Bonus Activity 1 Introduce Prime and Composite Numbers

Objective

Students will be able to:

1. Physically model prime and composite numbers using rectangular area models.
2. Classify numbers as prime or composite.

Vocabulary

prime number
composite number
factor
product
factor pair

Materials and Background Knowledge

- Prime Factor Tiles student sets

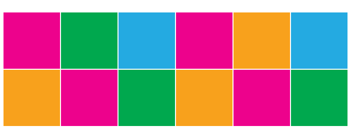
Students should be familiar with the area model for multiplication and know that factors multiply to yield a product.

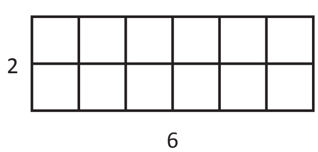
Overview

Students previously understood multiplication as the product of two factors. Now, students discover that different pairs of factors multiply to give the same product. Later, students will utilize multiple factor pairs when comparing the factors of two or more numbers.

Introduce the Activity

1. Model numbers with area models. *Say:* **Please flip the Prime Factor Tiles number side down. Then select any twelve tiles and arrange them in a rectangle.** (See diagram for one example.) *Say:* **Please sketch your rectangle and record the dimensions of the base and height.** *Say:* **Please write the area of the rectangle as a times-table fact from your dimensions.** *Say:* **These square tiles are 1 inch on each side, so the area of every rectangle will be in square inches.**



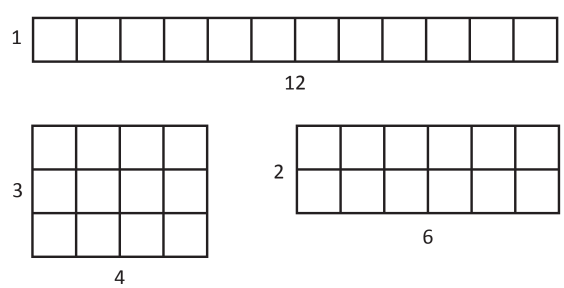


$2 \times 6 = 12$

2. *Ask:* **Would anyone like to share their rectangle with the group?** (One student shares their dimensions and the times-table fact for area.) *Ask:* **Did anyone make a rectangle with different dimensions?** (A second student shares dimensions and area.) Note for the example that 2 × 6 and 6 × 2 have the same dimensions and are the same rectangle regardless of orientation.

3. *Ask:* **Are any other rectangles possible with 12 tiles?** (The third possible rectangle is shared even if no student has constructed it.) *Say:* **We can make three different rectangles with exactly 12 square tiles: 1 × 12, 2 × 6, and 3 × 4. Each rectangle shared has an area of 12 square inches.**

4. *Say:* **Please sketch and label all three rectangles and write the associated times-table fact for each.** *Say:* **We were able to make three different rectangles because there are three ways to multiply to make 12. Each of these products represents a factor pair: 1 × 12, 2 × 6, and 3 × 4.** *Say:* **Please write all of the factors of 12 in ascending order: 1, 2, 3, 4, 6, 12.**



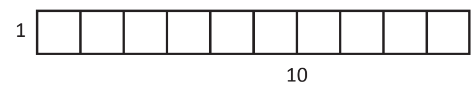
$1 \times 12 = 12$
$2 \times 6 = 12$
$3 \times 4 = 12$

5. *Say:* **Please remove one tile, so that only 11 tiles remain. Arrange the 11 tiles in a rectangle. Sketch and label your rectangle, then write the times-table fact for its area.** *Say:* **Please compare your rectangle with your neighbors'. What do you notice?** (Everyone built the same rectangle: 1 × 11.) *Ask:* **Why does everyone have the same rectangle?** (Because there is only one factor pair for eleven, 1 × 11.) *Say:* **The factors of eleven are 1 and 11.**

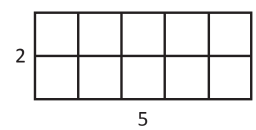
6. Define prime and composite numbers. *Say:* **A number is prime if it has exactly two factors: 1 and the number itself.** *Ask:* **Which of the two numbers we modeled is prime?** (11. The only factors of 11 are 1 and 11.) *Say:* **Please label your sketch of 11 tiles as "prime" and write the definition of prime.** *Ask:* **Can anyone name a prime number that is less than 11?** (2, 3, 5, 7) *Ask:* **What is different about 12?** (12 has six factors.)

7. *Ask:* **Would anyone like to define a composite number using 12 as an example?** (A composite number has more than 2 factors.) *Say:* **A composite number has three or more factors.** Say: **Please label your sketches of 12 as "composite" and write the definition of composite.** *Ask:* **Can anyone name a composite number that is less than 12 and list its factors?** (4: 1, 2, 4; 6: 1, 2, 3, 6; 8: 1, 2, 4, 8; 9: 1, 3, 9) *Say:* **Think about 1 and 0. How many factors does each have? Are 1 and 0 prime or composite?** (1 has exactly 1 factor, itself. Any number times 0 equals 0, but we don't consider every number a factor of 0.) *Say:* **Zero and one are neither prime nor composite by definition.**

8. Assign independent practice. *Say:* **Please select a random number of tiles and build each unique rectangle. Sketch each rectangle and label its dimensions. Next, record the multiplication fact for each rectangle and list all of the factors of the number in ascending order. Finally, classify each number you modeled with tiles as prime or composite.** *Say:* **Please repeat with a few different numbers by adding or removing just a few tiles.** Students should end up with area model sketches for both prime and composite numbers.

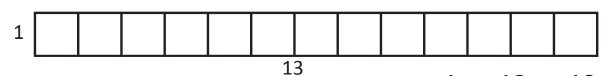


1 × 10 = 10



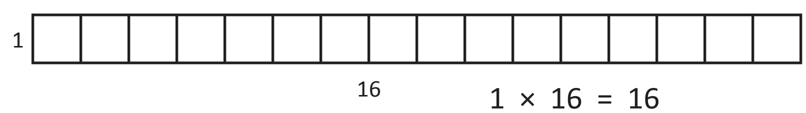
2 × 5 = 10

10 is composite: 1, 2, 5, 10

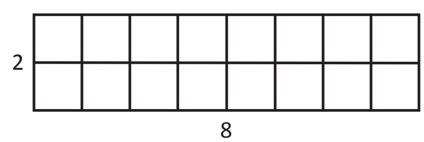


1 × 13 = 13

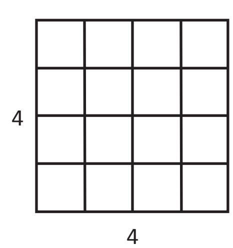
13 is prime: 1, 13



1 × 16 = 16



2 × 8 = 16



4 × 4 = 16

16 is composite: 1, 2, 4, 8, 16

Bonus Activity 2 Fractions Greater Than 1 and Mixed Numbers

Objectives

Students will:

1. Rename a fraction greater than one as a mixed number.
2. Rename a mixed number as a fraction greater than 1.
3. Transition from visual models and Prime Factor Tiles to a paper-and-pencil method.

Vocabulary

numerator
denominator
mixed number
improper fraction

Materials and Background Knowledge

- Prime Factor Tiles student sets; Factor Finder
- For teacher demonstration: fraction circles, ten-frames with counters and coins

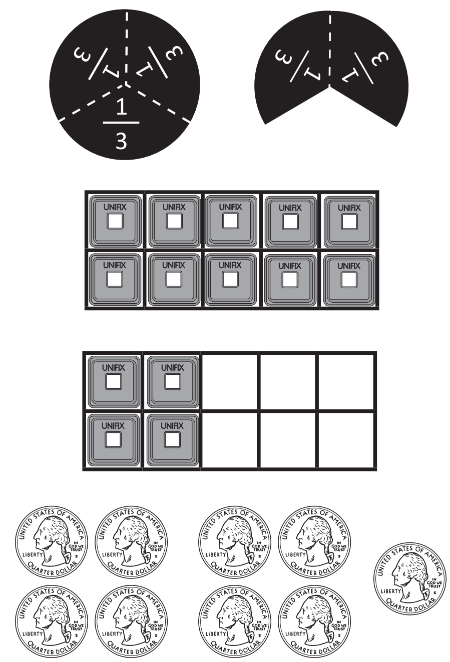
Students should understand the difference between the numerator and denominator of fractions. Students should be familiar with writing a fraction as the sum of two fractions with like denominators.

Overview

Students previously learned that a fraction is some portion of one whole. Now, students make conversions between fractions greater than 1 and mixed numbers. Later, students will replace mixed numbers with fractions for computations and replace fractions with mixed numbers for some answers.

Introduce the Lesson

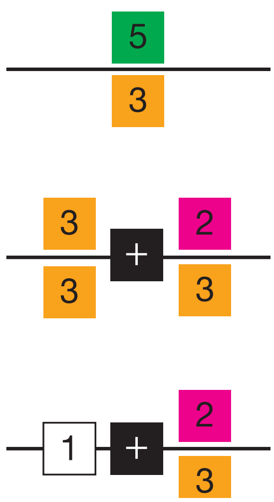
1. Demonstrate a fraction greater than 1. Display one group of five 1/3 sectors of a fraction circle, one group of 14 counters on two ten-frames, and one group of nine quarters.
2. *Ask:* **What fraction does each group represent and how they are different from what we usually think of as fractions?** (5/3, 14/10, 9/4; The number of parts is greater than the number of parts in one whole.)
3. *Say:* **A fraction greater than 1 is also called an *improper fraction*. In an improper fraction the numerator is larger than the denominator.**
4. For each representation, align the parts to form one or more wholes distinct from any remaining parts. *Ask:* **How could you write each improper fraction as a mixed number with whole and fractional parts?** (1 2/3, 2 4/5, 2 1/4)



5. *Ask:* **How do the models relate to the whole number and remainder in a long division problem?** (When a numerator greater than one is divided by the denominator, whole groups divide out as the whole number, and the remainder becomes the numerator of the fractional part.)

6. *Say:* **In the case of the coins, 2 whole groups of 4 quarters can be divided out, leaving a remainder of 1 quarter. As a mixed number, $\frac{9}{4} = 2\frac{1}{4}$.** (See graphic, previous page.)

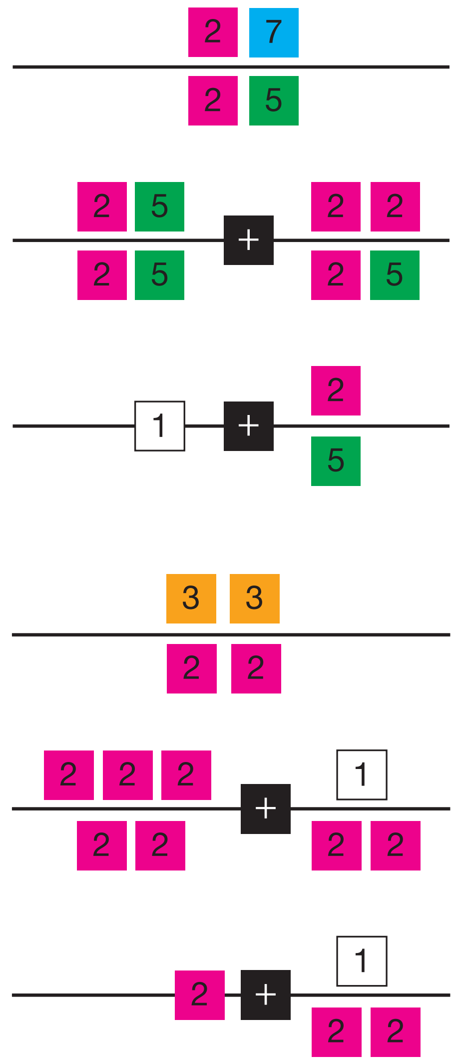
7. *Say:* **Please model 5/3 with Prime Factor Tiles (one green #5 tile above the fraction bar and one yellow #3 below).** *Say:* **Use the black addition tile to model 5/3 as the sum of two fractions. The first fraction must represent the number of whole groups of thirds in 5/3, and the second fraction will represent the remainder.** (3/3 + 2/3)



8. *Ask:* **How do we translate this sum into a mixed number?** (Replace 3/3 with the white #1 tile, and the sum is 1 + 1/3 or 1 1/3.) *Say:* **We can replace 3/3 with 1 because 3/3 = 1.**

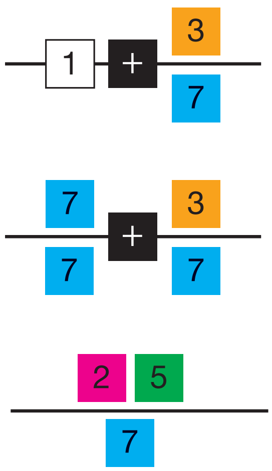
9. Assign practice converting fractions greater than 1 to mixed numbers using Prime Factor Tiles. *Say:* **Please model 14/10 as a sum of two fractions like we did in the example, then convert it to a mixed number. When you are finished, do the same with 9/4.** (See diagrams.)

   Note that with 14/10, the fractional part of the mixed number, 4/10, is reduced to 2/5 in the answer $1\frac{2}{5}$. In the second problem, 9/4, 2 is the resulting whole number part of the mixed number when 8/4 is reduced by canceling out two of the 2's in the numerator with both of the 2's in the denominator.

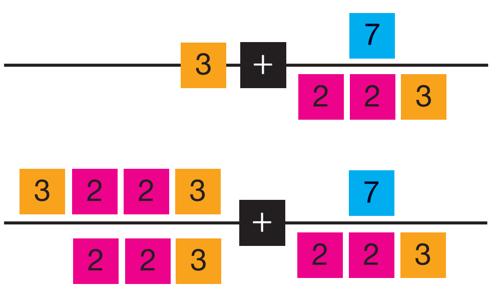


10. Have students convert mixed numbers to fractions greater than 1. Display $1\frac{3}{7}$. *Say:* **Using Prime Factor Tiles, please model the mixed number as a sum of the whole and fractional parts.**

11. *Ask:* **What fraction will need to replace the 1 to have fractions with like denominators that can be added?** (7/7 because 7/7 = 1) *Ask:* **What is $1\frac{3}{7}$ as a fraction greater than one?** (10/7 because 7/7 + 3/7 = 10/7)



12. Assign practice. *Say:* **Please use Prime Factor Tiles to convert these mixed numbers to improper fractions: $1\frac{5}{9}$, $2\frac{2}{15}$, $3\frac{7}{12}$, $2\frac{3}{10}$.** *Say:* **Each problem can be solved with Prime Factor Tiles, but the tiles may not be able to represent one or more numerators. Prime Factor Tiles are not intended to represent every number, only multiples of 2, 3, 5, and 7.** The solution to $3\frac{7}{12}$ is modeled below.



The solution $\frac{43}{12}$ cannot be modeled with Prime Factor Tiles! 43 is not a product of 2, 3, 5, or 7.

13. Teach a paper-and-pencil method. Display 25/9. *Ask:* **How many whole groups of nine parts are there in 25 parts?** (2 because 9 × 2 = 18) *Ask:* **How many parts remain when the 2 whole groups are removed?** (7 parts because 25 – 18 = 7)

14. Write out according to the diagram below as you *say:* **You can rewrite 25/9 as the sum of 18/9 + 7/9 and then simplify this to 2 7/9 because 18/9 = 2.** *Say:* **You could also use long division: 2 whole groups of 9 divide out of 25, leaving a remainder of 7. The mixed number is $2\frac{7}{9}$ because there are 2 whole groups and only 7 parts out of 9 parts needed to make one more whole.**

$2 \times 9 = 18$

$$\frac{25}{9} = \frac{18+7}{9} = 2\frac{7}{9}$$

$18 \div 9 = 2$

$$\frac{25}{9} \qquad \begin{array}{r} 2 \\ 9\overline{)25} \\ -18 \\ \hline 7 \end{array} \qquad 2\frac{7}{9}$$

2 groups of 9 in 25.
7 left over.

15. Display $3\frac{4}{5}$. *Ask:* **How many 5ths are there in 1 whole? How about in 3 wholes?** (One whole is 5/5 and three wholes is 3 × 5/5 or 15/5.) Write out according to the diagram below as you *say:* **Write a multiplication symbol between the whole number, 3, and the denominator, 5, to indicate they are multiplied together. Write the product, 15, above the 3 because 15 is the numerator of the 15/5 in three wholes. Write an addition symbol between the 15 and 4 to indicate that the numerators are summed.** *Ask:* **What is 3 4/5 as a fraction greater than one?** (19/5 because 15/5 + 4/5 = 19/5)

$$\overset{15\,+}{3}\frac{4}{\times 5} = \frac{19}{5}$$

$$\frac{3}{1} \times \frac{5}{5} = \underset{\text{whole}}{\frac{15}{5}} + \underset{\text{part}}{\frac{4}{5}} = \frac{19}{5}$$

Bonus Activity 3 **Proportional and Non-Proportional Relationships**

Bonus Activity 3 Proportional and Non-Proportional Relationships

Vocabulary

numerator
denominator
ratio
proportion

Objective

Students will be able to:

1. Determine whether relationships are proportional or non-proportional.
2. Generate equivalent ratios and solve proportions.
3. Find missing values in a ratio table.

Materials and Background Knowledge

- Prime Factor Tile student sets; Factor Finder
- For teacher demonstration: fraction circles and printed ratio table

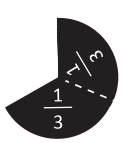
Students should be familiar with making equivalent fractions and reducing fractions to lowest terms.

Overview

Students previously ordered and compared fractions by making like denominators. Now, students determine whether relationships are proportional or non-proportional. Later, students will solve rational equations in algebra.

Introducing the Activity

1. Relate ratios to fractions. Display two-thirds using fraction circle pieces. *Ask:* **What is the name of the fraction, and what do the numerator and denominator represent?** (The fraction is 2/3. The numerator is 2 because there are 2 parts shown, and the denominator is 3 because it takes 3 parts to make one whole circle.) *Say:* **Using ratios, this same model could be interpreted in many different ways. For instance, 1:3 is the ratio of unshaded parts to total parts.** *Ask:* **What is the ratio of shaded parts to unshaded parts?** (2:1) *Ask:* **What does the ratio of 3:2 represent?** (total to shaded)



Fraction $\frac{2}{3}$ $\frac{\text{shaded parts}}{\text{parts in whole}}$

Sample ratios:

$\frac{2}{3}$ $\frac{\text{shaded}}{\text{total}}$ $\frac{1}{3}$ $\frac{\text{unshaded}}{\text{total}}$ $\frac{2}{1}$ $\frac{\text{shaded}}{\text{unshaded}}$

2. *Say:* **A fraction is a ratio that generally describes the part out of one whole.** *Ask:* **What do we call a fraction with a numerator greater than the denominator, and why do you think this is?** (It is called "improper" because it represents more than the whole instead of a portion of the whole.) *Say:* **Even though a ratio may not represent a part out of a whole, mathematically, we can compute with ratios the same way we compute with fractions.**

3. Introduce or review proportional relationships. *Ask:* **Why is it necessary to follow a recipe when baking cookies?** (You need the correct amount of each ingredient or the cookies won't come out right.) *Ask:* **What can you do if you need twice as many cookies as the recipe makes?** (Double the amount of each ingredient.) *Say:* **The ingredients in a baking recipe are always in specific ratio—you can alter the recipe to make more cookies or fewer, but the ratio of the ingredients must stay the same. If you double the amount of any one ingredient,**

you must double the amount of every other ingredient.

4. Make equivalent ratios with Prime Factor Tiles. *Say:* **Please represent 2/3 with Prime Factor Tiles and the fraction mat.** *Say:* **As a fraction we would read this as two parts out of a total of three parts. As a ratio, it represents a comparison: two parts to three parts—with five parts in all.** *Say:* **A proportion consists of equivalent ratios. In a "proportional" relationship, the ratios are equivalent; in a "non-proportional" relationship, the ratios are not equivalent.**

5. *Ask:* **How does keeping the same ratio between any two values, such as the ingredients in a recipe, relate to working with fractions?** (Making equivalent ratios is the same as making equivalent fractions!)

6. *Ask:* **How could you generate ratios equivalent to 2/3 ?** (Multiply the numerator and denominator by the same factor!) *Say:* **Please model some ratios equivalent to 2/3 with Prime Factor Tiles.** (See representative solutions.)

$$\frac{2}{3} \qquad \frac{2\ \ 2}{2\ \ 3} \qquad \frac{2\ \ 3}{3\ \ 3} \qquad \frac{2\ \ 5}{3\ \ 5}$$

7. Have students compare ratios to determine whether a relationship is proportional or non-proportional. *Say:* **Please model the relationship 10/12 and 15/18 with Prime Factor Tiles.** *Ask:* **Does this represent a proportional or non-proportional relationship? How can you demonstrate this with Prime Factor Tiles?** (Proportional; The ratios are equivalent because they reduce to the same ratio when factors common to the numerator and denominator are canceled out.)

$$\frac{\cancel{2}\ \ 5}{\cancel{2}\ \ 2\ \ 3} \qquad \frac{\cancel{3}\ \ 5}{2\ \ \cancel{3}\ \ 3}$$

$$\frac{5}{2\ \ 3} = \frac{5}{2\ \ 3}$$

8. *Say:* **To determine whether a relationship is proportional or non-proportional, simply reduce the ratios to lowest terms.**

9. Assign practice. Have students model the following relationships with Prime Factor Tiles and identify them as either proportional or non-proportional:

   12/15 and 6/10

   21/30 and 35/50

   21/36 and 28/48

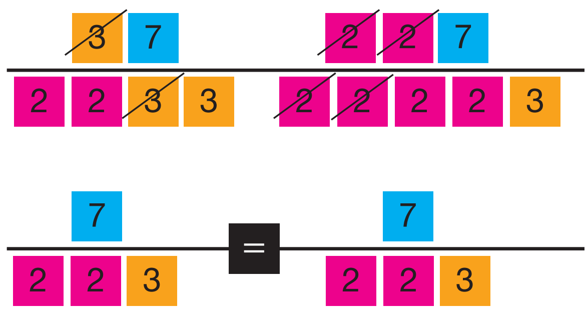
   (See diagrams below.)

$$\frac{2\ \ 2\ \ \cancel{3}}{\cancel{3}\ \ 5} \qquad \frac{\cancel{2}\ \ 3}{\cancel{2}\ \ 5}$$

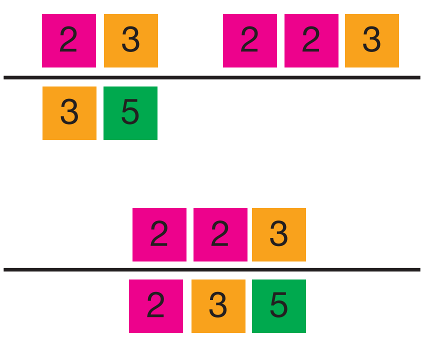
$$\frac{2\ \ 2}{5} \neq \frac{3}{5}$$

$$\frac{\cancel{3}\ \ 7}{2\ \ \cancel{3}\ \ 5} \qquad \frac{\cancel{5}\ \ 7}{2\ \ \cancel{5}\ \ 5}$$

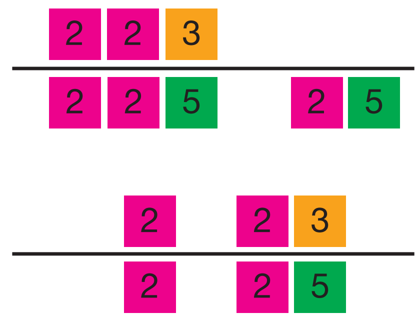
$$\frac{7}{2\ \ 5} = \frac{7}{2\ \ 5}$$



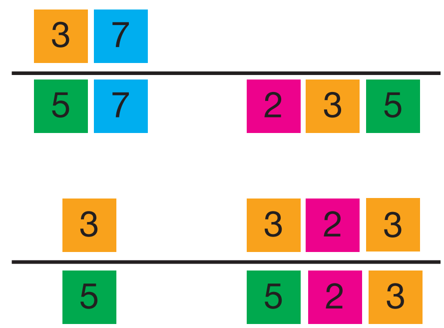
10. Have students solve proportions with Prime Factor Tiles. *Ask:* **How could you solve a proportion if one of the values in one of the ratios was missing?** (By making equivalent ratios.) *Say:* **Please model the following problem: 6/15 and 12/*n*.** (If students are not familiar with using variables, insert a "?" for "*n*" or leave the space blank.) *Ask:* **What is the missing value? How do you know?** (30; 12 is 6 × 2, so the missing value must be 15 × 2 for the ratios to be equivalent.)



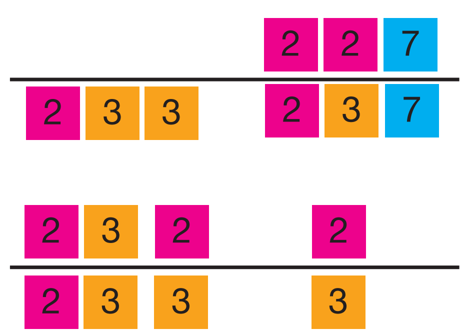
11. Assign practice. *Say:* **To solve for a missing value in a proportion, just identify the factors needed to make an equivalent ratio. It may be necessary to reduce the known ratio to lowest terms first.** *Say:* Solve the proportions with Prime Factor Tiles: 12/20 and *n*/10, 21/35 and *n*/30, *n*/18 and 28/42.



In comparison to the denominator in the first ratio, the denominator in the second ratio lacks one 2 as a factor. The numerator of the second ratio must also lack this 2 as a factor.



To begin, the first ratio is reduced to lowest terms. Next, in comparison to the denominator in the first ratio, the denominator of the second ratio has 2 and 3 as additional factors. The numerator of the second ratio must also have 2 and 3 as additional factors.



To begin, the second ratio is reduced to lowest terms. Next, in comparison to the denominator in the second ratio, the denominator of the first ratio has 2 and 3 as additional factors. The numerator of the first ratio must also have 2 and 3 as additional factors.

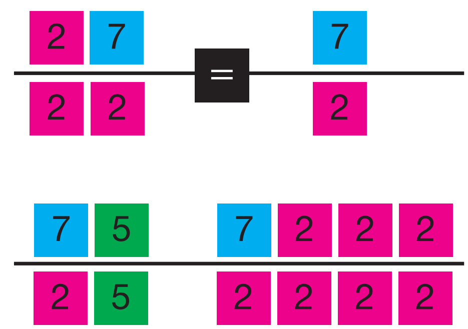
12. Introduce and have students complete ratio tables using Prime Factor Tiles. Display the simple ratio table shown below. *Say:* **Sometimes ratios are listed in tables to make it easy to find one value when you know some other value.** *Ask:* **According to the table, what is the ratio of pineapple juice to orange juice?** (2 to 3) *Ask:* **How many ounces of pineapple juice are needed for 9 ounces of orange juice?** (6 oz.) *Ask:* **How many ounces of orange juice are needed with 12 ounces of pineapple juice?** (18 oz.)

Punch Recipe						
Pineapple juice	**2**	**4**	**6**	**8**	**10**	**12**
Orange juice	**3**	**6**	**9**	**12**	**15**	**18**

13. Display the incomplete ratio table shown below. *Ask:* **How can you determine the missing values in the table using Prime Factor Tiles?** (Introduce and/or cancel out the factors necessary to make equivalent ratios.) *Ask:* **What is the simplest known ratio in the table? Can it be reduced further?** (14 to 4, or 14/4, can be reduced to 7/2.)

Work/Rest Balance						
Time working (minutes)	7	14	21		42	56
Time resting (minutes)		4	6	10	12	

14. *Say:* **Please represent the ratio 7/2 with Prime Factor Tiles.** (See diagram.) *Ask:* **How many minutes of working should accompany 10 minutes of resting? Why?** (35, because 35/10 is equivalent to 7/2) *Ask:* **How many minutes of resting should accompany 56 minutes of working? Why?** (16, because 56/16 is equivalent to 7/2)

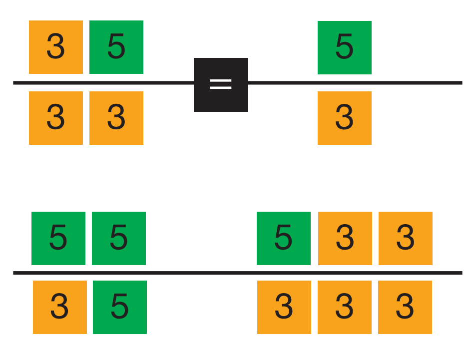


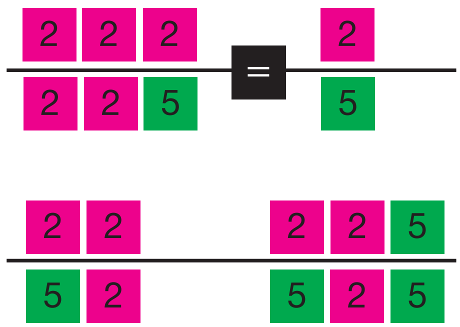
15. Assign more practice. *Say:* **Every ratio in a table can be generated by reducing any given ratio to lowest terms and then multiplying those two values by the same factor. Complete the following tables using Prime Factor Tiles to confirm that the ratios are equivalent.**

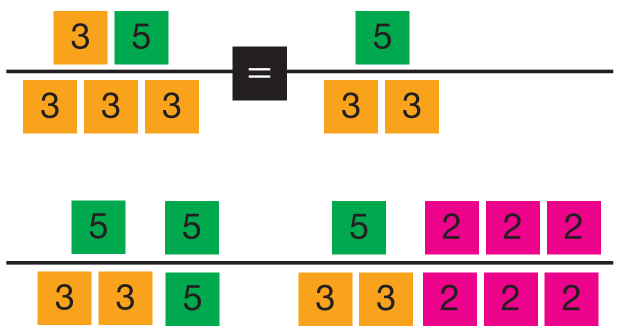
	15	25		60
3	9		27	36

2		8	20	28
	10	20		70

5	15		35	40
	27	45	63	







Activity 2 Prime Factorization of Composite Numbers

Objectives

Students will be able to:

1. Represent any composite number as a product of prime factors.
2. Model prime factorization with Prime Factor Tiles.
3. Create "factor hedges" as a superior alternative to factor trees.

Paper-and-Pencil Method

1. Introduce the "factor hedge" as a paper-and-pencil method for quickly determining and neatly recording the prime factorization of composite numbers. The factor hedge is superior to the more familiar "tree" because the prime factors are arranged in a neat horizontal row that lends itself nicely to identifying common factors between two or more numbers, reducing fractions to lowest terms, and more.

   Students may begin the factor hedge with ANY prime factor and will usually need a combination of divisibility tests and times-table facts to accomplish this. The order of the prime factors does not matter (due to the Commutative Property of Multiplication) and forcing a specific order of factors is counterproductive if the goal is speed and fluency.

2. Ask students to write down the number 63 and try to recall a times-table fact for 63. Most students should quickly respond with "7 × 9 = 63" either from memory or from the Factor Finder. Since one of the factors, 7, is prime, it can be listed after the equal sign next to the 63 (refer to the diagram below). Since the other factor, 9, is composite, it is written in smaller type above the multiplication sign following the 7. Nine is the product of two prime factors, 3 × 3, so 3 × 3 follows the 7. The prime factorization is easily confirmed as correct when the factor hedge is read from right to left: "3 × 3 = 9 and 9 × 7 = 63."

$$63 = 7 \overset{9}{\times} 3 \times 3$$

$$63 = 7 \overset{9}{\times} 3 \times 3$$

Read backwards:
3 × 3 = 9 and
9 × 7 = 63.

3. Ask the students to write the number 36 and recall a times-table fact. Most students will respond with "4 × 9 = 36" or "6 × 6 = 36." In these cases, none of the factors in either pairing is prime, so the times-table fact should be written in small type above the line where the prime factors will be recorded. The prime factorization of each composite factor from the times-table fact is then written directly below on the same line as 36 (refer to the diagram below). A student looking for the lowest prime factor may begin with 2 × 18 since 36 is even, but memorized times-table facts should be emphasized as the most efficient starting point.

$$36 = \overbrace{2 \times 2}^{4} \times \overbrace{3 \times 3}^{9}$$

$$36 = \overbrace{2 \times 3}^{6} \times \overbrace{2 \times 3}^{6}$$

$$36 = 2 \overset{18}{\times} 2 \overset{9}{\times} 3 \times 3$$

4. Ask the students to write the number 120 and recall some possible ways to multiply to get 120. Responses will vary but should include "10 × 12" (known from the 12 × 12 times table or as an obvious multiple of 10), "2 × 60" (because 120 is even and can be cut in half), or perhaps "3 × 40" or "4 × 30" (both of which derive from the times-table fact 3 × 4 = 12).

   Have students attempt to write the factor hedge for 120 whereby all prime factors are full size and on the same line as 120 and any intermediate composite factors are written in smaller type in a row above the primes (see diagram below). The factor hedges will differ according to the method chosen and the order in which times-table facts are recalled, but all of them should result in 120 = 2 × 2 × 2 × 3 × 5. (Exponential form of prime factorization is introduced in Activity 13.)

$$120 = \overbrace{2 \times 5}^{10} \overset{12}{\times} 3 \overset{4}{\times} 2 \times 2$$

$$120 = 2 \overset{60}{\times} 2 \overset{30}{\times} 3 \overset{10}{\times} 2 \times 5$$

$$120 = 3 \overset{40}{\times} \overbrace{2 \times 2}^{4} \times \overbrace{2 \times 5}^{10}$$

5. Additional practice problems: 48, 54, 90, 75, 81, 24, 150, 144, 210, 300. Representative solutions for 48 and 210 are shown here.

$$\begin{array}{l} \qquad\quad\; 6 \;\times\; 8 \\ 48 = 2 \times 3 \times 2 \overset{4}{\times} 2 \times 2 \end{array}$$

$$210 = 3 \overset{70}{\times} 7 \overset{10}{\times} 2 \times 5$$

Activity 4 Identify the GCF of Two or More Numbers

Objectives

Students will be able to:

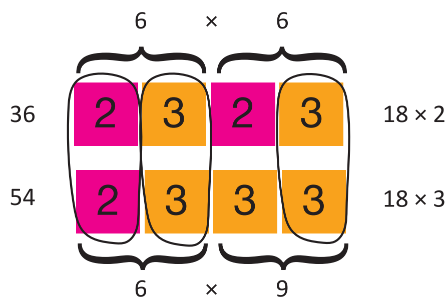
1. Use times-table facts and divisibility tests to determine factors of numbers.

2. Determine the greatest common factor (GCF) by comparing lists of factor pairs.

3. Determine the greatest common factor (GCF) by comparing prime factorization.

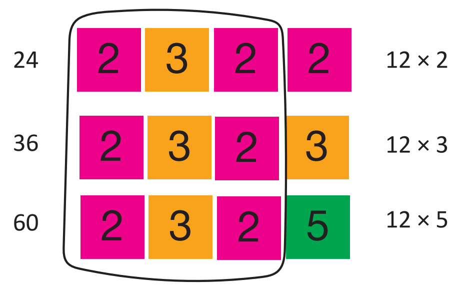
Additional Practice with Solutions

1. For additional practice, instruct the students to determine the GCF of 36 and 54 by modeling with the Prime Factor Tiles.



GCF is 18.
(2 × 3 × 3)

2. For a challenge, instruct students to determine the GCF of three numbers: 24, 36, and 60. Reinforce that the GCF of two or more numbers will always be the product of the common prime factors. Once again, show each of the numbers as the product of the GCF and any factors that were not common to the other numbers: 18 × 2 = 36 and 18 × 3 = 54, 12 × 2 = 24, 12 × 3 = 36, and 12 × 5 = 60.



GCF is 12.
(2 × 3 × 2)

Activity 6 Reduce Fractions to Lowest Terms

Objectives

Students will be able to:

1. Reduce a fraction to lowest terms by identifying and canceling out any factor(s) common to the numerator and denominator.
2. Recognize numerators and denominators as times-table facts and also as products of prime factors.
3. Identify the greatest common factor (GCF) of the numerator and denominator.
4. Transition from Prime Factor Tiles to a method using written-out times-table facts.

Paper-and-Pencil Method

1. Teach a simple paper-and-pencil method. *Say:* **Please write out 14/42 and next to it, a second fraction with the numerator and denominator written as times-table facts instead: $\frac{2 \times 7}{6 \times 7}$.** (See diagram below.) *Ask:* **What do you notice about the times-table facts?** (7 is a factor common to the numerator and denominator.) *Ask:* **What can we do with any factor common to the numerator and denominator?** (Cancel it out.) *Say:* **Cross out the 7 to leave 2/6.** *Ask:* **What do you recognize about 2 and 6 without even rewriting them?** (2 is a common factor.) *Ask:* **Using mental math, what is the fraction in lowest terms?** (1/3)

   Reduce with times-table facts:

   $$\frac{14}{42} = \frac{2 \times \cancel{7}}{6 \times \cancel{7}} = \frac{1 \times 2}{2 \times 3} = \frac{1}{3}$$

2. *Ask:* **What is the GCF of 14 and 42? How do you determine it?** (The GCF is 14 because 2 and 7 are factors common to the numerator and denominator and they multiply to equal 14.) *Say:* **To simplify a fraction, start with times-table facts or a divisibility test rather than prime factorization. Use the Factor Finder if necessary.**
3. *Say:* **Please write 15/45 followed by a fraction made up of times-table facts. Next, reduce the fraction to lowest terms, and identify the GCF of the numerator and denominator.** ($\frac{3 \times 5}{5 \times 9}$; 1/3; 15) *Ask:* **Who would like to share and explain their solution?** (The GCF is 15 because 5 and 3 canceled out and 5 × 3 = 15. 15 ÷ 15 = 1 and 45 ÷ 15 = 3.) *Say:* **If you had recognized that both numbers are multiples of 15, you could have written $\frac{1 \times 15}{3 \times 15}$ and simplified in one step**.
4. *Say:* **If you feel confident, you can cancel out common factors without rewriting the numerator and denominator as factor pairs.**

   Say: **Please write 42/56.** *Ask:* **Do you recognize a common factor?** (Both are multiples of 7.) *Ask:* **What is 42 ÷ 7?** (6) *Ask:* **What is 56 ÷ 7?** (8) *Say:* **With a red pen or colored pencil, cross out 42 and write 6, cross out 56 and write 8. We have reduced the fraction to 6/8.**

   Ask: **Can you reduce the fraction further with mental math? If so, write the fraction in lowest terms.** (See diagram.) *Ask:* **What is 42/56 in simplest form, and what is the GCF**

of the numerator and denominator? (3/4; The GCF is 14.)

Divide out ANY common factor instead of rewriting:

$$\frac{\cancel{42}^{\,6}}{\cancel{56}_{\,8}} = \frac{\cancel{6}^{\,3}}{\cancel{8}_{\,4}} = \frac{3}{4}$$

7 cancels out of 42/56 to leave 6/8, and 2 cancels out of 6/8 to leave 3/4.

5. Encourage the students to use a second color to show how the original problem has been altered as the result of performing an operation. Later they can look back on their work and follow the solution step-by-step.

Avoid the temptation to "save time but waste learning" by skipping straight to the quickest and most efficient algorithm, as it is important for students to see and practice the in-between steps that explain and justify rules or shortcuts. Students will be far less likely to confuse and misapply a rule if they first practice in a manner that promotes understanding of why the rule or shortcut makes sense.

Most efficient but least explanatory:

$$\frac{\cancel{42}\ \cancel{6}}{\cancel{56}\ \cancel{8}} = \frac{3}{4}$$

Activity 7 Identify the LCM of Two or More Numbers

Objectives

Students will be able to:

1. Identify the least common multiple (LCM) of two or more numbers using lists of multiples.

2. Identify the least common multiple (LCM) of two or more numbers by identifying common and non-common prime factors.

3. Transition to a paper-and-pencil method for finding LCM using factors.

Paper-and-Pencil Method

1. Teach a simple paper-and-pencil method. *Say:* **Please write 15 and 20 on separate lines, one below the other. To the side, rewrite each number as a product of the GCF and some other factor.** (See diagram below.)

GCF

15: 3 × 5

20: 4 × 5

GCF non-common

LCM = 5 × 3 × 4

or

15 × 4   20 × 3

60

easiest product to compute

2. *Say:* **There are three simple ways to calculate the LCM: Multiply the GCF, 5, by the non-common factors, 3 and 4; multiply 15 by 4 because 4 is the factor not common to 20; multiply 20 by 3 because 3 is the factor not common to 15. The LCM is 60: The products of 5 × 4 × 3, 15 × 4, and 20 × 3 are all 60.**

3. *Say:* **Always look for the product that is easiest to compute.**

4. Assign practice problems. *Ask:* **How can we determine the LCM of 18 and 28?** *Ask:* **What is the GCF of the two numbers?** (The GCF of 18 and 28 is 2.) *Ask:* **What is the LCM of 18 and 28 and how did you determine it?** (The LCM is 252. Possible methods: Multiply the GCF, 2, by the non-common factors, 9 and 14; or, using the factor pairs, 2 × 9 and 2 × 14, multiply 18 by the non-common factor 14 or multiply 28 by the non-common factor 9.) (See diagram.)

18: 2 × 9

28: 2 × 14

18 × 14

LCM: 2 × 9 × 14

28 × 9

28(10 – 1)

280 – 28 = 252

5. Assign additional practice with two numbers: 12 and 15 (LCM is 60), 14 and 21 (LCM is 42), 16 and 28 (LCM is 112), 18 and 24 (LCM is 72.)

6. Determine the LCM of three or more numbers using the same paper-and-pencil method. *Ask:* **How could we find the LCM of 8, 12, and 20?** (By identifying the GCF and any non-common factors) *Ask:* **As a starting point, what is the GCF of the three numbers?** (The GCF is 4.)

Say: **Rewrite each number as a product of 4 (the GCF) to find the LCM of 8, 12, and 20?** *Ask:* **What is the LCM and how did you compute it?** (The LCM is 120. Multiply the GCF by the non-common factors: 4 × (2 × 3 × 5).) Alternately, multiply any of the three numbers by the non-common factors of the other two numbers: 8 × (3 × 5), 12 × (2 × 5) or 20 × (2 × 3).) (See diagram.)

8: 4 × 2
12: 4 × 3
20: 4 × 5

8 × (3 × 5)
15

LCM: 4 × (2 × 3 × 5)
30

20 × (2 × 3)
6

12 × (2 × 5)
10

120

7. Assign practice problems. *Ask:* **How could we find the LCM of 24, 16, and 28?** (By identifying the GCF and any non-common factors) *Ask:* **As a starting point, what is the GCF of the three numbers?** (The GCF is 4.) *Ask:* **Rewrite each number as a product of 4 (the GCF) to find the LCM of 24, 16, and 28?** *Ask:* **What is the LCM and how did you compute it?** (The LCM is 672.) (See diagram.)

GCF

24: 4 × 6
16: 4 × 4
28: 4 × 7

GCF non-common

LCM = 4 × 6 × 4 × 7

or

$\underbrace{24 \times 4}_{96} \times 7$ 16 × 6 × 7 28 × 4 × 6

96 × 7

100 × 7 – 4 × 7
700 – 28

LCM = 672

8. Assign additional practice with three numbers: 6, 15, and 9 (LCM is 90), 8, 10, and 14 (LCM is 280), 12, 28. and 8 (LCM is 168).

Activity 8 Compare and Order Fractions

Objectives

Students will be able to:

1. Use factor pairs and prime factorization to determine the least common multiple (LCM) of two or more fractions.
2. Generate equivalent fractions according to the lowest common denominator (LCD) of the fractions.
3. Transition from Prime Factor Tiles to a paper-and-pencil method.
4. Understand that "cross multiplying" is a shortened form of making equivalent fractions with like denominators.

Paper-and-Pencil Method

1. Teach a paper-and-pencil method for generating equivalent fractions. Demonstrate with 3/8 and 5/12, written out according to the graphic below.
2. *Say:*

   Step 1: Rewrite each denominator as a times-table fact or product of primes to identify the common and non-common factors: 8 = 2 × 4 and 12 = 3 × 4.

   Step 2: To the side of each fraction, multiply both the numerator and the denominator by the factor(s) not common to that denominator: 3 is the factor missing from 8, and 2 is the factor missing from 12.
3. *For clarification, say:* **With Prime Factor Tiles, 8 would have 3 red #2 tiles and 12 would have only 2 red #2 tiles. Each denominator will now have identical factors and an identical product.**
4. *Say:*

   Step 3: Multiply the factors in each numerator and each denominator. Record the equivalent fractions beneath the original fractions.

   Step 4: Compare the equivalent fractions and insert the inequality symbol. *Say:* **3/8 < 5/12 because 9/24 < 10/24.**

$$\frac{3}{3} \times \frac{3}{\cancel{8}} \qquad \frac{5}{\cancel{12}} \times \frac{2}{2}$$

$$2 \times 4 \qquad 3 \times 4$$

$$\frac{9}{24} < \frac{10}{24}$$

5. *Say:* **Now it's your turn to compare 13/20 and 16/25.** *Ask:* **Where do you start?** (Rewrite the denominators as 4 × 5 and 5 × 5.) *Ask:* **What factor needs to be multiplied by each denominator to make them the same?** (5 × 20 and 4 × 25 to make both denominators 100) *Ask:* **What are the equivalent fractions that result?** (65/100 and 64/100) *Ask:* **How do the fractions compare and why?** (13/20 > 16/25 because 65/100 > 64/100)

$$\left(\frac{5}{5}\right)\frac{13}{\cancel{20}} \qquad \frac{16}{\cancel{25}}\left(\frac{4}{4}\right)$$

$$4 \times 5 \qquad 5 \times 5$$

$$\frac{65}{100} > \frac{64}{100}$$

6. Assign addition practice: 9/16 and 11/20, 7/24 and 5/16, 3/8 and 5/12.

7. Compare three fractions with the same method of making like denominators. *Say:* **Make equivalent fractions with like denominators to order 11/20, 7/12, and 8/15 from least to greatest.** *Ask:* **What times-table facts are best to substitute for the denominators and why?** (4 × 5, 3 × 4, and 3 × 5, because the common and non-common factors are easily recognized)

Ask: **What factors need to be multiplied by each denominator to make them all the same?** (3 × 20, 5 × 12, and 4 × 15, so that each denominator will be 3 × 4 × 5) *Ask:* **What are the equivalent fractions that result?** (33/60, 35/60, 32/60)

Ask: **What is the order from least to greatest and why?** (8/15, 11/20, 7/12, because 32/60 < 33/60 < 35/60)

$$\frac{3}{3} \times \frac{11}{\cancel{20}} \qquad \frac{5}{5} \times \frac{7}{\cancel{12}} \qquad \frac{4}{4} \times \frac{8}{\cancel{15}}$$

4 × 5   3 × 4   3 × 5

Missing 3   Missing 5   Missing 4

$$\frac{32}{60} < \frac{33}{60} < \frac{35}{60} \qquad \cancel{\frac{32}{60}}$$

Note: "Cross multiplication" is often taught as a shortcut to compare two fractions by simply multiplying the numerator of each fraction times the denominator of the other fraction. The result is numerators of equivalent fractions with like denominators, but without calculating the denominators! The Cross Products Property is intended for solving proportions. It should not be used to compare fractions because it creates unnecessary confusion later when multiplying and dividing fractions.

Cross-multiply vs. equivalent fractions

$$\frac{5}{9} \times \frac{7}{12} \qquad\qquad \left(\frac{12}{12}\right)\frac{5}{9} \qquad \frac{7}{12}\left(\frac{9}{9}\right)$$

12 × 5   9 × 7

60 < 63 $\frac{60}{108} < \frac{63}{108}$

No understanding   Complete understanding

Add and Subtract Fractions with Unlike Denominators

Objectives

Students will be able to:

1. Add and subtract fractions by generating equivalent fractions according to the lowest common denominator (LCD).
2. Understand and explain why fractions with unlike denominators cannot be added or subtracted.
3. Transition from Prime Factor Tiles to a paper-and-pencil method.

Paper-and-Pencil Method

1. Teach a paper-and-pencil method for adding or subtracting fractions by making equivalent fractions with identical factors in the denominators. Work out the example problem along with the students. Solicit student input for each successive step. (See diagram.)
2. *Say:* **Please write out 5/6 – 7/9. Next, cross out the denominators and replace them with times-table facts: 2 × 3 and 3 × 3.** *Ask:* **What do you notice about the factors in the two denominators?** (3 is a common factor to both denominators.) *Ask:* **What factor must each denominator be multiplied by to give identical factors in the denominators?** (3 times the 6 and 2 times the 9 for like denominators of 18)
3. To the side of each fraction, show multiplication by 3/3 and 2/2, respectively. Next, multiply and rewrite the equivalent fractions directly beneath the original fractions. *Ask:* **What is the difference of 5/6 and 7/9?** (1/18)

$$\frac{3}{3} \times \frac{5}{\cancel{6}} - \frac{7}{\cancel{9}} \times \frac{2}{2}$$

$$2 \times 3 \qquad 3 \times 3$$

$$\frac{15}{18} - \frac{14}{18} = \frac{1}{18}$$

4. Assign practice problems: 3/8 + 5/12, 5/24 + 7/16, 8/15 – 9/20. (See solutions below.)

$$\frac{3}{3} \times \frac{3}{\cancel{8}} + \frac{5}{\cancel{12}} \times \frac{2}{2}$$

$$2 \times 4 \qquad 3 \times 4$$

$$\frac{9}{24} + \frac{10}{24} = \frac{19}{24}$$

$$\frac{2}{2} \times \frac{5}{\cancel{24}} + \frac{7}{\cancel{16}} \times \frac{3}{3}$$

$$3 \times 8 \qquad 2 \times 8$$

$$\frac{10}{48} + \frac{21}{48} = \frac{31}{48}$$

$$\frac{4}{4} \times \frac{8}{\cancel{15}} - \frac{9}{\cancel{20}} \times \frac{3}{3}$$

$$3 \times 5 \qquad 4 \times 5$$

$$\frac{32}{60} - \frac{27}{60} = \frac{5}{60}$$

5. For a challenge, assign: 5/6 + 7/15 – 3/10.

$$\frac{5}{5} \times \frac{5}{\cancel{6}} + \frac{7}{\cancel{15}} \times \frac{2}{2} - \frac{3}{\cancel{10}} \times \frac{3}{3}$$

$$2 \times 3 \qquad 3 \times 5 \qquad 2 \times 5$$

$$\left(\frac{25}{30} + \frac{14}{30}\right) - \frac{9}{30}$$

$$\frac{39}{30} - \frac{9}{30} = \frac{30}{30} = 1$$

Activity 10 Multiply and Simplify Fractions

Objectives

Students will be able to:

1. Explain the multiplication of fractions with reference to a physical model.
2. Efficiently multiply fractions to yield products already reduced to lowest terms.
3. Transition from Prime Factor Tiles to a paper-and-pencil method.

Paper-and-Pencil Method

1. Reteach or review the simple, paper-and-pencil method introduced in **Activity 6: Reduce Fractions to Lowest Terms.** Work out the example problem along with the students. Solicit student input for each successive step.

2. *Say:* **Please write out $\frac{15}{28} \times \frac{14}{45}$. To the side, rewrite each fraction by replacing the numerators and denominators with times-table facts instead: $(\frac{3 \times 5}{4 \times 7}) \times (\frac{2 \times 7}{5 \times 9})$.** (See diagram.) *Ask:* **What do you notice about the factors in the times-table facts?** (5 and 7 are factors common to the numerator and denominator.) *Ask:* **What can we do with any factors common to the numerator and denominator?** (Cancel them out.)

3. *Say:* **Cross out the 5's and 7's to leave $\frac{3 \times 2}{4 \times 9}$.** *Ask:* **What do you recognize about the remaining factors without even rewriting them?** (2 is a factor of 2 and 4, 3 is a factor of 3 and 9.) *Ask:* **Using mental math, how does the fraction simplify further when you cancel factors common to the numerator and denominator?** ($\frac{1 \times 1}{2 \times 3}$). *Ask:* **What is the product in lowest terms?** (1/6)

Multiply and simplify by rewriting as factor pairs:

$$\frac{15}{28} \times \frac{14}{45} = \frac{\overset{1}{\cancel{3}} \times \cancel{5} \times \overset{1}{\cancel{2}} \times \cancel{7}}{\underset{2}{\cancel{4}} \times \cancel{7} \times \cancel{5} \times \underset{3}{\cancel{9}}} = \frac{1}{6}$$

4. Have students simplify using mental math instead of rewriting as factor pairs. *Say:* **You do not need to rewrite the fractions as products if you recognize factors common to the numerator and denominator. Cross out the composite numbers that are reduced and write in their place the factor of the pair that doesn't cancel.**

5. *Say:* **Please write the problem $\frac{21}{25} \times \frac{35}{48}$.** *Ask:* **Do you recognize any factor of the first numerator that is common to a factor of either denominator? If so, cross out the numbers and write the factors that did not cancel out. Repeat with the second numerator. What factors canceled out, and what factors remain?** (3 is common to 21 and 48, leaving 7 and 16. 5 is common to 35 and 25, leaving 7 and 5.) *Ask:* **What is the product of $\frac{21}{25} \times \frac{35}{48}$ in lowest terms?** (49/80) (See diagram.)

Multiply and simplify by canceling common factors:

$$\frac{\overset{7}{\cancel{21}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{7}{\cancel{35}}}{\underset{16}{\cancel{48}}} = \frac{49}{80}$$

6. *Say:* **When multiplying fractions, always simplify before multiplying! Canceling out factors that are common to the numerator and denominator avoids large and unfamiliar products that will only have to be reduced to lowest terms later.**

7. Assign the same practice problems students solved with Prime Factor Tiles:

   5/21 × 14/15

   2/3 × 9/10

   7/12 × 9/14

   24/25 × 35/36

   18/35 × 20/27

   30/49 × 14/45

8. Assign additional practice:

   15/28 × 14/45

   20/21 × 9/16

   24/33 × 77/80

   13/42 × 14/39

   16/45 × 27/64

   As an alternative, students can work in pairs to create and solve challenge problems of their own making.

Note: Paper-and-pencil problems are not limited to multiples of the prime factors 2, 3, 5, and 7, so all numbers are fair game! Using recall of times-table facts or divisibility tests to find factors will be quicker and more efficient than writing prime factorization.

Divide and Simplify Fractions

Objectives

Students will be able to:

1. Explain division by a fraction as multiplication by its reciprocal using simple examples.
2. Convert division problems into related multiplication problems before solving.
3. Transition from Prime Factor Tiles to a paper-and-pencil method.

Paper-and-Pencil Method

1. Teach a paper-and-pencil method for dividing and simplifying fractions by applying the principles used with Prime Factor Tiles. Work out the example problem along with the students. Solicit student input for each successive step. It is critical that written changes to the original problem are coordinated with the spoken words "multiplication" and "reciprocal." (See diagram.)
2. *Say:* **Please write out the quotient 2/9 ÷ 4/15.** *Say:* **Division is *multiplication* by the *reciprocal*,** crossing out the ÷ tile and 4/15 with a large "X" as you say "multiplication," and writing 15/4 after the "X" as you say "reciprocal."
3. *Say:* **We now have the related multiplication problem instead: 2/9 × 15/4.** *Ask:* **What factors are common to the numerator and denominator and can be canceled out?** (2 is common to 2 and 4. 3 is common to 15 and 9.) *Ask:* **What is the quotient of 2/9 and 4/15?** (4/7) (See diagram.)

"multiplication"

1st: $$\frac{2}{9} \;\cancel{\div \frac{4}{15}}\; \frac{15}{4}$$

"reciprocal"

2nd: $$\frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{4}}} = \frac{5}{6}$$

(Rewritten for clarity of steps only. Cancel and solve without rewriting is preferred.)

4. Assign the same problems used with Prime Factor Tiles for paper-and pencil practice. (See representative solution below.)

2/3 ÷ 7/6

5/7 ÷ 15/14

12/25 ÷ 9/20

3/14 ÷ 5/21

1st: $$\frac{2}{3} \;\cancel{\div \frac{7}{6}}\; \frac{6}{7}$$

reciprocal

2nd: $$\frac{2}{\underset{1}{\cancel{3}}} \times \cancel{\frac{7}{6}}\; \frac{\overset{2}{\cancel{6}}}{7} = \frac{4}{7}$$

(rewritten for clarity of steps only)

5. Assign additional practice problems from the following:

3/8 ÷ 5/12

5/24 ÷ 7/16

8/15 ÷ 9/20

27/35 ÷ 45/14

See representative solutions below.

$$\frac{3}{\underset{2}{\cancel{8}}} \div \cancel{\frac{5}{12}} \quad \frac{\overset{3}{\cancel{12}}}{5} \qquad \frac{9}{10}$$

$$\frac{5}{\underset{3}{\cancel{24}}} \div \cancel{\frac{7}{16}} \quad \frac{\overset{2}{\cancel{16}}}{7} \qquad \frac{10}{21}$$

$$\frac{8}{\underset{3}{\cancel{15}}} \div \cancel{\frac{9}{20}} \quad \frac{\overset{4}{\cancel{20}}}{9} \qquad \frac{32}{27}$$

$$\frac{\overset{3}{\cancel{27}}}{\underset{5}{\cancel{35}}} \div \cancel{\frac{45}{14}} \quad \frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{45}}} \qquad \frac{6}{25}$$