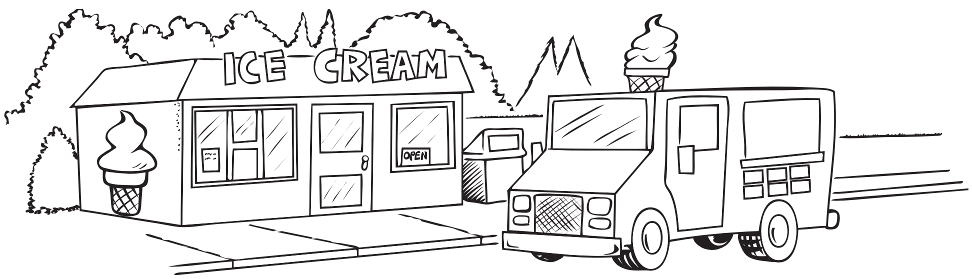
Use the information above to solve these problems.

1. Anna has 9 boxes and 24 cartons of ice-cream cones. How many cones does she have?

   ____

2. The ice-cream factory has 593 boxes and 70 loose cones still to be boxed. How many cartons will they need?

   ____



3. The truck delivered 48 cartons and 17 boxes of ice-cream cones to the snack bar. How many cones were delivered?

   ____

4. The truck has a load of 94 cartons and 713 boxes of cones. It delivers 5,600 cones. How many cones does it have left?

   ____

5. The ice-cream shop has 52 boxes and 17 cartons of cones. It receives a delivery of 600 cones. How many cones does it now have?

   ____

6. At the end of the day, the ice-cream shop has 840 cones. If there are 6 full cartons and no loose cones, how many extra boxes of cones does it have?

   ____

5.2 THE LIBRARY

The library has books, magazines and CD-ROMs. It has a total stock of 9,472 books, 315 magazines, and 143 CD-ROMs.

1. In the library, each shelf holds 100 books. How many shelves are needed to hold all of the books?

   ____

2. The cleaner has dusted 48 shelves. How many books have been dusted?

   ____

3. (a) At the start of the day, the library's computer showed there were 6,841 books currently in the library. During the morning, 275 books were taken out and 97 books returned, and during the afternoon 166, books were taken out and 134 returned. How many books are now in the library?

   ____

   (b) Later in the day, 6 boxes of books, 2 boxes of magazines, and 4 boxes of CD-ROMs were delivered to the library. If each box of books holds 65 books, how many books will now be in the library?

   ____

4. Every student in the fourth grade borrowed a book from the library. If there are 4 classes with 26 students in each class, how many books were borrowed?

   ____

5. During stocktaking, 284 old and damaged books were removed from the shelves to be packed into boxes. If each box can hold 10 books, how many boxes are needed to pack all of the books?

   ____

5.3 BOOKWORMS

Wanda's book starts on page 1 and has 1,380 pages. There are 10 chapters of equal length in the book, and she has read up to page 760.

1. How many pages are in each chapter?

2. Wanda's favorite page is 409. What chapter is it in?

3. Wanda reads 10 pages each morning and 10 pages each night. How long has she been reading the book?

4. Wanda's favorite chapter is Chapter 5. What pages are in Chapter 5?

5. The most exciting part was from page 619 to page 694. What chapters are these pages in?

6. How many pages has Wanda read past the middle of the book?

7. Wanda's friend is also reading this book. She has read 8 pages of Chapter 6. What page is she up to?

8. How many more pages does Wanda's friend need to read to finish the book?

9. How many more pages does Wanda need to read to finish the book?

10. If Wanda reads 10 pages per day from now on, how long will it take her to finish the book?

Problem-Solving Objective

To use logical reasoning and spatial visualization to solve problems

Materials

Counters, base 10 materials

NCTM Content Standards

- Number and Operations 1.1

Focus

This page explores problems based on a conceptual understanding of fractions. Writing the fractions using numbers and words is designed to help students focus on the number of parts as well as their comparative sizes and to lead them to consider ways of solving the problems other than by fraction calculations. One way of solving them is by backtracking from the answers. Counters can be useful, as they allow the parts to be considered while the whole problem is also kept in mind. Using a diagram is another method and is probably a different way of thinking about fractions than is used by many students and teachers.

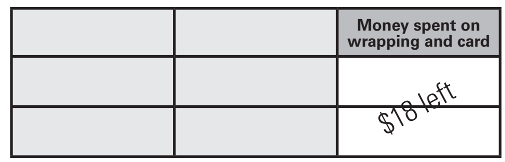
Discussion

Page 47 – Shopping

These problems can be used as a consolidation of the "analyze-explore-try" model of problem solving that has evolved with the various number, spatial, and measurement situations posed in this book. This model is discussed in detail in the Introduction. In these problems the information must be carefully analyzed to determine how much money was available at the beginning rather than at the end of a situation. Students could use counters or base 10 materials as they work through the questions either forwards or backwards. Using "try and adjust" is another possible way to find a solution.

Placing the amounts involved in Question 2 in a diagram (as shown below) can help. Peter originally spent two-thirds of his total amount on the present. This is represented by the two shaded blocks.

With the third left, he spent one-third of the amount on wrapping and a card. This is represented by the shading in the final block.



The $18 he could spend on himself is shown by the two unshaded parts of the diagram. This means that each of the two parts must be $9, and he took 9 × $9, or $81 in total, to shop with.

The last problem can be solved by using a similar diagram.

Possible Difficulties

- Not confident working with fractions
- Thinking that one-third and one-half gives one-fifth
- Thinking that two-thirds of Peter's initial money, together with one-third of what is left, must account for all of his money
- Not considering using materials or a diagram and attempting a solution on the basis of calculations alone

Extension

- Change the fraction amounts but leave the problem statements the same.
- Change the problem contexts but leave the fraction amounts the same.
- Have students make up problems of their own and challenge others to solve them using diagrams.

6.1 SHOPPING

1. On Saturday, Peta went to the mall to buy a new outfit to wear to her friend's birthday party. She spent half of her money on a dress and then one-third of what she had left on a pair of sandals. She was left with $60.00 in her purse. How much money did she have to start with?

2. On Sunday, her brother Peter went to the mall to buy a birthday present for the party. He spent two-thirds of his money on the present and then spent one-third of what he had left for a card and wrapping paper. That left him with $18 to buy something for himself. How much money did he originally have when he went shopping?

3. The next week Peta's friend Sharon did her holiday shopping. After she spent one-fifth of the money she had on her brother, she spent one-half of what she had left on her mother, and one-fourth of her remaining money on her father. She had only $15 left. How much money did Sharon originally take to spend?

Problem-Solving Objective

To solve problems involving money and make decisions based on particular criteria

Materials

Counters, play money, or a calculator

NCTM Content Standards

- Number and Operations 1.1, 1.3

Focus

This page explores reading for information, obtaining information from another source (the takeout menu), and using it to find solutions. The problems are about using money, making decisions based on money, and comparing amounts of money, rather than adding or subtracting. Solutions can be obtained using materials and comparing amounts. Counters, blocks, play money, or a calculator can be used if needed.

Discussion

Page 49 – Chicken Takeout

Students read the items in the menu and note how much each costs. In most cases, students need to buy two or more of an item and then determine the amount spent and the change that remains.

The first two problems involve determining how much is spent on particular orders. Students must take into consideration that more than one of some items have been ordered. The remaining problems analyze particular orders, with an additional focus on how much change would be received from $50.

Possible Difficulties

- Unfamiliarity with the $ symbol
- Not taking into account that they may need two or more of some items

Extension

- In pairs, students write their own questions based on the takeout menu and give them to other pairs to solve.

7.1 CHICKEN TAKEOUT

Chicken burger	$4.50	Small coleslaw	$2.50
Chicken & cheese burger	$5.00	Large coleslaw	$3.00
Chicken & bacon burger	$5.50	Small drink	$1.50
Chicken & coleslaw	$6.00	Large drink	$2.50

How much money did the people spend at the takeout restaurant?

1. Liam ordered ...
   1 chicken burger
   1 large coleslaw
   1 small drink
   Amount spent: ....

2. Rosanna ordered ...
   1 chicken & coleslaw
   2 chicken & bacon burgers
   3 large drinks
   Amount spent: ....

How much money did the following people spend, and how much change did they receive from $50?

3. Kayla ordered ...
   2 chicken & bacon burgers
   1 large coleslaw
   2 large drinks
   Amount spent: ....
   Change: ....

4. Samir ordered ...
   3 chicken & coleslaw
   2 chicken & cheese burgers
   5 small drinks
   Amount spent: ....
   Change: ....

5. Julian ordered ...
   3 chicken burgers
   3 large coleslaw
   3 large drinks
   Amount spent: ....
   Change: ....

6. Danny ordered ...
   2 chicken & bacon burgers
   2 chicken & cheese burgers
   4 large drinks
   Amount spent: ....
   Change: ....

7. Who spent the most? ____________________

Problem-Solving Objective

To identify and use concepts about numeration

Materials

Counters, blocks, or a calculator

NCTM Content Standards

- Number and Operations 1.1, 1.3

Focus

These pages explore solving problems involving number sense, magic squares, and logic. Students must carefully analyze the problems to locate information necessary to find the magic number or arrangement of numbers. Counters, blocks, or a calculator can be used, as these problems focus on the concepts of number sense and number logic rather than basic facts.

Discussion

Page 51 – Star Gaze

This activity investigates possible combinations of numbers to make a total. Students are given a list of numbers that make up that score. Number sense and the "try and adjust" strategy are needed to find solutions.

For example, the score of 32, using the digits 5 and 9, requires the combination of 9, 9, 9, and 5, while a score of 57, using 24, 7, and 13 requires the combination of 24, 7, 13, 13.

Page 52 – Magic Squares/Famous Magic Squares

This investigation involves the concept of magic squares. Simple 3-by-3 magic squares are used to establish the idea that each row, column, and diagonal adds to the same magic number. In this case, the concept is explored further as students investigate a famous magic square.

The Dürer magic square has a number of points of interest aside from being a magic square. Not only does each row, column, or diagonal add to the same number, but so do the four corners, the four middle numbers, and the four corner 2-by-2 grids. Also, it uses the numbers 1 to 16 and shows the year in which it was constructed.

Page 53 – Sudoku

This pages explores sudoku. The word *sudoku* roughly means "digits must occur only once." In this case, 4-by-4 and 6-by-6 grids are used. In Problem 1, every row, column, and minigrid must contain one of each of the digits 1 to 4, while the second set uses the digits 1 to 6. No addition or basic facts are involved and students need to use logical reasoning to find solutions.

Possible Difficulty

- Considering only rows or columns rather than rows, columns, and diagonals in the magic squares or the smaller grids in the sudoku puzzles

Extension

- Investigate other magic squares and magic numbers.
- Explore sudoku games in magazines, newspapers and on the Internet that involve 4-by-4 grids as well as 9-by-9 grids.

8.1 STAR GAZE

Star Gaze is a new computer game.
Each time you win, you collect stars.
Total the four stars to find your score.

1. Figure out these scores.

(a) 5, 9, 6, 3

Score: ____________

(b) 17, 19, 15, 4

Score: ____________

(c) 15, 33, 24, 33

Score: ____________

2. Add combinations of the numbers given to find the score.

(a) **Score: 27**
use only 3 and 8

(b) **Score: 32**
use only 5 and 9

(c) **Score: 26**
use only 6 and 7

(d) **Score: 16**
use only 5, 2 , 7

(e) **Score: 21**
use only 8, 1 , 4

(f) **Score: 28**
use only 9, 5 , 7

(g) **Score: 57**
use only 24, 7, 13

(h) **Score: 71**
use only 6, 19, 23

(i) **Score: 99**
use only 23, 37, 16

8.2 MAGIC SQUARES

In magic squares, all of the numbers in each row, column, and diagonal add to the same total.

7	0	5
2	4	6
3	8	1

1. This magic square has a magic number of ________.

Complete the magic squares below. Remember, all rows, columns, and diagonals must add to the same total.

2. (a)

		11
	9	
7		3

Magic number: ________

3.

80		
	50	70
		20

Magic number: ________

(b) What do you notice about each number in the square?

____

FAMOUS MAGIC SQUARES

The following magic square was constructed in 1514 by an artist named Albrecht Dürer. Complete the magic square and discover why it is so famous.

16			13
5	10		8
9		7	
	15		1

4. Magic number: ________
5. What do the four corners add to? ________
6. What do the four middle squares add to?

____

7. What do the four squares in each corner add to?

____

8. What numbers have been used? ________
9. Can you see anything to do with the date it was first constructed? ________________

8.3 SUDOKU

Every row, column, and minigrid of these Sudoku puzzles must contain one of each of the numbers 1, 2, 3, and 4. For example:

2	1	3	4

			4
			1
			2
			3

		3	4
		1	2

2	1	3	4
3	4	2	1
4	3	1	2
1	2	4	3

The completed sudoku has the numbers 1, 2, 3, and 4 in every row, column, and minigrid.

1. Complete each sudoku, using the digits 1 to 4.

(a)

	1	2	
2		3	
	2		3
	3	4	

(b)

	1		2
		3	1
2	4		
1	3		

(c)

3			2
		1	3
1	3		
4		3	

2. Complete each sudoku, using the digits 1 to 6.

(a)

2					
5				4	
		4		2	
3	6		4		
		3			4
			1		3

(b)

			1		4
	6		3	2	
	3		4		
				3	
		4		1	
5		6			

Problem-Solving Objective

To use strategic thinking to solve problems

Materials

0–99 number board

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

NCTM Content Standards

- Number and Operations 1.1
- Algebra 2.1

Focus

These pages explore students' understanding of the number system and their ability to solve questions about numbers. Students must coordinate the reading and writing of numerals with the symbols involved in writing the numbers 400–600.

Discussion

Page 55 – How Many Digits?
This problem explores students' ability to reason about the number system and keep track of the possibilities they find. Students need to discuss what it means to say a number as opposed to writing it.

When pronouncing certain numbers—for example, with numbers that include *four*—it is important for students to include numbers that include the pronunciation of the word part *four* in *fourteen*, *forty*, *forty-one*, and so on.

If students take the problem further and try other number ranges, as suggested, they will find a different pattern altogether for the three hundreds as opposed to the four hundreds. After the five hundreds, the patterns begin to repeat. When students notice this, they will have really come to terms with the strategic thinking needed to organize and solve problems with several interacting conditions.

Trying different number ranges or counting and writing in fives and 25s will provide different patterns as students coordinate what happens to the ones, tens, and hundreds digits.

Possible Difficulties

- Unable to keep track of the number of times they determine a digit or word
- Confusion between saying and writing the digits
- Not understanding that we say "four" in numbers that contain *forty*, even though it is written differently from *four* or *fourteen*
- Thinking that "five" is said for numbers that have *fifty* in the name
- Confusion about 44, 55, and so on, seeing the word *four* or *five* only once, whereas it actually occurs in both the ones and tens places

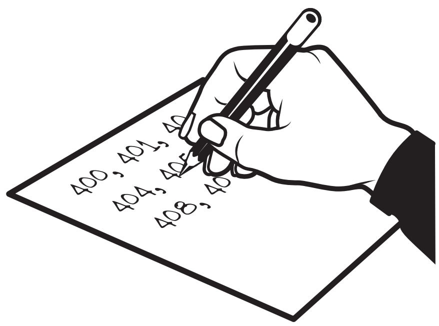
Extension

- Try other numbers ranges such as 300–500, 400–700, and so on.
- Investigate the number of times "five" is said and written when counting by fives or 25s.

9.1 HOW MANY DIGITS?

1. (a) How many times do you say “four” when you count from 400 to 600?

____



(b) How many times do you write “4” when you write all of the numbers from 400 to 600?

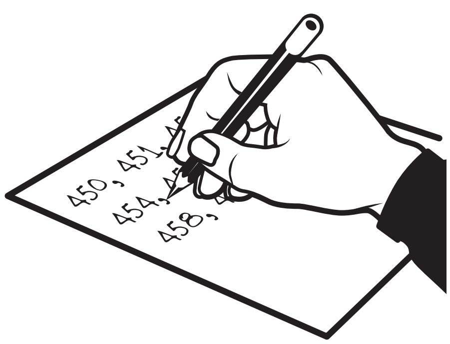
____

2. (a) If counting from 400 to 600, would you say “five” more or fewer times than you would say “four”?

____

(b) How many times would you say “five”?

____



(c) How many times would you write “5” when writing all of the numbers from 400 to 600?

____

3. Try other one-digit numbers. Can you see a pattern?

____

____

Problem-Solving Objective
To use patterns and logical reasoning to determine numbers in a table

Materials
Calculator

NCTM Content Standards
- Number and Operations 1.1
- Algebra 2.1

Focus
This page explores problems that use the ordering of numbers to discern patterns. In this way larger numbers can be determined without laboriously writing or counting all of the numbers up to the number asked for. The page also highlights the value of using factors and multiples when thinking about numbers.

Discussion
Page 57 – Numbers in Columns
In Questions 1–4, most students will observe that any number with a 5 in the ones place occurs in Column E. However, they may be surprised that a number with a 0 in the ones place does not also occur in this column. Both 87 and 34 are placed in Column D, and this may lead them to observe that each column contains two different types of numbers. Any number with 2 in the ones place occurs in Column B, so this is where 92 is; meanwhile, any number with 8 in the ones place, including 108, is in Column C.

The pattern is:

Digit in Ones Place	Column
1 or 0	A
2 or 9	B
3 or 8	C
4 or 7	D
5 or 6	E

The challenge now is to describe the row where each of the numbers occur! One pattern is that since the numbers occur in blocks of ten, then:

Digit in Tens Place	Row
0	1 and 2
1	2, 3, and 4
2	4, 5, and 6
3	6, 7, and 8
4	8, 9, and 10
5	10, 11, and 12
. . .	. . .

Some students may be able to describe this: Multiply the digit in the tens place by 2 to get the row with 0 in the ones place. The next row has the numbers with 1, 2, 3, 4, or 5 in the ones place; the one after for the numbers with 6, 7, 8, or 9 in the ones place.

For example, a number with 10 tens must be in row 20, 21, or 22—108 will be in row 22.

For Question 5, the patterns for the ones digit is:

Digit in Ones Place	Column
1 or 6	A
5 or 0	B
2 or 7	C
4 or 9	D
3 or 8	E

Numbers with 0 tens are in rows 1–4
1 tens are in row 4–7, etc.

In Question 6, comparing the two arrangements shows that only the digits 4 and 1 remain in the same columns. Some students may be able to observe that 20% of the numbers are unchanged.

Possible Difficulties
- Thinking that writing out all of the numbers is the only way to be sure of a solution
- Considering only the ones place when searching for a pattern

Extension
- Describe a pattern for the row in which a number occurs in the second arrangement.

10.1 NUMBERS IN COLUMNS

The counting numbers are arranged in five columns: A, B, C, D, and E.

A	B	C	D	E
1	2	3	4	5
10	9	8	7	6
11	12	13	14	15
		...	17	16

1. In which column will 75 appear?

   ____

2. In which column will you find:

   (a) 34? ____________

   (b) 61? ____________

   (c) 87? ____________

   (d) 92? ____________

   (e) 108? ____________

3. In which row will you find:

   (a) 34? ____________

   (b) 61? ____________

   (c) 87? ____________

   (d) 92? ____________

   (e) 108? ____________

4. Can you find and describe a pattern for the rows and columns to help you find where any number would be?

   ____

   ____

   ____

The arrangement of counting numbers was changed to:

A	B	C	D	E
1		2		3
	5		4	
6		7		8
	10		9	
11		...		

5. Can you describe a pattern for the columns to help you find numbers in this arrangement?

   ____

   ____

   ____

6. Do any numbers appear in the same column as they were in the first table?

   ____

   ____

Problem-Solving Objective

To use strategic thinking to solve problems

Materials

Counters in several different colors, coins

NCTM Content Standards

- Number and Operations 1.1, 1.2, 1.3
- Algebra 2.1

Focus

This page explores other complex problems in which the most difficult step is determining what each question is asking. Using materials to explore the situation is one way this can be done. Another is to use a diagram to assist in thinking backwards or using "try and adjust" to find a solution that matches all of the conditions.

Discussion

Page 59 – Puzzle Scrolls 1

The puzzle scrolls contain a number of different problems, all requiring strategic thinking to find possible solutions. In most cases, students will find that tables, lists, and diagrams are needed to manage the data while exploring the different possibilities. In Problem 2, combinations that include zero must be considered, since zero can be used in a personal identification number. Students also need to consider that a digit can be used more than once; for example, 4, 4, 4, and 7 could be used.

Page 60 – Scrapbooking

These problems require knowledge of division with remainders, multiples, and factors. There are many ways they can be solved—using counters to see what is happening, "try and adjust" using different numbers, placing the attempts in a table to organize an approach, and using some form of simple algebraic thinking. Analyzing Problem 1 shows that 4 x the number of pages + 7 must be the same as 5 x 1 less than the number of pages:

Number of pages	4 × number of pages + 7	5 × 1 less than number of pages
4	23	15
8	39	35
12	55	55

There are 12 pages and 55 photos.

Page 61 – Changing Coins

Problem 1 can be solved by using coins or counters to keep track of what is happening. Another way is to put the information in a table where what is occurring is readily seen:

10¢	10¢	10¢	10¢	10¢	10¢	10¢	10¢	10¢	10¢
	$1		$1		$1		$1		$1
		$2			$2			$2	
			50¢				50¢		
				25¢					25¢

He would have $8.70, and two dimes would not change.

This approach will readily solve the second problem as well. The pattern is that the coins in a prime number position (2, 3, 5, 7, 11, 13, …) only change twice; and only those in square number positions will show heads (1, 4, 9, 16, …).

Possible Difficulties

- Considering only some aspects of the puzzle scrolls
- Not taking into consideration the remainders when considering multiples
- Not being able to keep track of the changes in the coins

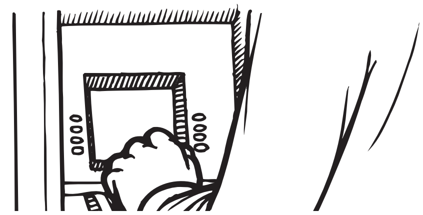
Extension

- Write problems based on the puzzle scrolls.
- Provide other combinations of pictures and pages for the scrapbook problem.
- Extend the coin problems to start with larger numbers of coins and look for patterns.

11.1 PUZZLE SCROLLS 1

1. A new dam was built, and it took 48 days for it to fill with water. If the amount of water it stored doubled each day, how long did it take to be half full?

2. My PIN has four digits that, when added, total to 19. They could be 1, 3, 6, and 9.



What else could they be?

3. (a) What two-digit number is 3 times the sum of its digits?

   (b) Can you find any that are four times the sum?

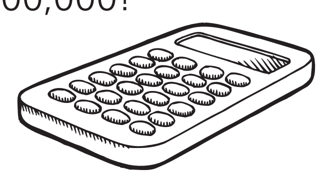
4. A math club starts with one member, who then recruits two new members the following week. Each new member then does the same each following week.

   How many members are there after 5 weeks?

5. An atlas has 6 consecutive pages of maps of the United States The sum of these page numbers is 237.

What are the page numbers?

6. What two whole numbers, neither of which contains a zero, have a product of 1,000,000?



How many can you find?

11.2 SCRAPBOOKING

1. Layla is organizing photos of her family to put in her scrapbook. If she puts 4 photos on each page, 7 photos will not be able to be included in her scrapbook. If she puts 5 photos on each page, 1 page will be empty. How many photos does she have to put in her scrapbook, and how many pages does the scrapbook have?

____

____

2. When he saw what his sister was organizing, Sammy decided to put pictures of his favorite cars in a scrapbook like Layla's. He had some large and some smaller pictures, so he decided to put 4 pictures on each page. However, this meant there were 3 pages without any pictures. When he changed the arrangement to 3 pictures on a page, there were 5 pictures that he had no space for. How many pictures did he have, and how many pages were in his scrapbook?

____

3. Sammy noticed that when the football players in his club were being organized to sit on benches for the end-of-season photo, if 5 players were seated on each bench, 4 players did not get a seat. When they moved closer together to fit 6 players on each bench, they now had enough room for the coach and assistant coach to sit down. How many football players are there in Sammy's club?

____

4. What is the smallest number that, when divided by 3, has a remainder of 1; when divided by 4, has a remainder of 2; when divided by 5, has a remainder of 3; and when divided by 6, has a remainder of 4?

____

11.3 CHANGING COINS

John put out a row of ten dimes. He asked his mother to exchange every second coin for a $1 bill, his father to exchange every third coin for a $2 bill, his brother to exchange every fourth coin for a half dollar, and his sister to exchange every fifth coin for a quarter.

1. (a) How much money did he have after all the exchanges were made?

   ____

   (b) How many of the original dimes were not changed? ______________

   (c) Were any coins changed three times? ______________

   (d) Which coins were changed twice? ______________

   Once? ______________

His sister, Jane, set out 16 dimes tails up in a row. Then she changed the coins by turning them over so that tails became heads or heads became tails.

The first time she turned over every coin, the next time every second coin, the next time every third coin, and so on, until she had turned the 16th coin five times.

2. (a) After she made all the changes, which coins displayed heads?

   ____

   (b) Which coins changed only once? ______________

   Twice? ______________

   (c) If she put out more coins and turned them following the same pattern, what would be the next coin to show heads? ______________

   (d) If she put out 25 coins, what other coins would be changed only twice?

   ____

   (e) Can you describe a pattern for the changes?

   ____

   ____

Problem-Solving Objective
To use spatial visualization and logical reasoning to solve problems

Materials
Cubes such as wooden, Multilink™, or Unifix™

NCTM Content Standards
- Geometry 3.1, 3.3, 3.4

Focus
These pages explore arrangements and dissections of three-dimensional shapes to determine how particular outcomes are formed. Students investigate all likely arrangements to ensure that the final forms match the given criteria or visualize a given shape in terms of its component parts.

Discussion
Page 63 – Stacking Shapes
Students must be able to visualize the arrangements of cubes, oranges, and cans stacked in several layers. Some will see the first drawing as consisting of one layer of 16 cubes, the next layer of nine cubes, the next of four cubes, and then one cube on top. Others will see slices going down the staircase—10, then 9, 7, and 4—or see these building upward. They will need 30 cubes to build the arrangement.

The stacks of oranges and cans have either a triangular or square base, with the shapes stacked on the intersection of the ones below rather than one on top of another.

The final shape does not follow a simple pattern, and students will need to visualize how the blocks are stacked and find a way to organize their count. This will influence how they build their own structures and how they keep track of the individual cubes.

Page 64 – Painted Cubes
Seeing how cubes make a larger structure is now extended to applying paint to a completed shape and visualizing the effect on the individual cubes. Students may need to build the structures with blocks to see what happens. Some students may need to place labels on certain faces of the cubes to help them come to terms with what the questions are asking.

Encourage the systematic analysis of the shapes when discussing the results: Could any cubes have all sides painted? Where would cubes with six, four, three, two, or one side(s) painted be found?

Page 65 – Cube Painting
The problems on this page extend the thinking needed to visualize patterns. A number of small cubes are hidden inside the larger structure, and careful analysis is needed to determine how individual cubes would be painted. A table of possibilities is one way of keeping track of what is happening.

Page 66 – Viewing Cubes
These investigations encourage students to see three-dimensional shapes in terms of the component two-dimensional forms. When viewed from above and below, the outline will appear the same. The views from opposite sides will be mirror images of each other. Understanding this can be difficult, and students must be encouraged to be systematic in constructing the shapes. When making a shape for another student, rather than draw the different views, students may take digital photos of the shapes for other students to build from.

Page 67 – Nets and Cubes
The activities on this page ask students to visualize how a two-dimensional representation can be folded to make a cube. Some students may need to cut out the nets from grid paper and fold them to see which form cubes. The die at the top of the page shows 6, 3, and 2 dots, so the opposite sides would have 1, 4, and 5 dots. (It is important to realize that the numbers on the opposite sides of a die always add to seven.) Seeing which dots would occur on the blank squares requires students to visualize how the squares fold and which faces would be opposite each other. There are several ways the numbers could be arranged on the blank die, with opposite sides adding to seven. Writing letters on the net to match the given views extends this thinking.

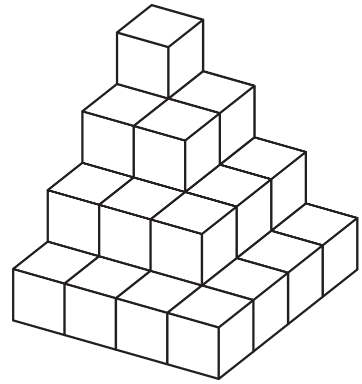
Possible Difficulties
- Unable to visualize the shapes from the two-dimensional representations
- Unable to keep track of the possibilities in any of the investigations

Extension
- Have some students make stacks using different arrangements of cubes and have others use isometric paper to draw the stacks and see how many cubes were used.
- Extend the cube painting problem by asking: What if the large block had been cut into 64 or 125 smaller cubes?
- Ask students to write a six-letter word on a net and then challenge a partner to draw them on a representation of the cube.

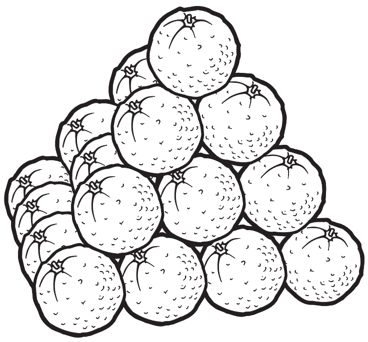
12.1 STACKING SHAPES

1. A set of steps was made by stacking one cube on top of another. How many cubes were needed to make the staircase?

____



Find as many cubes as you think you need and make the staircase.

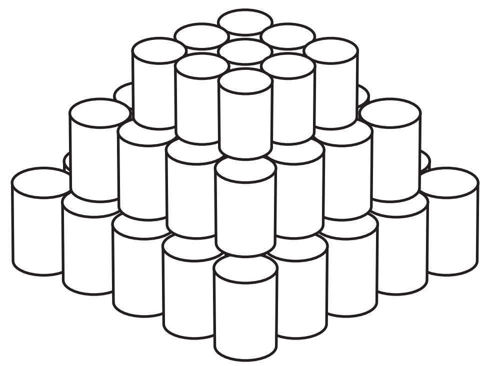


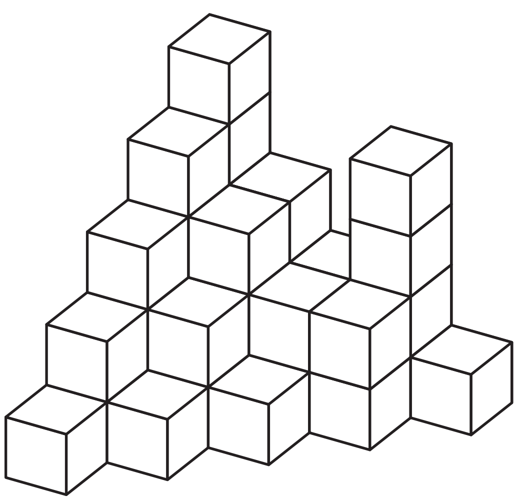
2. Greengrocers often stack fruit in a pyramid. How many oranges are in this stack?

____

3. At the supermarket, you sometimes see cans of soup stacked for a display. How many cans are in this stack?

____



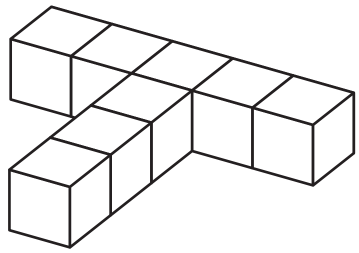


4. These cubes have been stacked one on top of another. Some of the cubes are hidden behind or beneath others and cannot be seen. How many cubes were used to build the shape?

____

Get some cubes and build your own stack. Write the number of cubes you used and challenge a friend to figure out how many you used without pulling your stack apart.

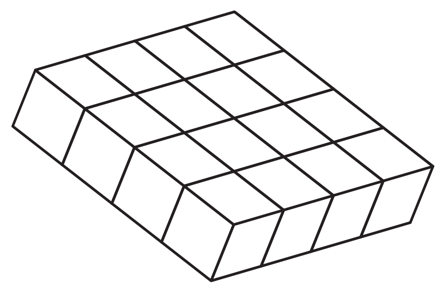
12.2 PAINTED CUBES



This shape was made using 8 cubes. After they were joined together, the final shape was painted green on all of its sides, including the top and bottom. When it was taken apart again, some of the faces of the cubes had green paint and others did not.

1. How many of the individual cubes would have green paint on exactly 4 faces? ______________________
2. Would any of the individual cubes have 5 faces painted? How many? ______________________
3. Would any have 3 faces painted? How many? ______________________

A wooden block is 8 cm long, 8 cm wide, and 2 cm high. The block is painted blue on all 6 faces and then cut into 16 cubes, each with sides of 2 cm.



4. How many of the cubes would have blue paint on 4 faces?

   ____

5. How many of the cubes would have only 3 faces painted?

   ____

6. How many of the cubes would have only 2 faces painted?

   ____

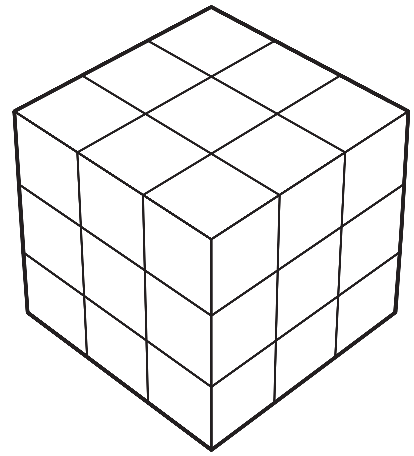
7. Why would there be no cubes with only 1, 2, or 6 faces painted?

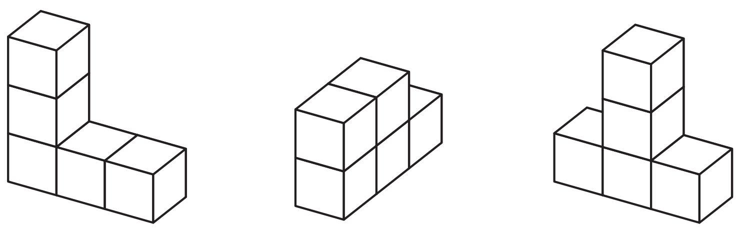
   ____

   ____

12.3 CUBE PAINTING

In the art gallery, a large polystyrene cube was hung from a string and spray-painted red on all 6 faces. It was then cut into 27 smaller cubes to be used by children to make shapes of their own.





1. With the large cube cut, how many of the individual cubes would have red paint on only 3 faces?

____

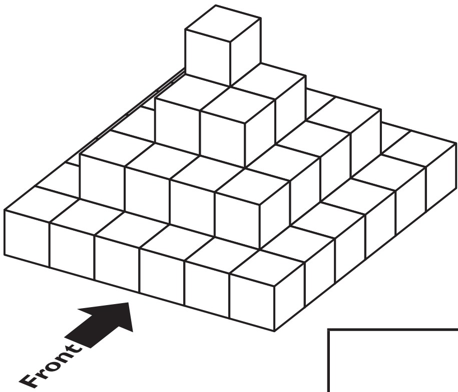
2. Would any of the cubes have no red paint? ____________________________

3. How many? ______________________________

4. Complete the table to describe the cubes.

Number of Faces Painted	Number of Cubes
	Total =

12.4 VIEWING CUBES



Draw what this shape would look like viewed from above? Would it look any different if viewed from below?

1. (a) Draw it from each of the sides in the space provided.

	Top	Bottom	
Left	Front	Back	Right

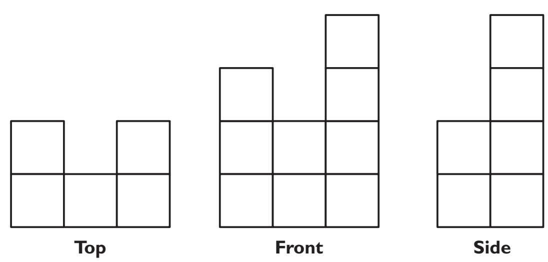
(b) How do the views compare?

____

Get some cubes and make the shape.

2. How many cubes did you need? __________

These are the views of an arrangement of cubes from the top, front, and side.



Get some cubes and make the shape.

3. How many cubes did you need? __________

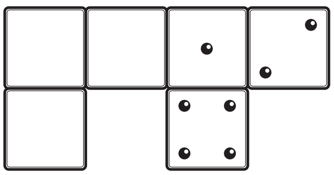
4. Make and draw a shape of your own. Ask a friend to construct the shape using your drawing. Compare the result with your drawing.

12.5 NETS AND CUBES

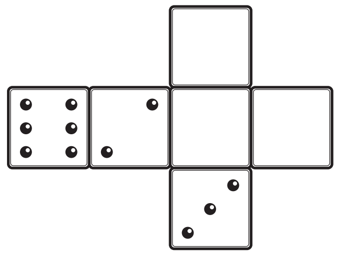
Most dice are made in the shape of a cube, with the dots on each square face arranged so the sum of the dots on opposite faces is always 7. An arrangement of squares that can be folded to make a cube is called the net of the cube.

1. Which of the arrangements of squares forms a net for a die? Circle the ones that form a correct net and draw the missing dots on the blank squares.

(a)



(b)



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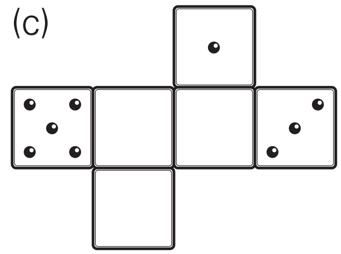

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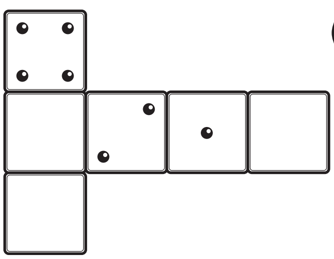

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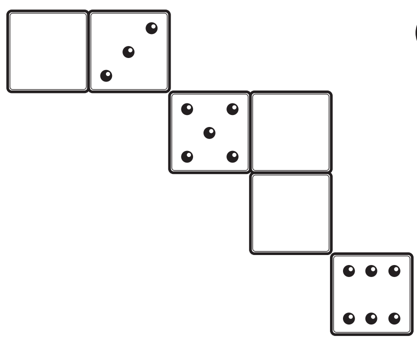

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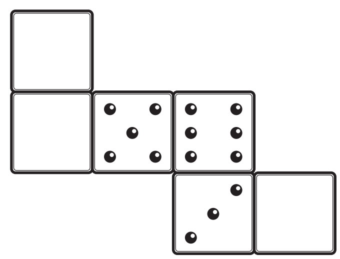
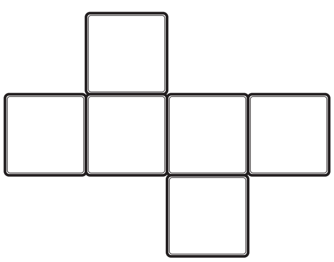
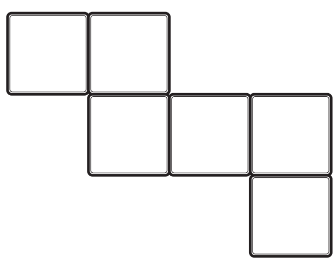
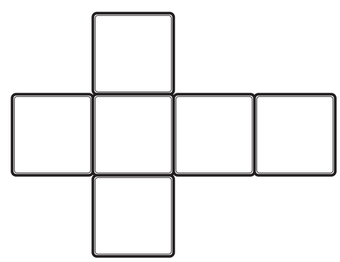


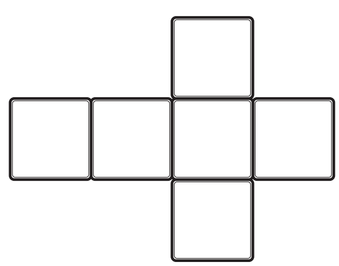
2. Put digits on the squares on the nets to make dice. Make sure the sum of the digits on the opposite faces is 7.







3. Here are three views of a die on which Lauren wrote one letter of her name on each face. Can you draw a net to show how the letters would be arranged on the squares?



Problem-Solving Objective

To use spatial visualization and measurement to solve problems

Materials

Paper to make and fold squares and equilateral triangles, triangular and square grid paper

NCTM Content Standards

- Number and Operations 1.1
- Geometry 3.4

Focus

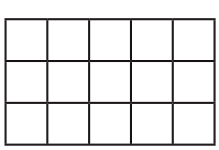
These pages explore ideas of perimeter by using knowledge of squares and equilateral triangles to visualize shapes and to determine the lengths of sides within or composed of the shapes. Spatial and logical thinking, as well as numerical reasoning and organization, are involved as students investigate the relationships among the shapes to determine the required distances.

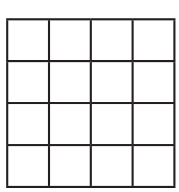
Discussion

Page 69 – Squares and Perimeters

Students must be able to understand that the perimeter of the first shape (made from five squares) consists of 12 sides. This can be done by counting all of the sides in a systematic way or by seeing the shape made up of symmetric parts—two sections with five sides on the top and bottom and two in the middle, for example. Since the perimeter is 36 in., the side of each small square must be 3 in. long. It may need to be explained to students that the shape is not to scale.

This reasoning is then applied to the other shapes, with the number of sides multiplied by 3 in. For Questions 2 and 3, several possible shapes can be made with a perimeter of 48 in. The only criterion is that each perimeter must use 16 sides of the smaller squares—for example, a 3-by-5 rectangle or a 4-by-4 square:

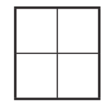




Page 70 – More Perimeters

Students need to be able to visualize the way in which the squares or triangle are folded and how the perimeter of the final shape relates to the whole shape, if unfolded. Students may need to use paper cut into squares.

In Problem 1, folding the square in half and then in half again shows 4 small squares when unfolded.



Page 71 – Perimeters in Squares

This page continues the students' investigation of perimeter. While the first problem can be solved by thinking of the rectangle as being made up of two whole 9-inch sides and two half 4.5-inch sides, this does not readily generalize to the other ways the square is partitioned. Thinking of it as six half-sides allows the next shapes to be seen as, respectively, eight third-sides and 10 fourth-sides to readily solve the perimeter of each of the original squares. Note that thinking of these as fraction symbols (1/3 and 1/4) makes the solution considerably more difficult to determine, since it requires working with fractions rather than whole numbers. Another way to visualize solutions is to divide the squares and rectangles into smaller squares and see the results directly, as on the preceding pages.

Possible Difficulties

- Uncertain of definition of perimeter
- Does not understand that the sides in a square or equilateral triangle are of equal length
- Unable to visualize the sides of the smaller shapes within the large shapes
- Cannot keep track of the number of sides that must be used

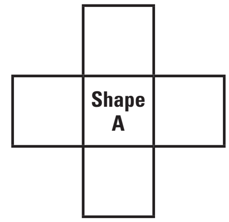
Extension

- Have students investigate shapes made of small equilateral triangles in the same way as those made of small squares.
- Ask students to create their own examples of perimeters in squares that are folded into five, six, or more rectangles.

13.1 SQUARES AND PERIMETERS

Perimeter is the distance around the boundary of a shape.

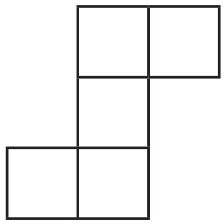
Small squares, all of the same size, have been used to make these shapes.



1. Shape A has a perimeter of 36 in. What is the perimeter of each of these shapes?

(a)

____

(b)

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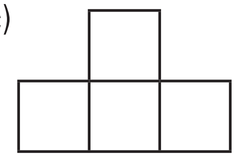
(c)

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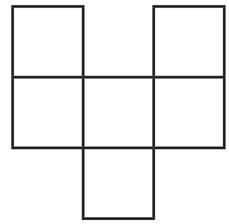
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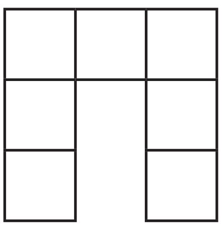
(e)

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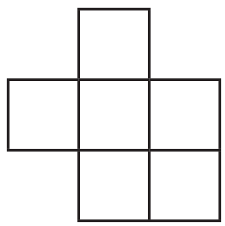
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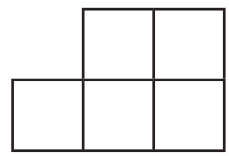
(g)

____

2. Can you draw a shape with a perimeter of 48 in.?

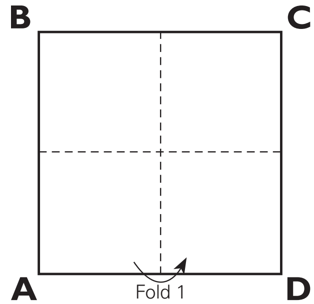
3. How many different shapes with a perimeter of 48 in. can you make? Draw them.

13.2 MORE PERIMETERS

ABCD is a square sheet of paper with a perimeter of 16 in.

Imagine that you fold the paper so that corner A folds onto corner D and corner B folds onto corner C.

Now imagine that you fold it again so that A folds onto C.

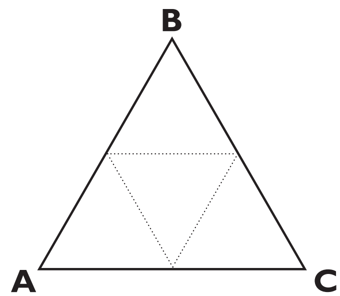


1. (a) After folding, what new shape would be created?

   ____

   (b) What would its perimeter be? ______________________________

ABC is a triangle.



2. If it were folded along the dotted lines as shown, what would the new shape be? ______________________________

The perimeter of the new shape is 9 in.

3. What would the perimeter of the original triangle (ABC) have been?

   ____

4. When a larger square is folded in the same way as square ABCD, the resulting shape has a perimeter of 24 in. What is the perimeter of the original square?

   ____

13.3 PERIMETERS IN SQUARES

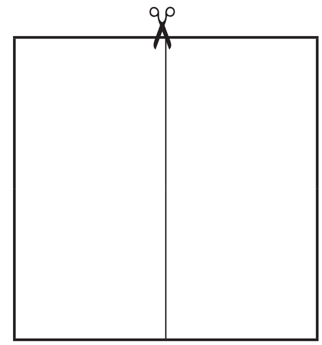
Perimeter is the distance around the boundary of a shape.

1. A small sheet of paper has been folded in half and then cut along the fold to make two rectangles.

   The perimeter of each rectangle is 18 in.

   What was the perimeter of the original square sheet of paper?

   ____

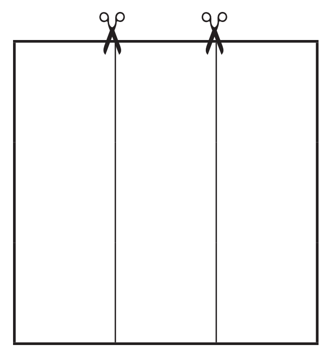


2. A larger sheet of paper has been folded into thirds and then cut along each fold to make three rectangles.

   The perimeter of each rectangle is 48 in.

   What was the perimeter of the original square sheet of paper?

   ____

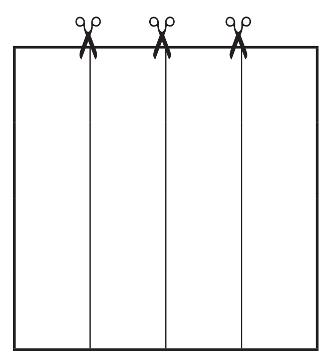


3. A large square of plastic has been folded into fourths and then cut along each fold to make four rectangles.

   The perimeter of each rectangle is 50 in.

   What was the perimeter of the original square of plastic?

   ____



Problem-Solving Objective

To use spatial visualization and measurement to solve problems

Materials

Paper to make and fold square and equilateral triangles, triangular and square grid paper

NCTM Content Standards

- Geometry 3.1, 3.4

Focus

These pages use students' knowledge of squares and triangles to explore the area and perimeter of smaller shapes within larger squares and triangles. Spatial and logical thinking as well as numerical reasoning and organization are involved as students investigate these interrelationships.

Discussion

Page 73 – Area

Students must be able to see that the shape in Problem 1 is made of three squares, each of which is made of four smaller squares. The shape can be seen as the sum of two large squares and two small squares, or one large square and two shapes made of three small squares, or three large squares with an area of half a large square (or two small squares) removed. Since the length of each side of the large square is 12 ft, the area of a large square is 144 ft^2 and the area of a small square is 36 ft^2.

In Problem 2, half of the large rectangle is made of two smaller rectangles, each of which contains two small triangles. This means that the smaller triangle must be seen as half of the area of the larger triangle and one-eighth of the area of the whole shape. In the third problem, each small square has an area of 100 cm^2 and the new area is found by extending the pattern to find the number of small squares.

Page 74 – Area and Perimeter 1

In Problem 1, students must be able to visualize how the shapes are made of squares and how the perimeter of the final shape uses only some of the sides of the squares, each of which has a side length of 4 cm. Problem 4 requires visualizing the smaller triangles created by the overlapping equilateral triangles and the hexagon shape of the overlap that make up the garden and pond to see the area and perimeter of the shapes.

Page 75 – Area and Perimeter 2

This page extends the thinking about areas. For Problem 1, trying to calculate the area of the path alone would be tedious and require a level of mathematics beyond the capacity of most students at this age. Instead, it is better to think of the park as being a square with a path and four triangular garden beds. From the information provided, we know that the length of the long side of each triangle is 12 meters. These four gardens can be put together to form a square with an area of 12 m by 12 m, which equals 144 m^2. Each side of the park is 18 m in length, so the park has an area of 324 m^2. Subtracting the area of the garden beds from the size of the park shows that the area of the paths is 180 m^2.

Problems 2 and 3 can be solved in a similar way by determining what part the triangles are of the total area. Rather than using complex mathematics, students can calculate the area of the triangles once they know the area of the entire shape—in this case 160 m^2. Because the triangles span the midpoints of the squares, we know that they are a certain fraction of the entire area; for example, in Problem 2, the garden bed is one-eighth of the entire area, or 20 m^2. Using the same thinking as in Problem 1, the gardens form one-quarter of the square, or 40 m^2.

Possible Difficulties

- May confuse area and perimeter because of a reliance on rules in place of understanding
- Does not understand that the sides of a square or equilateral triangle are of equal length
- Unable to visualize the smaller shapes within the large shapes
- Cannot keep track of the number of sides needed to determine perimeter

Extension

- Have students investigate shapes made of overlapping equilateral triangles in the same way as those made of overlapping squares.
- Investigate paths and garden beds within different-shaped parks.

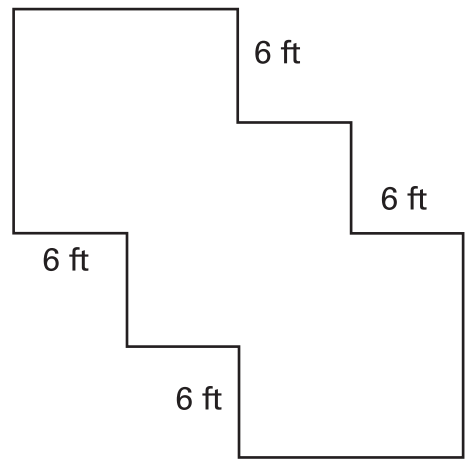
14.1 AREA

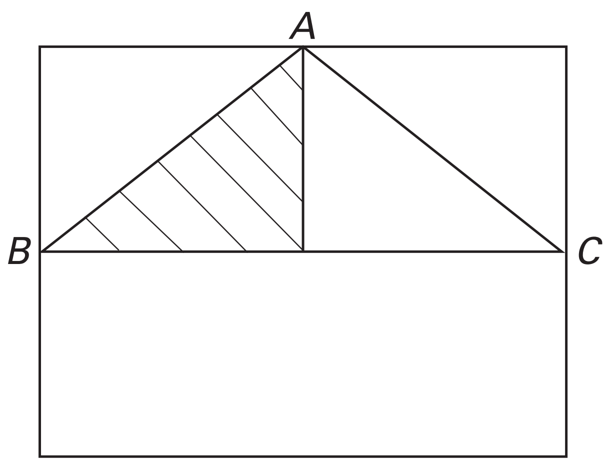
Area is the amount of surface there is of a shape.

This shape is made of three overlapping squares that are all the same size and all have sides of 12 feet in length.

1. What is the area of the shape?

____





This rectangle has an area of 144 square inches. The larger triangle *ABC* had been drawn by connecting the midpoints of three of the sides of the rectangle.

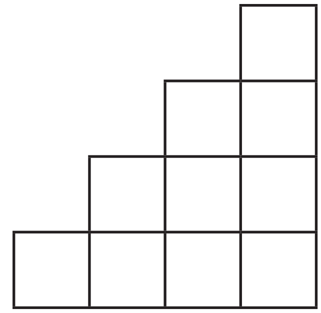
2. What is the area of one of the smaller triangles?

____

This shape is made of 10 small squares and has a height of 40 cm. The shape is continued until it has a height of 120 cm.

3. What is the area of the new shape?

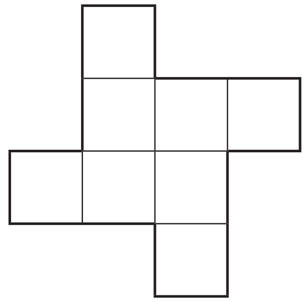
____



14.2 AREA AND PERIMETER 1

Perimeter is the distance around the boundary of a shape.

1. This shape is made of eight small squares that are equal in size. It has an area of 128 cm^2. What is the perimeter of the shape?



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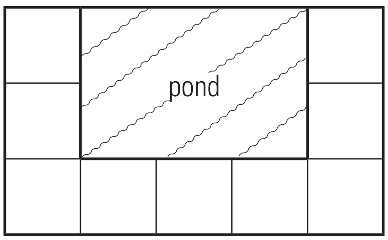
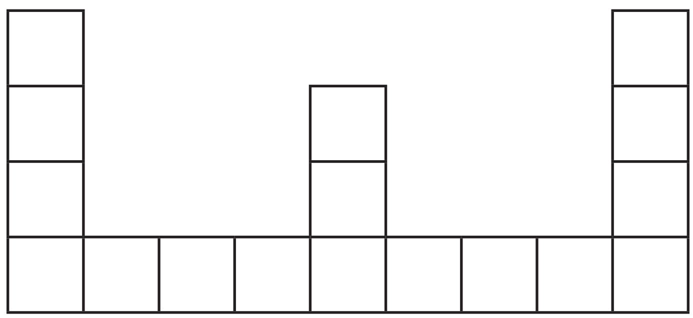


2. Nine equal-sized square were used to form a terrace around a large goldfish pond. If the perimeter of the terrace is 160 ft, what is the area of the terrace?

____

3. This garden is made of 17 square pieces of native grass turf of equal size. If the length of the boundary of the garden is 180 ft, what is the area of the garden?



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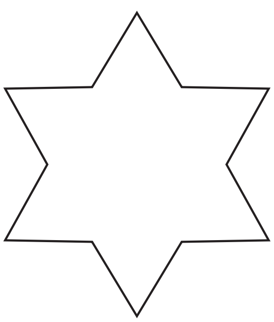


4. (a) A star-shaped garden was made by overlapping two equilateral triangles. If the perimeter of each triangle is 7 m 20 cm, what is the perimeter of the garden?

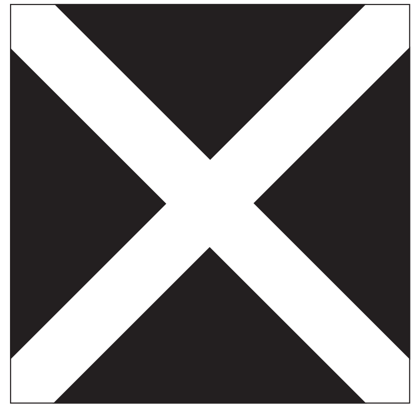
____

(b) The gardener has decided to turn the center of the garden (where the two equilateral triangles overlap) into a lily pond. What will be the shape of the pond and what is the length of the pond wall he needs to make?

____

14.3 AREA AND PERIMETER 2

1. A small park in the city is the shape of a square, with two diagonal paths and four triangular garden beds. On each border of the park, the length of the garden bed is 12 m, while the distance from one corner of the park to the garden bed is 3 m.

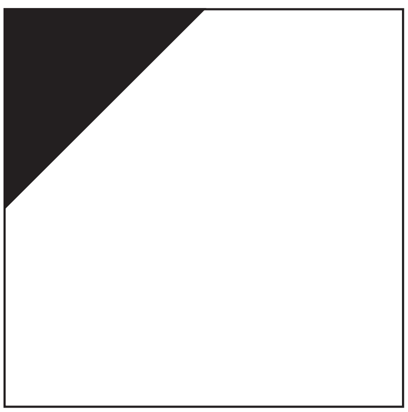


   (a) What is the area of each of the four garden beds?

   ____

   (b) What is the area of the paths?

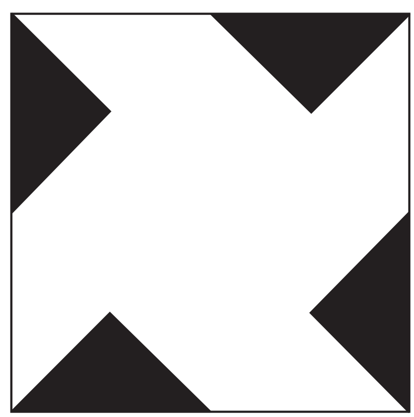
   ____



2. The city council plans to put a memorial garden in another of the city's squares. They decide to construct the garden in one corner of the square and leave the rest of the square as lawn for children to play on. The long edge of the garden bed will extend from the midpoints of two of the adjacent sides of the square. If the area of the square is 160 m^2, what will be the area of the garden?

   ____

3. Some local people protested at the small size of the garden. They suggest an alternative with four small triangular garden beds from the corners to the midpoint of the sides of the square.



   (a) What is the total area of the gardens now proposed?

   ____

   (b) How much less lawn is there compared to the first proposal?

   ____

TEACHER NOTES 15

Measurement: Area

Problem-Solving Objective

To use spatial visualization and logical reasoning to solve problems

Materials

Tangram sets

NCTM Content Standards

- Number and Operations 1.1
- Geometry 3.1, 3.4

Focus

This page explores arrangements and dissections of two-dimensional shapes to explore the relationships among the area of the individual pieces and the whole. Spatial as well as logical thinking and organization are required as students investigate all likely arrangements of the pieces to visualize the area of each shape in terms of its component parts.

Discussion

Page 77 – Tangram Areas

At first this problem may seem impossible, as the square cannot simply be placed inside the large triangle to directly determine its area. Instead, the problem must be broken down into a series of smaller problems that together will answer the initial question.

By placing the two small triangles on top of the small square, students can see that the area of the small triangle is half that of the small square: an area of 5 cm^2. The two triangles can also be arranged to form the parallelogram and the medium-sized triangle, so each of these has an area of 10 cm^2. When combined, the two small triangles, square, parallelogram, and medium triangle have an area of 40 cm^2. This is also the area of half of the original tangram, so the area of one large triangle must be 20 cm^2. Some students may immediately see that the size of a large triangle is the same as the square and the two small triangles (or that of two squares).

These relationships can now be used to solve Problem 7. If the area of the original tangram is now calculated to be 144 cm^2, the area of half the tangram is 72 cm^2. This means that the area of a large triangle is 36 cm^2 and the area of the square is half of this, or 18 cm^2. Alternatively, half of the tangram is formed by the two small triangles, square, parallelogram, and medium-sized triangle. The area of half the whole tangram, 72 cm^2, is four times the area of the small square, so the small square is 18 cm^2.

In Problem 8, since the area of a large triangle is the same as twice the area of the medium-sized triangle, and the area of the medium-sized triangle is the same as the area of the parallelogram, 8 cm^2, the area of a large triangle must be 16 cm^2.

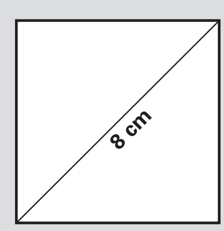
Possible Difficulties

- Unable to visualize the tangram parts in relation to each other
- Try to work with the numbers to calculate areas rather than visualize the part/whole relationships that let the areas be determined directly from each other

Extension

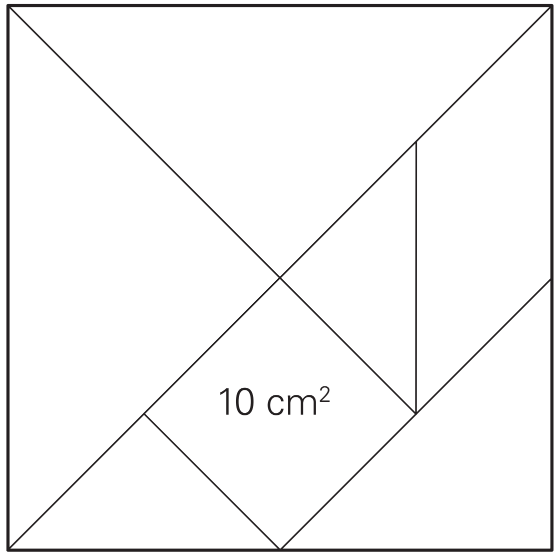
- Ask what the area of the other parts and the original square would be if the area of the parallelogram was 15 cm^2, the area of the large triangle was 28 cm^2, or the area of the original square was 16 cm^2.
- Have students make up their own values for particular areas and challenge others to find the areas of each part and the whole square.
- Ask students to express the areas of each of the parts as sums of the area of the small triangle
- Challenge the students to express each part as a fraction of the original square.
- Another challenge! If a square has a diagonal of length 8 cm, what is the area of the square? This builds on Problem 1, page 75.

Hint: Use the four large triangles from two tangram sets to explore dissecting and rearranging the area of the original square.



15.1 TANGRAM AREAS

The area of the small square in this tangram is 10 cm^2.



1. What is the area of one of the small triangles? ______________________
2. What is the area of the parallelogram? ______________________
3. What is the area of the medium-sized triangle? ______________________
4. When the two small triangles, square, parallelogram, and medium-sized triangle are placed together, what is the combined area?

   ____

5. How much of the tangram do they form? ______________________
6. (a) What is the area of one of the large triangles?

   ____

   (b) How did you work out the answer?

   ____

7. If the original tangram had an area of 144 cm^2, what would be the area of the square? ______________________
8. If the parallelogram had an area of 8 cm^2, what would be the area of one large triangle?

   ____

Problem Solving

To solve problems involving time or coordinates and to make decisions based on particular criteria

Materials

Digital clock, 0–99 number board

NCTM Content Standards

- Geometry 3.2

Focus

These pages focus on reading for information, obtaining information from a number of sources, and using it to find solutions. The problems involve thinking and working with time and coordinates.

Discussion

Page 79 – DVD Rentals

This worksheet requires students to read for information and decide which information meets different criteria. The concepts of "earliest" and "latest" are used in a number of the problems rather than exact time. The ideas of the "longest opening day" as well as the "longest opening times" are explored.

Page 80 – Drive Time

These problems involve following directions and using coordinates. With Question 1, students plot the path of a car using the given coordinates. They must keep in mind that the first digit in each pair is across (*x*-axis), while the second digit is up (*y*-axis). Question 2 requires the students to provide the coordinates that describe a given path. The final question requires them to coordinate drawing a path and recording the coordinates.

Page 81 – Clock Watching

This page explores students' understanding of 24-hour digital time as they investigate the ways in which the digits can be placed to show possible times and determine the time closest to noon and midnight. The way in which zero is used on a digital clock also must be considered. In Problem 2, an understanding of two-digit numbers must be coordinated with an understanding of how and when the digits change on a digital clock. Thinking about the two-digit numbers suggests where "2" will occur in the ones or tens place and how long, the "2" will remain displayed until it changes to a "3." For example, when "2" occurs in the hour display (for example, 2:00, 12:00, 20:00), it remains unchanging for the whole of the hour, so it should be counted just once.

Extension

- Use the information and timetable to write other questions.
- Construct a similar timetable for two pizza shops, where one opens for lunch and dinner and the other just for dinner.
- Have students call out the coordinates they have drawn to other students to construct the path and compare results.

16.1 DVD RENTALS

DVD Easy		Hours open
Monday	CLOSED	
Tuesday	11:00 a.m. to 8:00 p.m.	
Wednesday	11:00 a.m. to 9:00 p.m.	
Thursday	10:00 a.m. to 9:00 p.m.	
Friday	11:00 a.m. to 10:00 p.m.	
Saturday	10:00 a.m. to 10:00 p.m.	
Sunday	11:00 a.m. to 9:00 p.m.	
	Total hours	

DVD Rentals		Hours open
Monday	11:00 a.m. to 9:00 p.m.	
Tuesday	11:00 a.m. to 9:00 p.m.	
Wednesday	CLOSED	
Thursday	11:00 a.m. to 9:00 p.m.	
Friday	11:00 a.m. to 11:00 p.m.	
Saturday	10:00 a.m. to 11:00 p.m.	
Sunday	11:00 a.m. to 9:00 p.m.	
	Total hours	

1. What is the difference between the amount of time the two stores are open in one week?

____

2. What is the latest time that a DVD can be rented on the weekend?

____

3. What is the earliest time that a DVD can be rented during the week?

____

4. Which store has the day with the longest open hours?

____

5. Which day has the longest open hours?

____

6. Which store could I use if I wanted to rent a DVD on Tuesday at 2:00 p.m.?

____

7. Which store could I use if I wanted to rent a DVD on Monday at 11:00 a.m.?

____

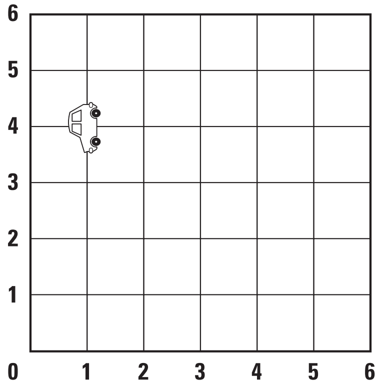
8. Circle the correct answer. **Can I rent a DVD on:**

(a) Wednesday at 3:00 p.m. from DVD Rentals?	YES	NO
(b) Saturday at 10:00 a.m.?	YES	NO
(c) Friday at 10:30 p.m.?	YES	NO
(d) Wednesday at DVD Easy?	YES	NO
(e) Thursday at 10:30 p.m.?	YES	NO

16.2 DRIVE TIME

1. Follow the coordinates to draw the paths of the cars.

(a) ***Car One***



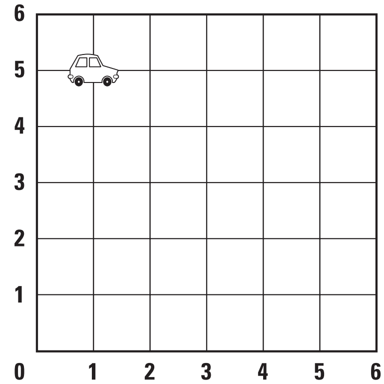
Car One

(1,4)(1,1)(3,1)(3,3)
(2,3)(2,5)(5,5)(5,1)

Car Two

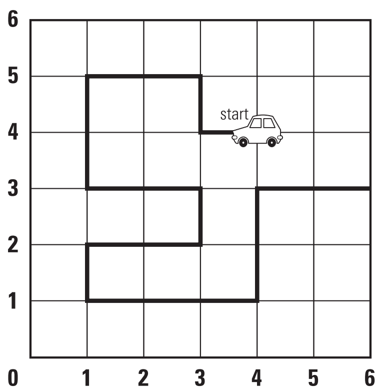
(1,5)(2,5)(2,3)(4,3)
(4,4)(3,4)(3,5)(5,5)
(5,1)(3,1)(1,1)(1,2)

(b) ***Car Two***



2. Write the coordinates of each car's path.

(a) ***Car Three***



Car Three

____

____

____

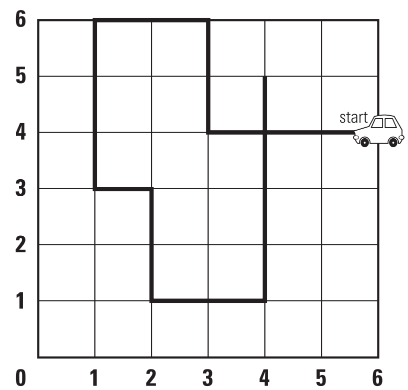
Car Four

____

____

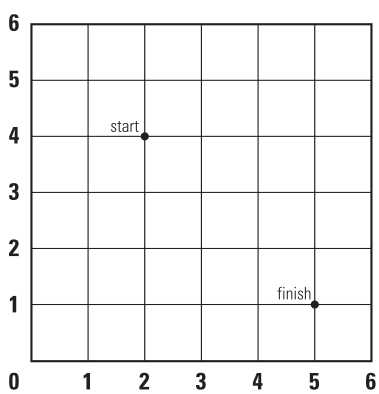
____

(b) ***Car Four***



3. Draw your own path and write the coordinates to match.

(a) ***Car Five***



Car Five

____

____

____

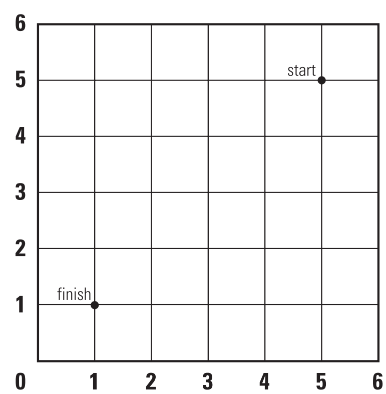
Car Six

____

____

____

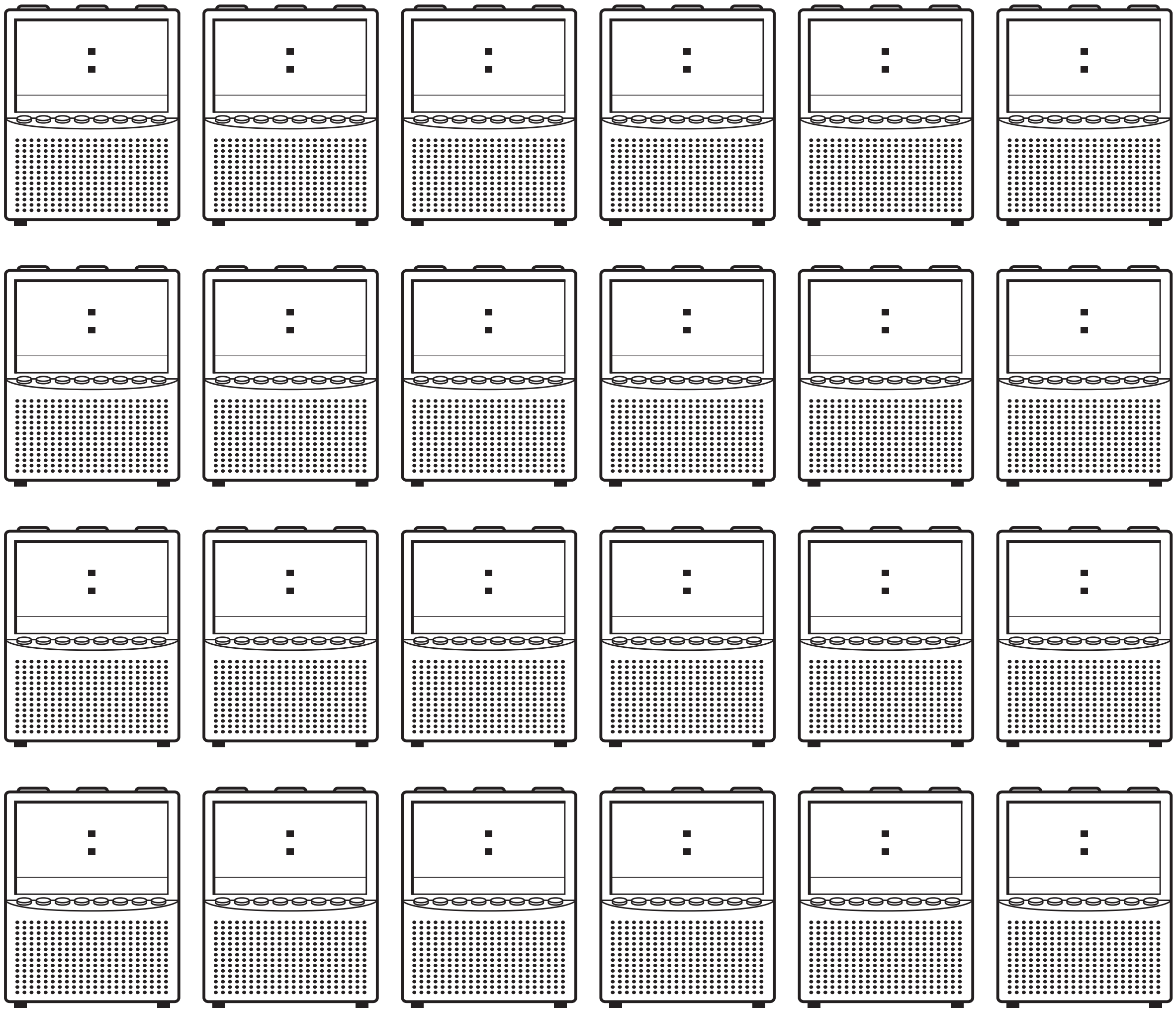
(b) ***Car Six***



16.3 CLOCK WATCHING

A digital clock has been set to show 24-hour time.

1. (a) What times can be shown using only the digits 0, 1, 2, and 9 (without repeating any digits)?



(b) What time is closest to noon? ______________

(c) What time is closest to midnight? ______________

2. If you watched this clock all day, how many times would the digit "2" be displayed:

(a) to show minutes each hour? ______________

(b) to show minutes in 24 hours? ______________

(c) to show hours in 24 hours? ______________

3. Total number of times "2" is displayed in 24 hours. ______________

Problem-Solving Objective

To use strategic thinking to solve problems.

Materials

Counters in two different colors (1 of one color, 17 of another color), grid paper, calculator

NCTM Content Standards

- Number and Operations 1.1
- Geometry 3.2
- Data Analysis and Probability 4.1

Focus

These pages explore more complex problems in which the most difficult step is to find a way of coming to terms with what the problem is asking. Using materials to explore the situation is one way in which this can be done. Another is to use a diagram to assist in thinking backwards or making trials and adjusting to find a solution that matches all of the conditions.

Discussion

Page 83 – The Big Race

These problems can be solved in several ways. One way is to work backwards from the final position. For example, Lucy finishes in eighth position—she must pass 11 cars and then be passed by five cars to get back to her original position. Counters can be used to model the process of cars passing and being passed. Using a counter of one color to represent Lucy's car and counters of a second color to represent the other 17 cars helps students keep track of who's passing who. Other students might prefer to base their solution on the diagram on the page to model what has happened.

Alternatively, students may choose a position for Lucy and work through each of the events in the race. If Lucy does not end up in eighth place, an adjustment can be made to determine the original starting position.

Page 84 – Serial Numbers

These problems challenge students' understanding as they investigate the ways the digits can be placed according to set criteria. Some students will use the given information to discard combinations until only the correct number remains, while other students may prefer to try each number in turn against all of the criteria until they find one number that answers all conditions. The last question has two possible answers.

Page 85 – Up and Down

In these problems, the information must be carefully analyzed to determine the distance at the beginning rather than at the end of a situation. Some students may choose to use counters to represent the changing positions as the situation is worked through either forwards or backwards. Using a diagram or "try and adjust" are other alternatives to find a solution.

In Problem 1, the notional "middle" of the ladder has to be taken for granted and the effects of moving up and down the ladder are measured against this position. This means that the firefighter has moved up 12 rungs and then 14 rungs to get to the roof. There are 26 rungs from the middle to the top, so there must also be 26 rungs from the middle to the bottom; therefore, the ladder has 53 rungs. Similar reasoning provides the path of the ball, remembering that it only drops the full distance once, and then each additional distance twice until the last distance, which is also measured only once—6.75 m in total. The elevator and airplane problems are solved in the same manner.

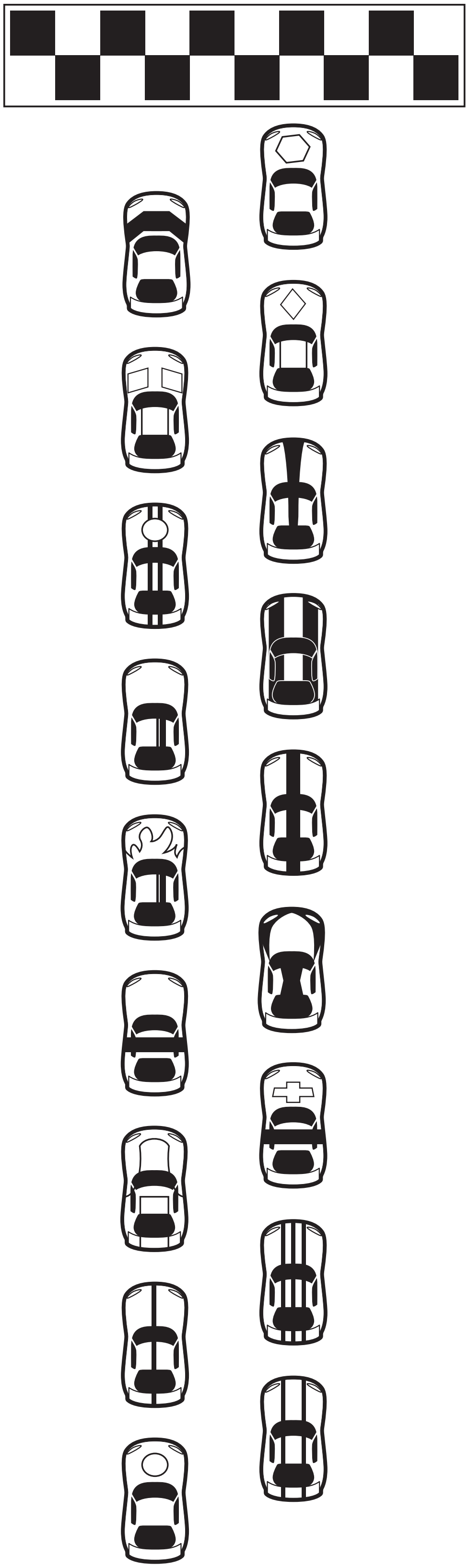
Possible Difficulties

- Using only the 11 cars that Lucy passed to determine her starting position
- Not taking into consideration all the criteria for the serial numbers
- Not considering the initial position to obtain an appropriate starting point for the movements—for example, thinking there are 52 rungs on the ladder instead of 53
- Not realizing that a ball will bounce both up and down before getting to its new level
- Unable to use a diagram or materials to come to terms with the problems
- Simply working on the basis of calculations with the numbers in the problem to obtain incorrect answers

Extension

- Students write their own car race, ladder, ball, elevator, and plane problems based on the questions.
- Students write their own criteria for figuring out a serial number.

17.1 THE BIG RACE



At the start of the race, the cars line up in their starting positions.

1. Lucy is driving one of the 18 cars in the race. During the race she passes 11 cars before being passed by 5 cars. She finished eighth in the race.

   In what position did Lucy start the race?

   ____

2. Theo started in fifth place. During the race 10 cars passed him.

   How many cars does he need to pass to win the race?

   ____

3. Paula won the race. During the race 4 cars passed her, and then she passed 12 cars.

   In what position did she start the race?

   ____

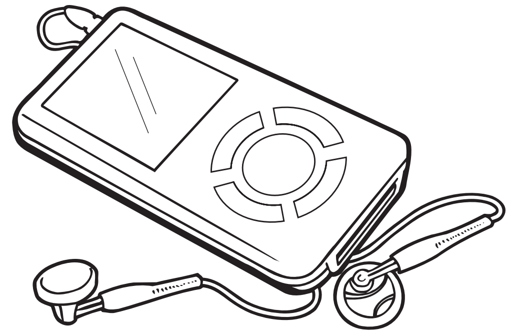
4. Luke is driving one of the 18 cars in the race. During the race he passes 10 cars before being passed by 2 cars. He finishes third in the race.

   In what position did Luke start the race?

   ____

17.2 SERIAL NUMBERS

1. The serial number of my MP3 player is a four-digit number less than 6,000 and has the digits 3, 4, 6, and 9.



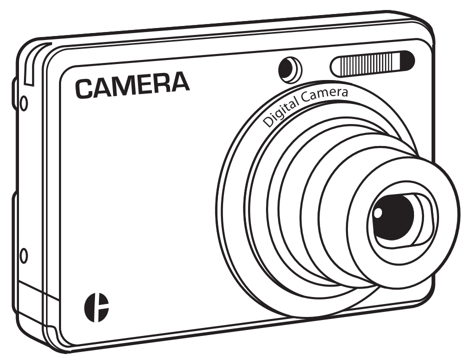
The 4 is next to the 9.

The 3 is not next to the 4.

The 6 is not next to the 3.

What is the serial number? ______________________

2. The serial number of my camera is a four-digit number more than 5,000 and has the digits 5, 2, 7, and 3.



The 2 is next to the 7.

The 5 is not next to the 2.

The 3 is not next to the 5.

What is the serial number? ______________________

3. The serial number of my sister's MP3 player is a four-digit number less than 7,000 and has the digits 2, 3, 5, and 8.

The 3 is next to the 8.

The 2 is not next to the 3.

The 5 is not next to the 2.

What could be the serial number? ________________

17.3 UP AND DOWN

1. A firefighter stood on the middle rung of an extended ladder, spraying water onto a burning apartment building. As he succeeded in dampening down the fire, he climbed seven rungs. A sudden flare-up sent him down 12 rungs. After it died down, he moved up 17 rungs. When the fire was put out, he climbed the remaining 14 rungs to the top of the ladder and got onto the roof. How many rungs did the extended ladder have?

____

2. Roberta drops a ball from the porch to the patio 2.4 m below. The ball then bounces up and down until her friend, Shen, catches it at the peak of the bounce—a height of 15 cm from the ground. It is a bouncy ball, at each bounce traveling half the distance of the previous bounce. How far did the ball travel from the time Roberta dropped it until it was caught by Shen?

____

3. You enter the elevator at a certain floor of a tall building. Then the elevator moves up 16 floors, down 19 floors, and up 11 floors. You are now at Floor 31. At which floor did you enter the elevator?

____

4. A plane is flying at its usual cruising altitude when turbulence occurs and the pilot flies to a new altitude 6,900 feet higher to get above the rough weather. Then a lightning storm hits, and he descends 14,400 feet to avoid it. To be safe, when the storm has finished, he ascends 9,300 feet to be at a height of 33,600 ft. What is the usual cruising altitude for the plane?

____

Problem-Solving Objective

To interpret and organize information found in a series of interrelated statements and to use logical thinking to find solutions.

NCTM Content Standards

- Number and Operations 1.1, 1.2, 1.3
- Geometry 3.2
- Data Analysis and Probability 4.1

Focus

These pages explore interrelated statements within a problem situation involving averages, distance, and payments. Students must read the stories carefully and consider a number of different criteria. Tables and lists can be used to help manage the various criteria.

Discussion

Page 87 – How Many?
Each problem requires students to consider all of the information provided in a series of interrelated statements to find a solution. The use of a table or list may be very helpful in managing the data. For example, in the first problem, a table listing the various years can be used as a starting point. The problem states how many visitors came in 2003, and this information can be used to work out the number of visitors in 2007 (twice as many), and in turn this can be used to figure out the numbers for 2006.

A similar table or list can be used for the other problems. The last problem contains additional information about the number of caves and the most visited caves, none of which is needed to find a solution.

Page 88 – How Far?
These problems explore the concept of average distance traveled over a period of time. In many cases the solution is not necessarily exact but rather an approximate time or distance; for example, Problem 2 states that Susie swims 100 meters in "about 2 minutes." This is not an exact time and would vary from lap to lap, so the solution of how far she has swum would again be an approximate distance.

Problem 4 deals with the concept of distance traveled over a period of time. Some students may reason that Brian runs 144 miles per week. Others may think of four weeks as having 20 weekdays and 8 weekend days.

As an extra dimension, Problem 5 contains additional information about lunch as well as stopping and starting times that are not needed to find a solution.

Page 89 – How Much?
Each problem must be read carefully to determine what is being asked. In the first question, it is necessary to work out how many shirts were purchased (four) in order to determine that eight are bought the next day (twice as many).

Problems 2 and 4 present two different ways of getting paid, and both options must be analyzed to work out which option is best. Again, a table showing both options can be used to assist with the problem-solving process.

Possible Difficulties

- Not using a table or list to manage the data
- Not understanding the term "average"
- Confusion when dealing with approximate times and distances

Extension

- Construct a table to show the running distance and how it varies from month to month.
- Write other problems using the same form of complex reasoning for other students to solve.

18.1 HOW MANY?

1. A large number of tourists visited Canyonlands National Park during 2007. There were twice as many visitors in 2007 in 2003. There were 6,530 more visitors in 2007 than in 2006.

   If there were 298,460 visitors in 2003, how many were there in 2006?

   ____

2. During September, 258,000 tourists visited the Great Barrier Reef. April had twice as many visitors as January, but 4,000 fewer than August. August had 8,000 more visitors than September.

   How many visitors were there in January?

   ____

3. The Jenolan Caves Touring Company in the Blue Mountains currently offers tours to 11 different caves. Many of the visitors tour the Lucas, River, and Chiefly caves. Due to storms, May was a poor month for visitors and only 19,970 people visited. June had 6,230 more tourists than July, and July had 3,150 more than March. March had 4,020 more visitors than May.

   How many visitors were there in June?

   ____

18.2 HOW FAR?

1. Erin caught a bus from New York to Cleveland. The bus left at 4 p.m. and, due to traffic, averaged 42 miles per hour for the first 2 hours. Once on the highway, the bus averaged 63 mph for the next 6 hours.

   How far had Erin traveled? ______________________________

2. Susie swims each morning in a 50 m pool. She can swim 100 meters in about 2 minutes. She usually swims for an hour, has a short break, and then swims for another hour.

   Approximately how far does she swim?

   ____

3. The train from New York to Chicago travels at an average speed of 85 miles per hour. If Chicago is about 1,000 miles from New York, how long will it take for the train to arrive?

   ____

4. Brian trains each day for the marathon. During the week he runs 8 miles in the morning and 12 miles in the afternoon. On the weekend he runs 22 miles each day.

   How far does he run in four weeks?

   ____

5. Kim-Ly drove from Dallas to Phoenix. She left at 9 a.m. and averaged a speed of 74 miles per hour for 3 hours. She stopped for lunch and started again at 1 p.m. She drove for four hours and averaged 72 mph.

   How far had she traveled?

   ____

18.3 HOW MUCH?

1. Derek paid $32 for shirts that cost $8 each. The next day he saw the same shirts for $6 each, so he bought twice as many as the day before. How much did he spend on shirts altogether?

____

2. Michael delivered 652 newspapers in 7 hours. He can be paid by the number of newspapers he delivers or by the hour. The rate per newspaper is 5.5¢, and the hourly rate is $4.25 for the first hour and $3.50 for each other hour. Which way pays more money?

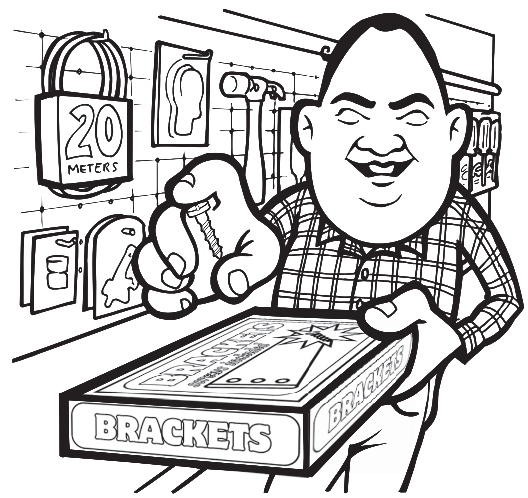
____

3. Alison bought apples from the market at a price of 6 apples for $2. She then sold them at her fruit store at a price of 4 for $2 and made a profit of $10. How many apples did she sell?

____

4. The hardware store sells shelf brackets both with and without screws. The brackets with screws cost $9.90 each and the ones without screws cost $6.50 each. Screws cost 80¢ each. If 4 screws are needed for each bracket, which is the cheapest option and by how much?

____



5. Wendy can be paid either by the day or by the number of trees she plants. She gets $1.20 per tree or $34.70 per day on weekdays and $48 per day on weekends. On average, she can plant about 37 trees per day. If she works 8 days starting on Sunday and ending on Sunday, which is the best payment option?

____

Problem-Solving Objective

To use strategic thinking to solve problems

Materials

Counters, calculator

NCTM Content Standards

- Number and Operations 1.2
- Geometry 3.2
- Data Analysis and Probability 4.1

Focus

These pages explore problems that may have several answers. Further analysis of the connections among the data is needed to see whether this is the case or whether there is only one solution. A process of "try and adjust" can be used; however, using logical reasoning to think about the different possibilities and employing a table, diagram, or materials to organize the possibilities will be more productive.

Discussion

Page 91 – On the Farm

These problems can be solved in several ways. Emus have two legs and alpaca have four legs. Since there are 38 heads, there must be 38 animals altogether. If all of the animals are emus, there are only 76 legs. The remaining 24 legs must belong to the alpacas. Since an alpaca has two more legs than an emu, there are 12 alpacas and 26 emus (a total of 100 legs).

Another way would be to put multiples of two or four in a table or diagram and systematically check the remaining numbers until a solution is reached. Counters could also be used to model the problem, again focusing on groups of two and four.

The second problem can be solved in the same way, while a table or counters can be used in solving Problem 3. Since the farmer sells twice as many emus as alpacas, she must receive $72 for each set of one alpaca and two emus. Since 1,080 divided by 72 is 15, she sold 15 alpacas.

Page 92 – In the Barn

The first two problems can be solved in the same way as the problems on page 91. (To solve Problem 2, students must know that spiders have eight legs and beetles have six.) However, Problem 3 does not give a second condition, and there are several possibilities. Since the animals have either two or four legs, an odd number of chickens is not possible.

Page 93 – Market Day

These problems require careful reading to understand how the information must be used. Some children may simply try to subtract the smaller number from the larger for the first two problems. However, it is the expression *more than* that is critical in each case. Subtraction gives the difference between the two prices, and the actual amounts must be found to match both the total and the difference.

In Problem 3, many students may at first just divide the money among the four brothers, whereas these amounts must match both the condition of equaling $1,200 and differing from the next highest amount by $100.

Possible Difficulties

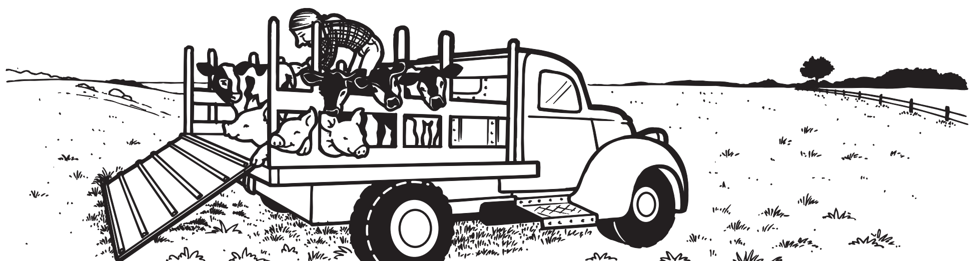
- Not using a table or diagram to manage the data
- Not considering all the possible answers—there may be more than one possibility
- Keeping only one condition in mind when there are two aspects to consider and reconcile

Extension

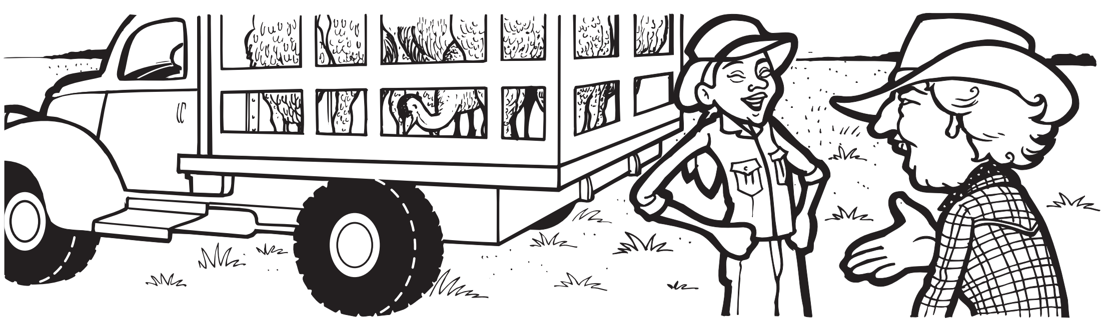
- Discuss the various methods used by students to solve the problems. Include the ones discussed above. Ask them to solve each problem using a different method. Encourage them to use a diagram rather than simply calculate.

19.1 ON THE FARM

1. A farmer had a number of emus and alpacas in one paddock. When she counted, there were 38 heads and 100 legs. How many emus and how many alpacas were in the paddock?



2. The farmer decided to buy some young calves and piglets at the market. She paid $60 for each piglet and $95 for each calf. She paid for the 10 animals she bought with a check for $740. How many calves and how many piglets did she buy?

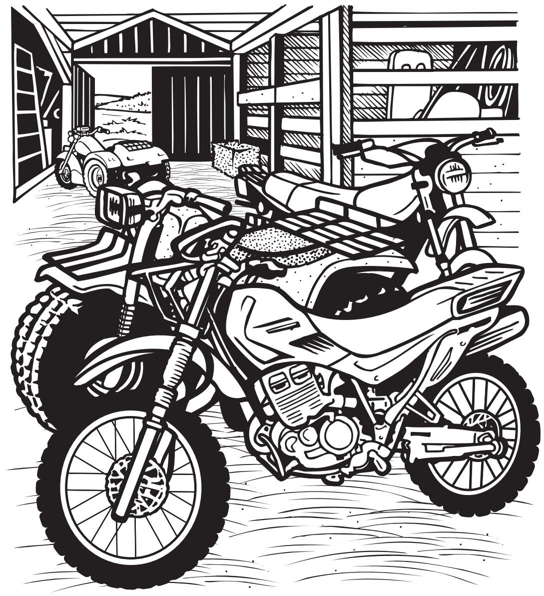


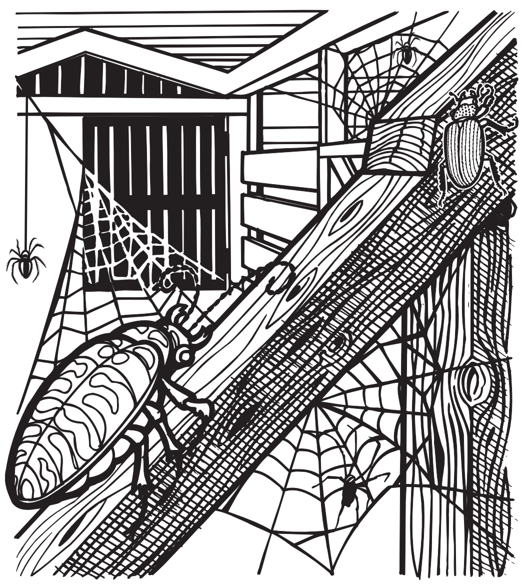
3. Her neighbor needed to buy more stock, so the farmer sold him some alpacas for $48 and twice as many emus for $12 each. She received a total of $1,080. How many alpacas and how many emus did she sell?

19.2 IN THE BARN

1. The farm workers keep their dirt bikes in the barn. Some of the workers have bikes with 2 wheels and some have bikes with 3 wheels, but all of the bikes have 2 handles. Peter, one of the farmhands, counts the handles on the bikes and gets a total of 50. He also counts a total number of 64 wheels on the bikes. How many of the bikes have 3 wheels and how many have 2 wheels?

____





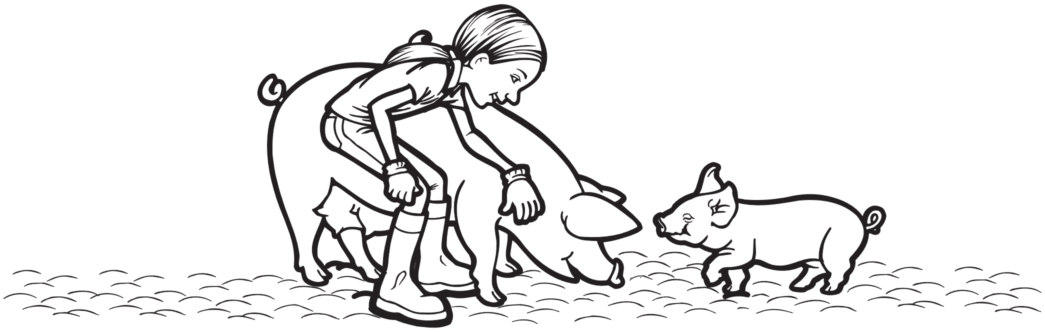
2. There are many spiders and beetles in the barn. One of the workers collects some of each. She notices that there are 200 legs and 29 bodies. How many spiders and how many beetles are in her collection?

____

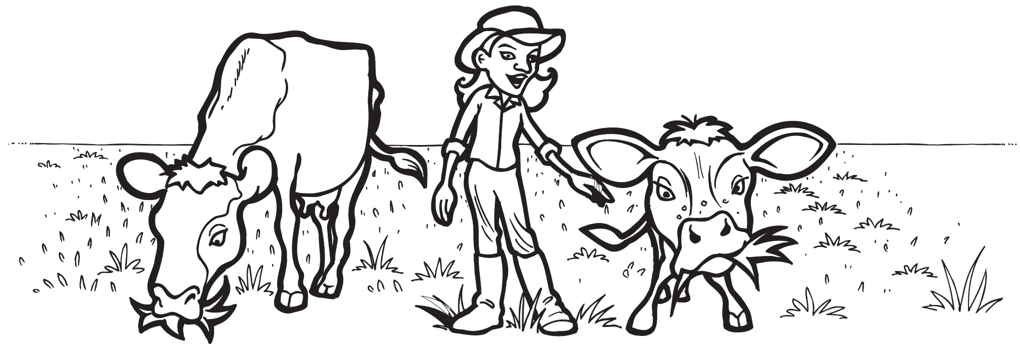
3. When the weather turns cold, the farmer puts her young calves and chickens in the barn to keep warm. As she puts them in the barn, she notices that there are a total of 28 legs. How many calves and how many chickens could there be?

____

19.3 MARKET DAY



1. At the market, the farmer bought a pig and a piglet for $300. If the pig cost $250 more than the piglet, what did she pay for the pig?



2. When the farmer sold a cow and her calf, she received $480. She noticed that the cow sold for $278 more than the calf. How much did the calf sell for?

3. The farmer had four sons, aged 12, 14, 15, and 16. In their spare time, they took the vegetables they grew to the market and sold them for $1,200. Since they did not all work equally hard in the vegetable garden, they decided to divide the money so that each brother got $100 more than his next-younger brother. How much did the youngest brother get?

Problem Solving: Tables and Diagrams

Problem-Solving Objective

To use spatial visualization, logical reasoning, and measurement to solve problems

Materials

Counters

NCTM Content Standards

- Number and Operations 1.1
- Geometry 3.2
- Data Analysis and Probability 4.1

Focus

These pages explore different ways of visualizing problems and analyzing the possibilities that make up the whole solution. Logical reasoning is required, as well as an understanding of metric measurement (kilometers, meters, and centimeters). In each situation, diagrams can be used to organize, sort, and explore the data.

Discussion

Page 95 – Farm Trails

In these problems, students must visualize the paths that the farmer and dog take as they travel around the outside of the paddock and garden. The length of one of the sides for each shape must be determined from the diagram, and an ability to convert from kilometers to meters or from centimeters to meters is required.

In the first problem, the farmer rides around the paddock more than twice, passing by corners B, C, and D three times. The distance around the paddock is 4,300 meters, which can be used, along with the distance from A to D, to calculate the distance traveled to return to D. For Problem 2, interpreting the diagram to determine the lengths is more complex, although some students may realize that finding the distance all the way around three times and then subtracting the distance from G to A is simpler.

Page 96 – Balancing

The problems on this page are essentially solved the same way; however, providing the image of a balance for the first problem makes it is easier to see how the mango from the top picture can be substituted using the scales in the second picture. Problem 2 requires more complex thinking.

One approach is to create a table of possible values using a process of "try and adjust":

6 rolls and 1 loaf cost $4.20.					
Roll	6 Rolls	Loaf	4 Rolls	2 Loaves	Total
0.50	$3.00	1.20	2.00	2.40	$4.40 – too little
0.40	$2.40	1.80	1.60	3.60	$5.20 – too little
0.35	$2.10	2.10	1.40	4.20	$5.60 – too little
0.30	$1.80	2.40	1.20	4.80	$6.00
A loaf of bread costs $2.40. A bread roll costs 30¢.					

Page 97 – Distanced Traveled

Using a diagram or counters to model the first two problems will help students understand the different pieces of information presented. In Problem 1, Kevin must walk as far as the second, fourth, sixth (and so on) tree, and back to the start. However, he can finish watering at the last tree and does not need to return to the tap. (Next time, he can do the same in reverse, starting from the last tree.)

Clearly, the answer to the second problem is not that the cyclists returned at the same time! A diagram or counters can be used to sort out the difficulty posed by the problem. In the last problem, the information about the ascent and descent of the balloon is not relevant. Rather, students must understand that the path traces a rectangle because the directions are at right angles to each other and that the final point is one side of the rectangle, or 750 m from her starting point.

Possible Difficulties

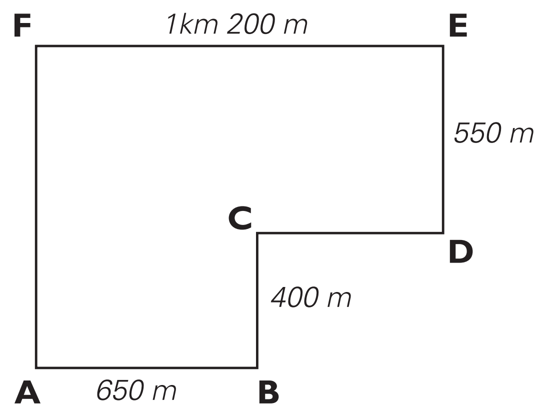
- Unable to convert centimeters to meters or meters to kilometers
- Not using the given data to determine the length of the sides for which the length is not stated
- Unable to see how the objects in the balancing problems can balance
- Unable to visualize the paths taken by the person watering the garden, the cyclists, or the balloonist

Extension

- Students could write their own problems involving distance around an irregular shape, in which some of the lengths have to be worked out from the information in the diagrams.
- Students could write their own problems involving cyclists, balloons, or items on a balance and give them to other students to solve.

20.1 FARM TRAILS

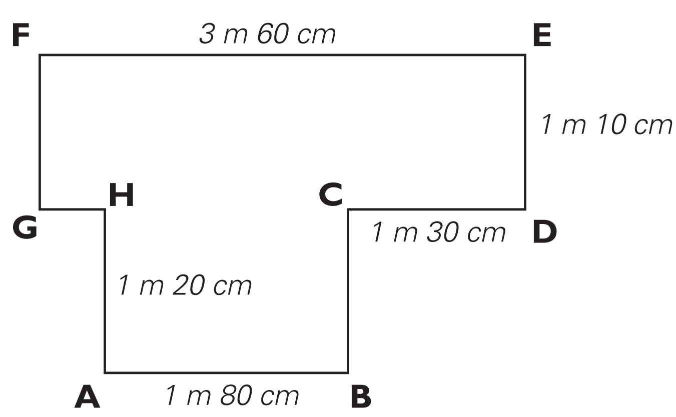
1. A farmer started at corner A of the paddock and rode his tractor around the perimeter to see if there were any gaps in the fence. He found one hole at D and continued all the way around to A, where he picked up his tools to fix the fence. He then rode counterclockwise to D and fixed the hole.



To be sure the fence was completely free of holes, he rode all the way around once more until he arrived at D again.

How far did he ride to check and fix the fence? ____________________

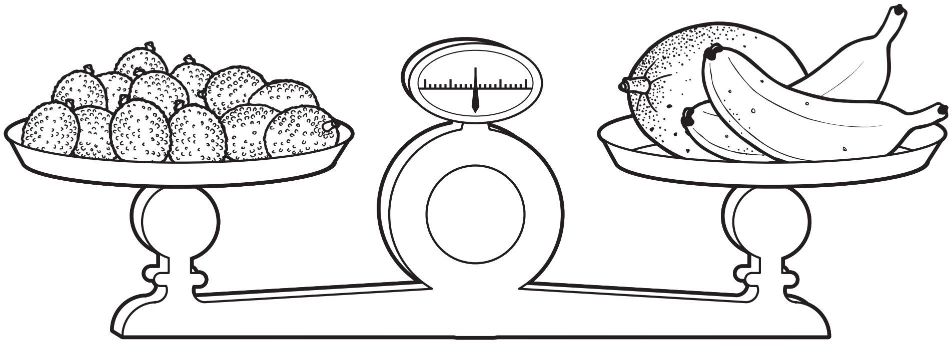
2. When he got back to the farmhouse, he picked some vegetables from the garden. His dog, Blue, wasn't allowed in the garden, so Blue ran around the outside fence, starting at A. He ran around and around counterclockwise, only stopping when he reached G for the third time.



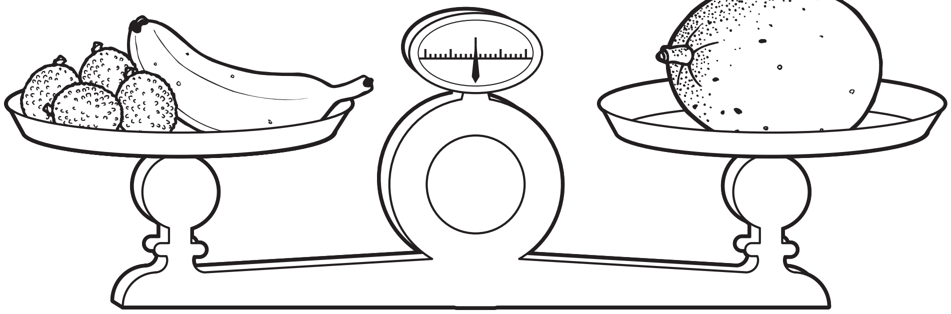
How far did Blue run? ____________________

20.2 BALANCING

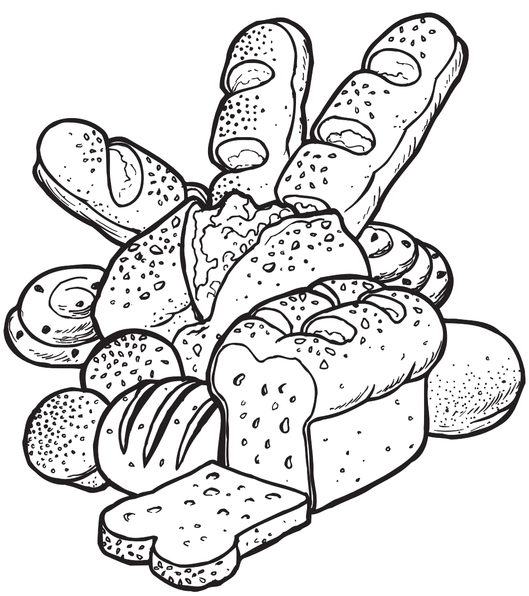
13 kiwis weigh as much as two bananas and one mango.



Four kiwis and one banana have the same weight as one mango.



1. How many kiwis are needed to balance one mango? ______________



Six bread rolls and one loaf of bread cost $4.20.

Four bread rolls and two loaves of bread cost $6.00.

2. What is the price of a loaf of bread?

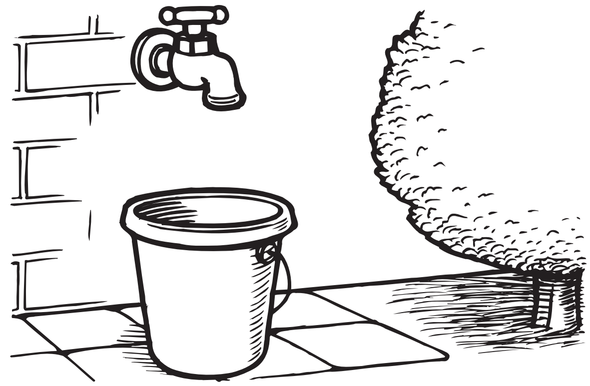
    ____

3. What is the price of a bread roll?

    ____

20.3 DISTANCE TRAVELED

1. Kevin has planted 16 small trees in a row to provide a hedge along the side of his property. He planted them 9 feet apart, with the first tree next to a garden tap. Because of a drought, he is only allowed to water them using a single bucket filled at the tap. If the bucket contains only enough water for 2 trees, how far does he need to walk to water all the trees?



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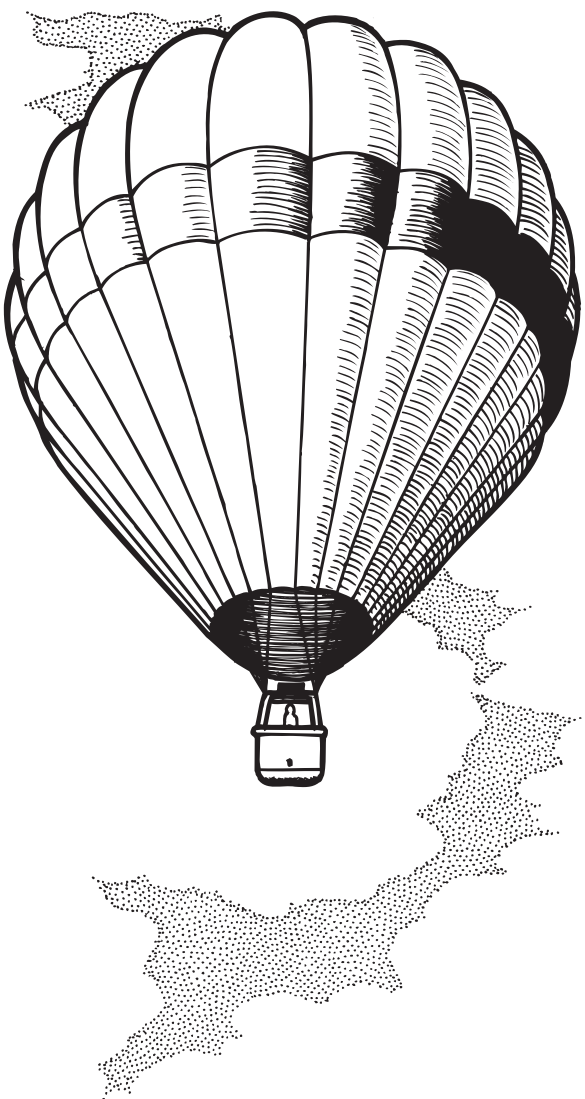


2. Two cyclists set out from the clubhouse, cycled to the bay, and returned to the clubhouse without stopping. The first cyclist traveled at the same speed the whole way, but the second cyclist started at half the speed and returned at twice the speed. Which cyclist returned first?

____

3. An adventurer made a flight in her balloon. First she ascended 300 meters. Then she flew 1,500 meters toward the northwest, descended 100 meters, and flew 750 meters to the northeast. After that, she flew 1,500 meters toward the southeast and descended 100 meters. How far was she from her starting point?

____

Problem Solving: Tables and Diagrams

Problem-Solving Objective

To use logical reasoning, spatial visualization, and measurement to solve problems involving a calendar or clock

Materials

Counters, calculator, calendar, clock with hands, analog clock

NCTM Content Standards

- Number and Operations 1.1
- Geometry 3.2
- Data Analysis and Probability 4.1

Focus

These pages explore different ways of coming to terms with problem situations and analyzing the possibilities that make up the whole solution. Logical reasoning as well as an understanding of measurement concepts, such as a clock, a calendar, length, perimeter, area, and direction, are required. Diagrams can be used to organize, sort, and explore the data.

Discussion

Page 99 – Calendar Calculations

This page explores students' understanding of the days in a week and how they are arranged in a calendar. For Problem 1, while it might seem that the variable days in a month might affect the result, all that is really being asked is to divide 100 by 7 and to find the remainder to see how many days later the birthday will be. The other problems use a similar understanding of the cyclical nature of the days of the week. An actual calendar, counters, or timeline will help with the second problem. Problem 3 calls on an understanding of multiples, while the last problem requires a careful elimination of the possibilities, so a table may help.

Page 100 – Taking Time

Each of these problems requires a good understanding of time: the hours and minutes in a day and how time is shown on a clock. Access to a clock with moveable hands will help students understand what happens when time is gained or lost on a clock, as well as the way an analog clock shows time over a 24-hour period. Using a list or table will help students keep track of the changes and possibilities.

Page 101 – Puzzle Scrolls 2

The puzzle scrolls contain a number of different problems, all involving strategic thinking to find possible solutions. In most cases, students will find that diagrams and lists are needed to manage the data while exploring the different possibilities. Concepts of space and measurement are explored in each scroll, and students may use a number of different ways to find possible solutions, including the "try and adjust" strategy.

Possible Difficulties

- Using only 12-hour and not 24-hour time in considering the correct time
- Thinking that the hands on a clock always line up on the hour
- Including 12 o'clock as a time when the hands line up on a clock
- Unable to draw or interpret diagrams to see the relationships among perimeter, side length, and area

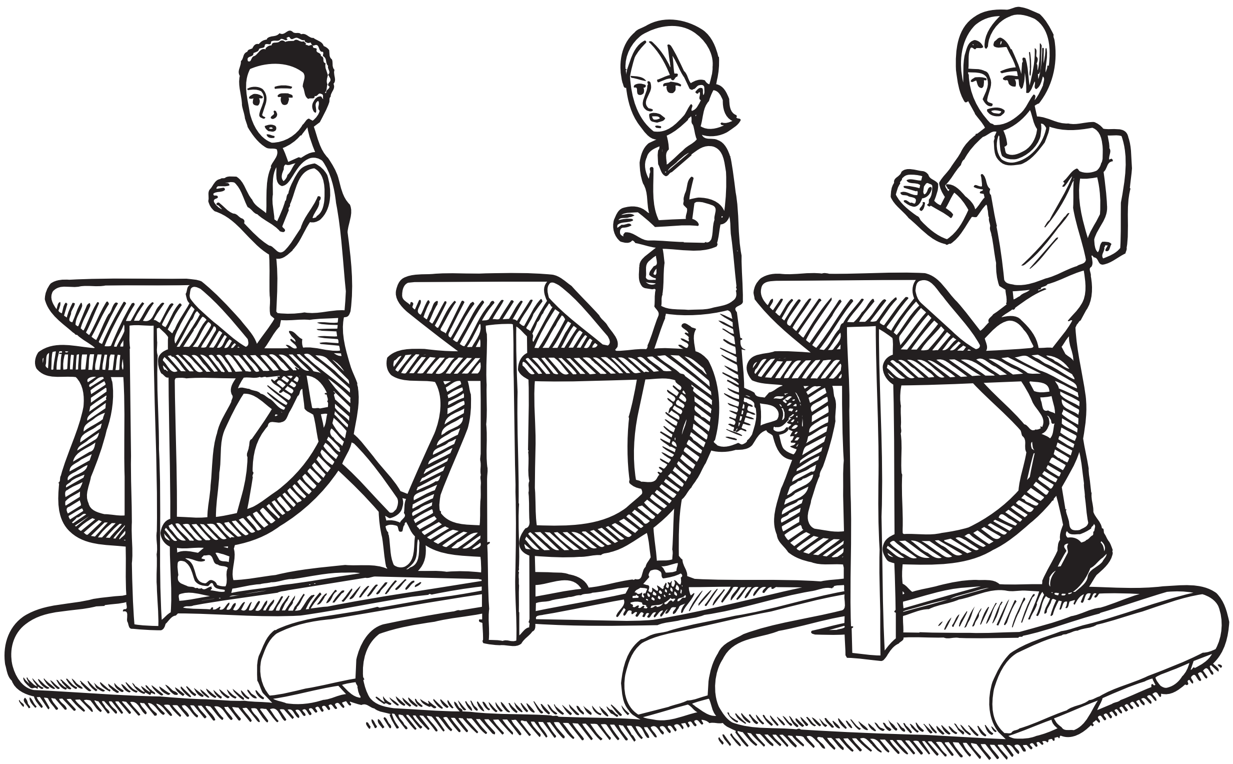
Extension

- Have students make up their own problems based on the calendar and days in a week.
- Students could write other complex problems involving clocks that lose or gain time.
- Students could write other complex problems involving the area, side lengths, and perimeter of a pentagon, hexagon, or other polygon.

21.1 CALENDAR CALCULATIONS

1. Today is my birthday. My friend's birthday is exactly 100 days after mine. If my birthday is on a Wednesday this year, what day of the week will she have her birthday?

2. What day of the week was yesterday if 4 days before the day after tomorrow was Monday?



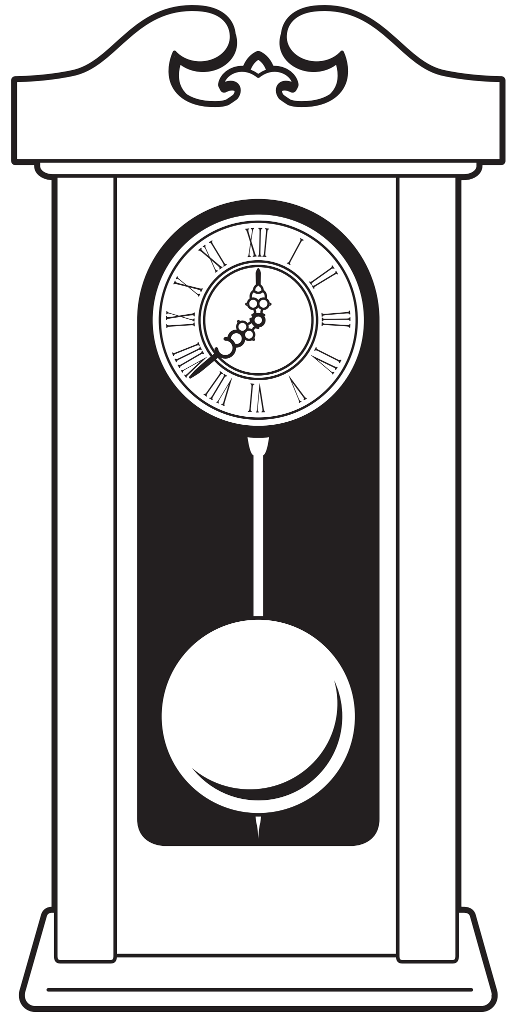
3. Three friends visit the gym on different days: the first boy once every 3 days, his girlfriend once every 4 days, and the second boy once every 5 days. They were last together at the gym on a Friday. In how many days will they again be together at the gym, and what day of the week will it be?

4. June has 30 days. One year, June had four Mondays. On which days of the week could June 30 not have occurred that year?

21.2 TAKING TIME

1. Judy bought an old-fashioned clock with hands for $10 at the flea market. It was a bargain, but it does not keep accurate time and gains 1 minute every hour. If the clock showed the correct time when she bought it, when will it next show the correct time?

____



2. Grandma's clock is so old that a key is used to wind it up to keep time. Unfortunately, the clockwork spring has stretched with time, and it now loses 6 minutes every hour. When I last visited Grandma, I set it to the correct time of 10:30 a.m. What will be the correct time when her clock shows 3 p.m. that afternoon?

____

3. A fast clock gains 1 minute per hour and a slow clock loses 2 minutes per hour. At a certain time, both clocks are set to the correct time. Less than 24 hours later, the fast clock registers 9 o'clock at the same moment that the slow clock registers 8 o'clock. What is the correct time at that moment?

____

4. How many times a day do the hands of an analog clock lie directly opposite each other?

____

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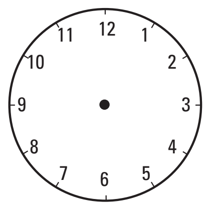
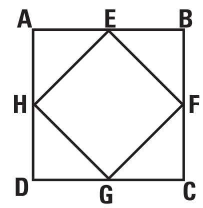
21.3 PUZZLE SCROLLS 2

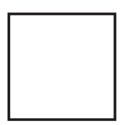
1. Divide the face of a clock into three parts. Make the numbers in each part add to the same total.



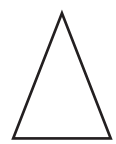
2. ABCD is a square with sides that measure 2 in. in length. E, F, G, and H are the midpoints of the sides. What is the area of EFGH?



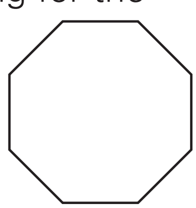
3. The area and perimeter of a small square are numerically equal. What is the length of one side of the square?



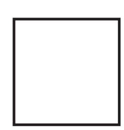
4. An isosceles triangle has two sides of equal length. If the base is half the length of one of the other two sides and the perimeter is 1 m 35 cm, how long is the base?



5. A man built a fence around an octagonal garden. On each side there are 7 posts. How many post holes did he have to dig for the posts?



6. If you double the length of each side of a square, do you double the area of the square?



SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

1.1 THE WATER PARK...................................... page 25

1. 301 more adults
2. 433 people swimming in the pools
3. 108 people lying on their towels
4. 100 people floating on floats
5. 150 people
6. 37 surfers did not catch the wave.

1.2 LILY PADS AND FROGS .... page 26

1. 261 flowers
2. 192 lily pads
3. 111 frogs
4. 125 lily pads are in flower.
5. Yes, there are 375 flowers.
6. 333 lily pads
7. 410 frogs

1.3 SAMANTHA'S FLOWER SHOP................. page 27

1. 54 pots are available for sale.
2. (a) Yes (b) 68 roses to sell
3. Yes, with 22 roses left over.
4. (a) 35 lilies were left over.
   (b) 75 lilies

1.4 THE HERB MARKET................................... page 28

1. 155 more fresh herbs
2. Simon has 144 pots.
3. Simon needs 360 seeds.
4. He can sell 195 bunches.
5. He has 106 bunches left.
6. none
7. He has unpacked 60 plants.

1.5 AT THE BAKERY .... page 29

1. She needs to bake 226 more cakes.
2. 316 whole-wheat loaves were sold.
3. (a) There are 180 pies available to sell.
   (b) 97 pies were not sold.
4. There are 44 brownies available to sell.

2.1 THE PLANT NURSERY .... page 31

1. 2,844 seedlings were planted.
2. $3,078 was made selling bark mulch.
3. $1,710 profit from selling bark mulch was made.
4. 6,565 plants were watered.
5. 2,124 seedlings were planted.

2.2 THE CITRUS ORCHARD .... page 32

1. 273 trees are watered each day. It will take 4 days.
2. 37,440 oranges have been picked.
3. 2,176 crates were packed after 6 acres.
4. 4,352 crates were packed altogether.

2.3 ANIMAL WORLD .... page 33

1. 6 days (and there are enough for another half-day)
2. 1,725 cars are in the parking lot.
3. It would cost $1,260 to feed the giraffe.
4. 728 bags of seed are used in one year.
5. 162 more people can watch the show.
6. The seals eat 2,600 pounds of fish per year.

2.4 BEADING.. page 34

1. Judy has 223 beads left.
2. Manu has 263 beads.
3. Clarence bought 576 beads.
4. Marty gave away 504 beads.
5. Ned has 145 beads.
6. No, 5 sets require 450 beads, and Zena has only 442 beads.

2.5 THE COMMUTER TRAIN .... page 35

1. 1,170 can travel on the train and 846 can sit.
2. 989 people on the train
3. 991 people are needed to fill the train.
4. 10 people are not sitting.
5. 668 people were on the train.
6. 13 buses are needed.

3.1 THE FRUIT FARM .... page 37

1. 82 trays and one peach left over
2. 24 boxes, 96 trays are needed
3. (a) 45 mangoes in each box
   (b) 15 mangoes in each tray
4. 1680 apricots and 816 pears are packed.
5. about 142 peaches per hour

3.2 AT THE DELI .... page 38

1. $41 profit on each container
2. 4 containers must be bought.
3. Loose olives are the better buy.
4. $17 more profit in summer
5. 168 packets have been unpacked.

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

3.3 THE SUGAR MILL .... page 39

1. 764 bins were transported.
2. 6 train trips
3. 3,229 bins and 23 trains were used.
4. 970 metric tons per month
5. about 1,122 metric tons of raw sugar
6. Two trucks would make 38 trips and one truck would make 37 trips.

4.1 CALCULATOR PROBLEMS .... page 41

1. 5,675
2. 6,976
3. 13,396
4. 9,658
5. 7,739
6. 6,095

5.1 ICE-CREAM CONES .... page 43

1. Anna has 2,490 cones.
2. They need 60 cartons.
3. 4,970 cones were delivered.
4. 10,930 cones are left.
5. The shop has 2,820 cones.
6. The shop has 24 extra boxes.

5.2 THE LIBRARY .... page 44

1. 95 shelves are needed.
2. The cleaner dusted 4,800 books.
3. (a) There are 6,631 books.
   (b) The library has 7,021 books.
4. 104 books were borrowed.
5. 29 boxes are needed.

5.3 BOOKWORMS .... page 45

1. 138 pages in each chapter
2. Page 409 is in Chapter 3.
3. She has been reading the book for 38 days.
4. 553–690
5. Some of these pages are in Chapter 5 and some in 6.
6. 70 pages
7. Her friend is up to page 698.
8. Her friend needs to read 682 pages.
9. Wanda needs to read 620 pages.
10. It will take her 62 days to finish the book.

6.1 SHOPPING.. page 47

1. $180
2. $81
3. Sharon took $50 to spend.

7.1 CHICKEN TAKEOUT.................................... page 49

1. $9.00
2. $24.50
3. Spent: $19.00, change: $31.00
4. Spent: $35.50, change $14.50
5. Spent: $30.00, change: $20.00
6. Spent: $31.00, change: $19.00
7. Samir

8.1 STAR GAZE... page 51

1. (a) 23 (b) 55 (c) 105
2. (a) 3, 8, 8, 8 (b) 5, 9, 9, 9 (c) 6, 6, 7, 7
   (d) 2, 2, 5, 7 (e) 8, 8, 4, 1 (f) 7, 7, 9, 5
   (g) 13, 13, 24, 7 (h) 23, 23, 19, 6 (i) 23, 23, 37, 16

8.2 MAGIC SQUARES...................................... page 52

1. 12
2. (a)

15	**1**	11
5	9	**13**
7	**17**	3

27 (b) All odd numbers

3.

80	**10**	**60**
30	50	70
40	**90**	20

24

4.

16	**3**	**2**	13
5	10	**11**	8
9	**6**	7	**12**
4	15	**14**	1

34

5. 34

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

6. 34
7. 34
8. 1–16
9. 1514 in the middle two squares in the bottom row.

8.3 SUDOKU .... page 53

1.

3	1	2	4
2	4	3	1
4	2	1	3
1	3	4	2

3	1	4	2
4	2	3	1
2	4	1	3
1	3	2	4

3	1	4	2
2	4	1	3
1	3	2	4
4	2	3	1

2.

2	4	6	5	3	1
5	3	1	6	4	2
1	5	4	3	2	6
3	6	2	4	1	5
6	1	3	2	5	4
4	2	5	1	6	3

2	5	3	1	6	4
4	6	1	3	2	5
6	3	2	4	5	1
1	4	5	6	3	2
3	2	4	5	1	6
5	1	6	2	4	3

9.1 HOW MANY DIGITS? .... page 55

1. (a) 140
   (b) 140
2. (a) fewer (you would exclude "fifty" and "fifteen")
   (b) 118
   (c) 140
3. 2 and 3 – say: 18 times, write: 40 times; 6 to 9 – say: 40 times; write: 40 times

10.1 NUMBERS IN COLUMNS .... page 57

1. E
2. (a) D (b) A (c) D (d) B (e) C
3. (a) 7 (b) 13 (c) 18 (d) 19 (e) 22
4. Answers will vary—see page 56.
5. Answer will vary—see page 56.
6. Yes, numbers with 4 in the ones place are still in Column D; numbers with 1 in the ones place are still in Column A.

11.1 PUZZLE SCROLLS 1................................ page 59

1. It was half full after 47 days
2. Answers will vary. Some are:
   1, 4, 5, 9; 5, 2, 3, 9; 1, 0, 9, 9; 2, 4, 7, 6
3. (a) 27 (b) 12, 24, 36, 48
4. 63 members after 5 weeks
5. Page numbers 37, 38, 39, 40, 41, 42
6. 64 x 15,625; there are no others.

11.2 SCRAPBOOKING .... page 60

1. 12 pages, 55 photos
2. 17 pages, 56 pictures
3. There are 34 players.
4. 58

11.3 CHANGING COINS .... page 61

1. (a) $8.70 (b) two (c) No
   (d) Twice – 4th, 6th, 8th, 10th; Once – 2nd, 3rd, 5th, 9th
2. (a) 1st, 4th, 9th, 16th coins
   (b) Once – 1st coin; Twice – 2nd, 3rd, 5th, 7th, 11th, 13th coins
   (c) 25th coin
   (d) 17th, 19th, 23rd
   (e) Prime number positions change only twice and only square number positions will show heads.

12.1 STACKING SHAPES................................ page 63

1. 30 cubes
2. 20 oranges
3. 50 cans
4. 36 cubes

12.2 PAINTED CUBES .... page 64

1. 4 cubes
2. yes; 3 cubes
3. yes; 1 cube
4. 4 cubes
5. 8 cubes
6. 4 cubes
7. No cube has only 1, 5, or 6 faces on the outside of the block.

12.3 CUBE PAINTING...................................... page 65

1. 8 cubes
2. Yes
3. 1 cube
4.

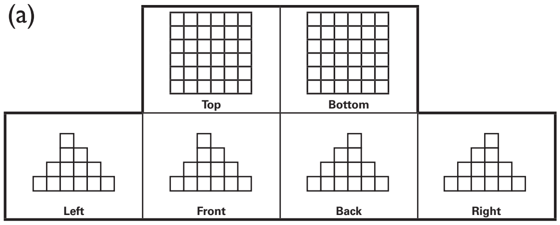
No. of Faces Painted	No. of Cubes
0	1
1	6
2	12
3	8
	Total = 27

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

12.4 VIEWING CUBES .... page 66

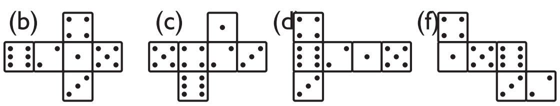
1. (a)



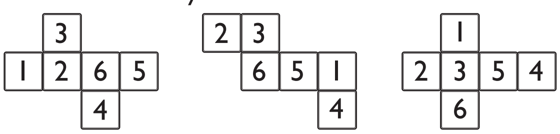
   (b) The views pair up as mirror images of each other.
2. 57
3. 11
4. Teacher check

12.5 NETS AND CUBES .... page 67

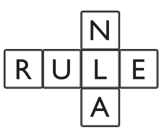
1. Students should have circled b, c, d, and f.



2. Answers will vary. Possible answers:



3.



13.1 SQUARES AND PERIMETERS................ page 69

1. (a) 30 in. (b) 36 in. (c) 30 in. (d) 42 in.
   (e) 48 in. (f) 36 in. (g) 30 in.
2. Teacher check
3. Answers will vary.

13.2 MORE PERIMETERS .... page 70

1. (a) A smaller square (b) 8 in.
2. A small triangle
3. 18 in.
4. 48 in.

13.3 PERIMETERS IN SQUARES .... page 71

1. 24 in.
2. 72 in.
3. 80 in.

14.1 AREA .... page 73

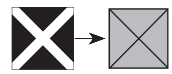
1. Area of shape is 360 ft^2
2. Area of smaller triangle is 18 $in.^2$
3. Area of new shape is 7,800 cm^2

14.2 AREA AND PERIMETER 1 .... page 74

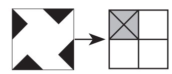
1. The perimeter is 64 cm.
2. The area is 576 ft^2.
3. The area of the garden is 425 ft^2.
4. (a) The perimeter of the garden is 9 m 60 cm.
   (b) A hexagon—the pond wall is 4.8 m long.

14.3 AREA AND PERIMETER 2 .... page 75

1. (a) 36 m^2
   (b) 180 m^2



2. 20 m^2
3. (a) 40 m^2
   (b) 20 m^2



15.1 TANGRAM AREAS .... page 77

1. The area of small triangle is 5 cm^2.
2. The area of the parallelogram is 10 cm^2.
3. The area of the medium-sized triangle is 10 cm^2.
4. The combined area is 40 cm^2.
5. They form half of the tangram.
6. (a) The area of a large triangle is 20 cm^2.
   (b) It is $\frac{1}{4}$ of the whole square, which is 80 cm^2.
7. The area of the square would be 18 cm^2.
8. The area of a large triangle will be 16 cm^2.

16.1 DVD RENTALS... page 79

DVD Easy		Hours Open
Monday	CLOSED	0
Tuesday	11:00 a.m. to 8:00 p.m.	9
Wednesday	11:00 a.m. to 9:00 p.m.	10
Thursday	10:00 a.m. to 9:00 p.m.	11
Friday	11:00 a.m. to 10:00 p.m.	11
Saturday	10:00 a.m. to 10:00 p.m.	12
Sunday	11:00 a.m. to 9:00 p.m.	10
	Total hours	63

SOLUTIONS

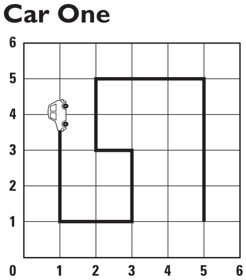
Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

DVD Rentals		Hours Open
Monday	11:00 a.m. to 9:00 p.m.	**10**
Tuesday	11:00 a.m. to 9:00 p.m.	**10**
Wednesday	CLOSED	**0**
Thursday	11:00 a.m. to 9:00 p.m.	**10**
Friday	11:00 a.m. to 11:00 p.m.	**12**
Saturday	10:00 a.m. to 11:00 p.m.	**13**
Sunday	11:00 a.m. to 9:00 p.m.	**10**
	Total hours	**65**

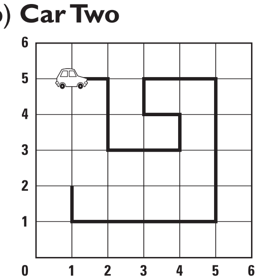
1. 2 hours
2. 11:00 p.m.
3. 10:00 a.m.
4. DVD Rentals
5. Saturday
6. either store
7. DVD Rentals
8. (a) No (b) Yes (c) Yes (d) Yes (e) No

16.2 DRIVE TIME.. page 80

1. (a) **Car One**



(b) **Car Two**



2. (a) (4, 4) (3, 4) (3, 5) (1, 5) (1, 3) (3, 3) (3, 2) (1, 2) (1, 1) (4, 1) (4, 3) (6, 3)
   (b) (6, 4) (3, 4) (3, 6) (1, 6) (1, 3) (2, 3) (2, 1) (4, 1) (4, 5)
3. Answers will vary.

16.3 CLOCK WATCHING .... page 81

1. (a) 0:12, 0:21, 0:19, 0:29, 1:02, 1:20, 1:09, 1:29, 2:01, 2:10, 2:09, 2:19, 9:01, 9:10, 9:12, 9:21, 9:02, 9:20, 10:29, 12:09, 19:02, 19:20, 20:19, 21:09
   (b) 12:09
   (c) 0:12
2. (a) 7
   (b) 168
   (c) 4
3. 172

17.1 THE BIG RACE.. page 83

1. 14th place
2. 14 cars
3. ninth place
4. 11th

17.2 SERIAL NUMBERS.................................. page 84

1. 3946
2. 5723
3. 2835 or 5382

17.3 UP AND DOWN .... page 85

1. The extended ladder has 53 rungs.
2. The ball traveled 6.75 meters.
3. You entered the elevator at Floor 23.
4. The usual cruising altitude is 31,800 feet.

18.1 HOW MANY?... page 87

1. 590,390 visitors in 2006
2. 131,000 visitors in January
3. 33,370 visitors in June

18.2 HOW FAR?.. page 88

1. Erin traveled 462 miles.
2. Susie swims about 6 km (6,000 m).
3. It will arrive about 11 hours and 45 minutes after leaving.
4. Brian runs about 576 miles.
5. Kim-Ly traveled 510 miles.

18.3 HOW MUCH? .... page 89

1. Derek spent $80 on shirts.
2. Payment per paper ($35.86) pays more than hourly ($25.25).
3. Alison sold 60 apples.
4. Brackets without screws are 20¢ cheaper.
5. Payment per tree ($355.20) is better than per day ($317.50).

19.1 ON THE FARM.. page 91

1. 26 emus and 12 alpacas
2. 4 calves and 6 piglets
3. 15 alpacas and 30 emus

SOLUTIONS

Note: *Many solutions are written statements rather than simply numbers. This is to encourage teachers and students to solve problems in this way.*

19.2 IN THE BARN .... page 92

1. three wheels = 14, two wheels = 11
2. 13 spiders and 16 beetles
3. Answers will vary, since there are multiple answers. Answers are displayed in the table:

Number of Cows	Number of Chickens	Number of Legs
6	2	28
5	4	28
4	6	28
3	8	28
2	10	28
1	12	28

19.3 MARKET DAY .... page 93

1. She paid $275 for the pig.
2. The calf sold for $101.
3. $150

20.1 FARM TRAILS .... page 95

1. 5,900 m or 5 km 900 m
2. 3,370 cm or 33 m 70 cm

20.2 BALANCING .... page 96

1. 7 kiwis
2. $2.40
3. 30 cents

20.3 DISTANCE TRAVELED .... page 97

1. He walks 1,017 feet to finish at the last tree.
2. The first cyclist would return first.
3. 750 meters northeast of the starting point

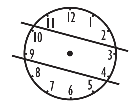
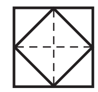
21.1 CALENDAR CALCULATIONS .... page 99

1. She has her birthday on Friday.
2. Tuesday
3. They will be together again on Tuesday.
4. Monday or Tuesday

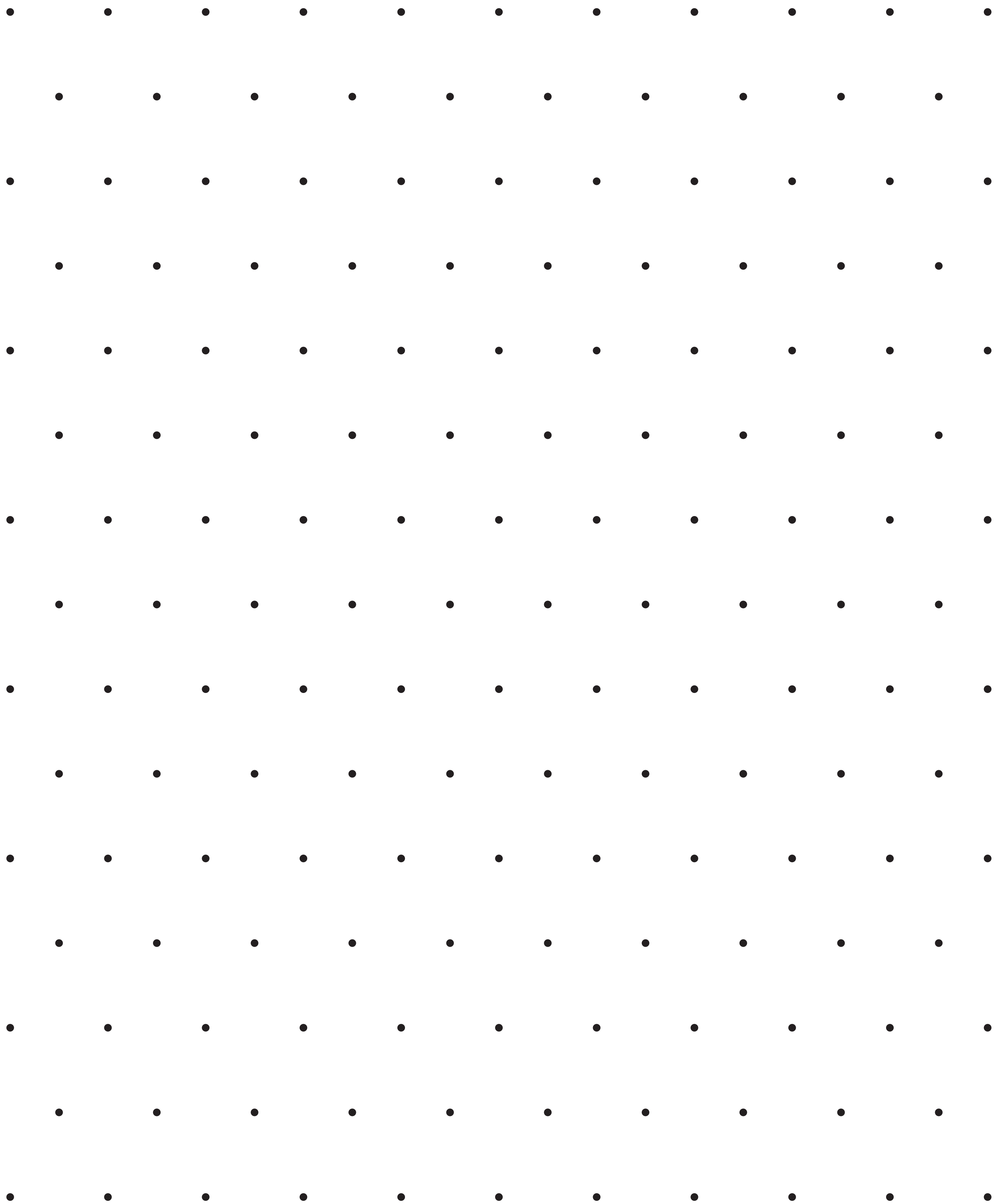
21.2 TAKING TIME .... page 100

1. 30 days
2. 3:30 p.m. is the correct time.
3. 8:40 is the correct time.
4. The hands are opposite 22 times.

21.3 PUZZLE SCROLLS 2 .... page 101

1. 
2. The area is half that of the large square, so it is 2 in.^{2} 
3. 4
4. 27 cm
5. He needs 48 holes to have 1 hole at each corner and 5 more along each side.
6. No, you increase it by four times.

ISOMETRIC DOTS



0–99 BOARD

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

4-DIGIT NUMBER EXPANDER (x 5)

ones

tens

hundreds

thousands

10 mm x 10 mm GRID

TRIANGULAR GRID