Another method we can use to find the area of a polygon is called the “chop method.” Here is an example of how the chop method works:

Take the figure $ABCD$, an irregular polygon:

### Step 1:
Build a rectangle completely around the shape. Note that the area of this rectangle, $FABE$, is 12 square units.

### Step 2:
Begin chopping:
- Area of $GAD = \frac{1}{2}$ area of $GAJD = 1$ square unit.
- Area of $HDC = \frac{1}{2}$ area of $HDKC = 1$ square unit.
- Area of $CBE = \frac{1}{2}$ area of $CLBE = 1\frac{1}{2}$ square units.
- Area of $FGDH = 1$ square unit.

Altogether, this chops $4\frac{1}{2}$ square units off the 12-square-unit rectangle $FABE$.

This means the area of the original polygon $ABCD$ is $7\frac{1}{2}$ square units.
Find the Area Using the “Rectangle Method”

Make these shapes on your geoboard and find their areas.

1. Area = _______ sq. units
2. Area = _______ sq. units
3. Area = _______ sq. units

4. Area = _______ sq. units
5. Area = _______ sq. units
6. Area = _______ sq. units

7. Area = _______ sq. units
8. Area = _______ sq. units
9. Area = _______ sq. units
**Find the Area Using the “Chop Method”**

Make these shapes on your geoboard and find their areas.

Example:

![Geoboard Diagram](image)

Area of $ABCD = 4$

- Area of $AED = 1 \frac{1}{2}$
- Area of $EBC = \frac{1}{2}$

Area of $DEC = 2$

1. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

2. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

3. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

4. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

5. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

6. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

7. ![Geoboard Diagram](image)
   
   Area = _______ sq. units

8. ![Geoboard Diagram](image)
   
   Area = _______ sq. units
Find the Area

Make these shapes on your geoboard and find their areas using either the rectangle method or the chop method.

1. 
2. 
3. 

Area = ______ sq. units  
Area = ______ sq. units  
Area = ______ sq. units

4. 
5. 
6. 

Area = ______ sq. units  
Area = ______ sq. units  
Area = ______ sq. units

7. 
8. 
9. 

Area = ______ sq. units  
Area = ______ sq. units  
Area = ______ sq. units
What’s the Area?

Make a shape on your geoboard—any shape you want. Find the area of the shape you made. Sketch your shape on the blank geoboard below if it will help.

Ask a classmate to find the area of your shape. If you disagree on what the area is, look at your solutions together to determine the correct answer.
A triangle is a polygon with three sides and is classified by the measures of its angles or the lengths of its sides. If classified by the lengths of its sides, a triangle is called scalene, isosceles, or equilateral:

**Scalene** – No two sides are equal in length.
**Isosceles** – At least two sides are equal in length.
**Equilateral** – All three sides are equal in length.

If classified by the measures of its angles, a triangle is called acute, obtuse, or right:

**Acute** – An acute triangle has three acute angles. *Acute* means less than 90 degrees, or less than a right angle.
**Obtuse** – An obtuse triangle has one obtuse angle. *Obtuse* means greater than 90 degrees. *Obtuse* means greater than 90 degrees but less than 180 degrees.
**Right** – A right triangle has one right angle—that is, an angle of 90 degrees.
**Equiangular** – All angles are equal in measure.

Have students use their geoboards to explore the properties of triangles in the activities that follow.
Scalene, Isosceles, and Equilateral Triangles

Use your geoboard to make the triangles on this page, and complete the table on the next page. (Hint: Fill in the corresponding row of the table as you make each triangle.)
**Scalene, Isosceles, and Equilateral Triangles (cont.)**

Refer to the triangles on the previous page and complete the following table. Measure the lengths of the sides with your ruler.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Pegs Inside</th>
<th>Scalene</th>
<th>Isosceles</th>
<th>Equilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
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<tr>
<td>B</td>
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</tr>
</tbody>
</table>
Triangles

Use your geoboard to make the triangles on this page, and complete the table on the next page. (Hint: Fill in each row of the table as you make each triangle.)
Refer to the triangles on the previous page and complete the this table. Classify each triangle by its angles and then by its side lengths.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Pegs Inside</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral</td>
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<tr>
<td>Scalene</td>
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<tr>
<td>Isosceles</td>
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<td>Right</td>
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<td>Obtuse</td>
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<td>Acute</td>
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<tr>
<td># Pegs Inside</td>
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</tbody>
</table>
Triangles

Look back at the tables on pages 12 and 14 and answer the following questions. Circle the correct answer. (You may need to make the triangles on your geoboard again to solve these problems.)

1. Are all isosceles triangles acute triangles?  
   Yes  No

2. Are all scalene triangles acute triangles?  
   Yes  No

3. Can you make a right triangle that is isosceles on your geoboard?  
   Yes  No

4. Can you make a right triangle that is obtuse?  
   Yes  No

5. Can you make a right acute triangle?  
   Yes  No

6. Are all right triangles acute?  
   Yes  No

7. Is it possible to make an equilateral triangle on your 25-pin geoboard?  
   Yes  No

8. Triangles with pegs inside are always larger in area than triangles without pegs inside.  
   True  False

9. Right triangles cannot have pegs inside.  
   True  False

10. Acute triangles are always smaller in area than obtuse triangles.  
    True  False
How Many Pins?

Below are three sets of exercises that will show you a new way to find areas on the geoboard. Each set asks you to make a shape with a specific number of pins inside the shape. For each shape you make, record the area and number of pins touched by the rubber band that forms the shape. A wonderful pattern exists. See whether you can discover it!

1. Make as many shapes as you can with NO pins inside the shape. Record your results in the table.

<table>
<thead>
<tr>
<th>Area (A)</th>
<th>Number of Pins Touched by Rubber Band (N)</th>
<th>Number of Pins Inside the Shape (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

2. Make as many shapes as you can with only 1 pin inside the shape. Record your results in the table.

<table>
<thead>
<tr>
<th>Area (A)</th>
<th>Number of Pins Touched by Rubber Band (N)</th>
<th>Number of Pins Inside the Shape (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
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<td>1</td>
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<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
How Many Pins? (cont.)

3. Make as many shapes as you can with just 2 pins inside the shape. Record your results in the table.

<table>
<thead>
<tr>
<th>Area (A)</th>
<th>Number of Pins Touched by Rubber Band (N)</th>
<th>Number of Pins Inside the Shape (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
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<td>2</td>
</tr>
</tbody>
</table>

4. What patterns did you discover? _____________________________________________________________
   _____________________________________________________________
   _____________________________________________________________
   _____________________________________________________________
   _____________________________________________________________

5. Write the formula that describes the pattern. ____________________________________________
Perimeter Word Problems

Using what you discovered about perimeters with your geoboard, solve the following problems.

Here is an example:

Find the perimeter of this rectangle.

\[
\begin{align*}
\text{Write the formula.} & \quad P = 2l + 2w \\
\text{Substitute the data.} & \quad P = (2 \times 60) + (2 \times 40) \\
\text{Solve the problem.} & \quad P = 120 + 80 \\
& \quad P = 200
\end{align*}
\]

The perimeter of the rectangle is 200 meters.

1. If you want to build a rectangular fence around your garden that is 12 ft. wide and 10 ft. long, how much fencing will you need?

2. A rectangular picture measures 39 centimeters by 50 centimeters. How much trim is needed to go around the picture?
The Same but Different

In the shapes shown below, notice that the area ($A$) remains the same but the perimeter ($P$) changes.

$$A = 6; \quad P = 12$$

$$A = 6; \quad P = 10$$

$$A = 6; \quad P = 14$$

Make some shapes on your geoboard and see whether the perimeter always changes when the shape is different but the area is the same. Sketch your findings on the blank geoboards below. No right triangles allowed!

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$

$$A = \text{_____} \quad P = \text{_____}$$
Chapter 1: Finding Area on the Geoboard

Page 5:
All answers are in square units.

1. 5  2. 6  3. 6  4. 8  5. 8  6. 12  7. $4\frac{1}{2}$  8. 10  9. 8

Page 6:
All answers are in square units.

1. $4\frac{1}{2}$  2. $3\frac{1}{2}$  3. 5  4. 4  5. 8  6. 2  7. 4  8. 7

Page 7:
All answers are in square units.

1. 4  2. 4  3. 3  4. 8  5. 7  6. 6  7. 9  8. 2  9. 5

Page 8: Teacher check.

Page 12:

<table>
<thead>
<tr>
<th>Shape</th>
<th># Pegs Inside</th>
<th>Scalene</th>
<th>Isosceles</th>
<th>Equilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0</td>
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<td>X</td>
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<td>D</td>
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<td>I</td>
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</tr>
</tbody>
</table>
Page 14:

<table>
<thead>
<tr>
<th>Shape</th>
<th># Pegs Inside</th>
<th>Acute</th>
<th>Obtuse</th>
<th>Right</th>
<th>Isosceles</th>
<th>Scalene</th>
<th>Equilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>X</td>
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<td>X</td>
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</tbody>
</table>

Page 15:


Page 17:

No, it is not a rhombus. It does not have four congruent sides. Adjacent sides are not equal in length.

Page 18:

Answers may vary. Possible answers include:

1. (A) It is a parallelogram, (B) has 4 right angles, and (C) adjacent sides are equal.

2. (A) It is a parallelogram, (B) has 4 right angles, and (C) adjacent sides do not have to be equal.

3. (A) It is a quadrilateral, (B) with only one pair of parallel sides, and (C) always has an obtuse angle.

4. (A) It is a parallelogram, (B) with all four sides equal in length, and (C) has two obtuse angles.

5. No

6. Yes, but not on the geoboard.

7. Yes
Page 29–30:

1. Teacher check.
2. Teacher check.
3. Teacher check.
4. Teacher check.
5. $A = \frac{1}{2} N + (I - 1)$, if $A = \text{area}$, $P = \text{the number of pins touched by the rubber band}$, and $I = \text{the number of pins inside the shape}$.

Page 32:

1. The area of the square on the hypotenuse equals the sum of the areas of the other two squares: $\sqrt{2}$. The sum of the squares of the lengths of the two sides equals the square of the length of the hypotenuse.
2. 13 units
3. 24 units
4. 5 units

Page 34:

1. 8 units
2. 16 units
3. 10 units
4. 14 units
5. 16 units
6. 14 units
7. $3 + \sqrt{5}$
8. $5 + \sqrt{17}$
9. $4 + \sqrt{8}$

Page 35:

1. 44 ft
2. 178 cm

Page 36: Teacher check.

Page 37:

1. 6
2. Yes
3. Yes: 1, 2, 4, 5, 8, 9; No: 3, 6, 7
4. Yes. Possible answers:

5. Yes, a $4 \times 4$ square has an area of 16 and a perimeter of 16.