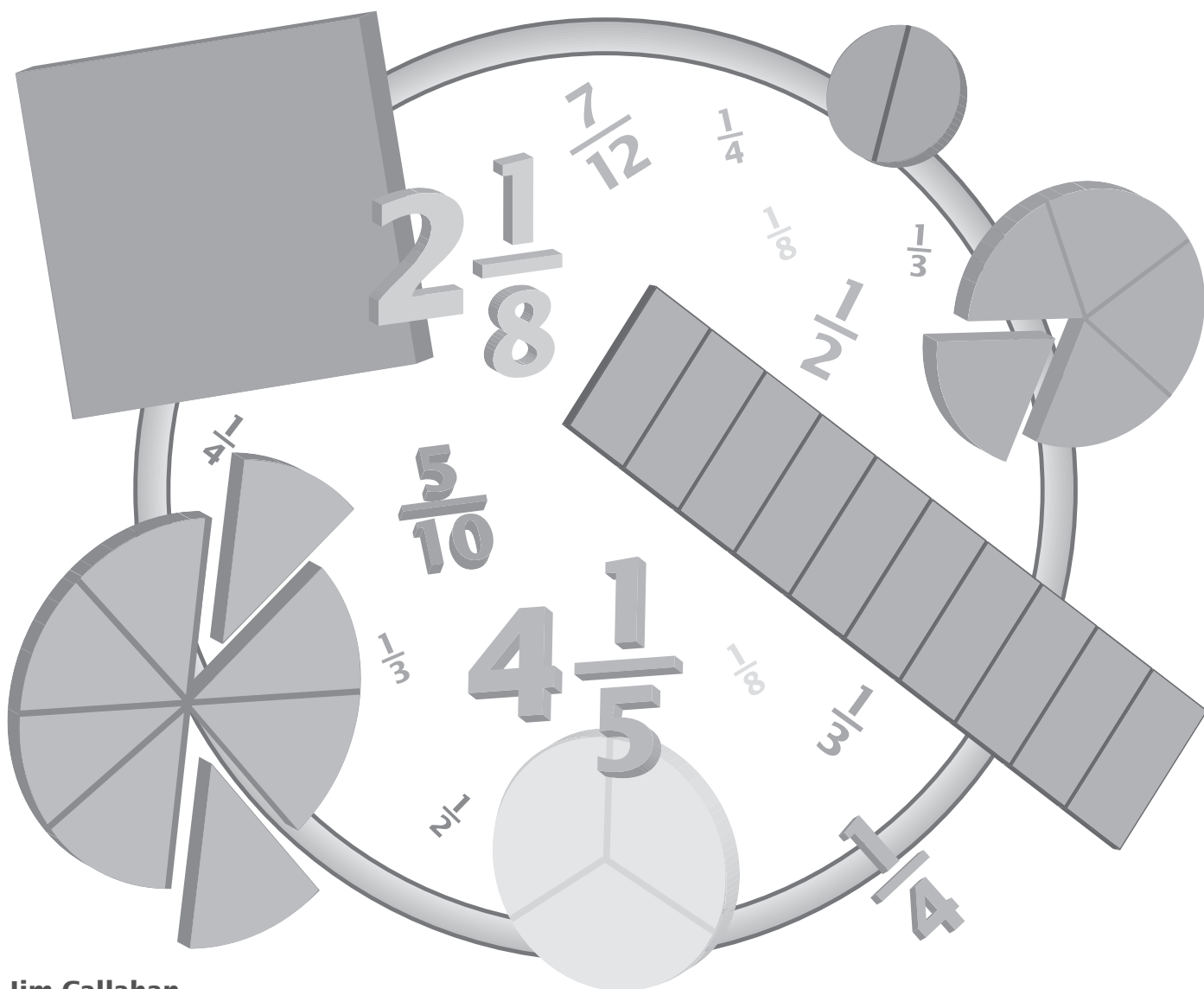


Fractions

Understanding Fractions with Visual Models

Made Easy



Jim Callahan

Standards-Based Instruction, Practice, and Assessment

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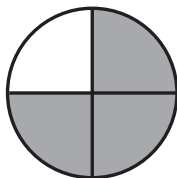
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Understanding Fractions

NAME: _____ DATE: _____

1. Look at this fraction:

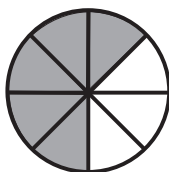


How many equal parts are there? _____

How many equal parts are shaded? _____

Circle the name of this fraction: $\frac{3}{4}$ $\frac{4}{1}$ $\frac{1}{4}$ $\frac{4}{4}$

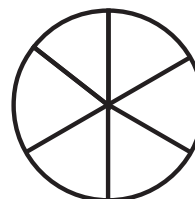
2. Look at this fraction:



Write this fraction:

Circle the name of this fraction: **three-fifths**
five-eighths
three-fourths
two-eighths

3. Shade the circle to show the fraction named. $\frac{5}{6}$

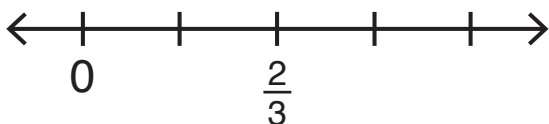




Fractions on a Number Line

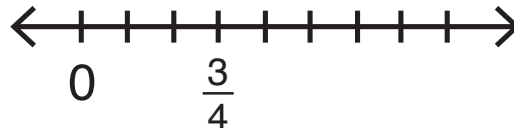
NAME: _____ DATE: _____

1.



- What is the unit fraction? _____
- Find and label one whole.
- How many unit fractions are there between 0 and 1? _____
- Label each fraction between 0 and 1.

2.



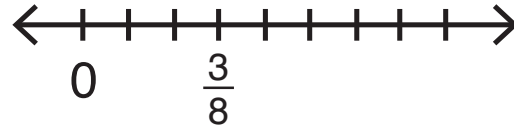
- What is the unit fraction? _____
- Find and label one whole.
- How many unit fractions are there between 0 and 1? _____
- Label each fraction between 0 and 1.

3.



- What is the unit fraction? _____
- Find and label one whole.
- How many unit fractions are there between 0 and 1? _____
- Label each fraction between 0 and 1.

4.



- What is the unit fraction? _____
- Find and label one whole.
- How many unit fractions are there between 0 and 1? _____
- Label each fraction between 0 and 1.

Section 1:

Discovering Equivalent Fractions with Visual Fraction Models

Mathematical Standard (CCSS) Addressed

4.NF.1: Extend understanding of fraction equivalence and ordering.

Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

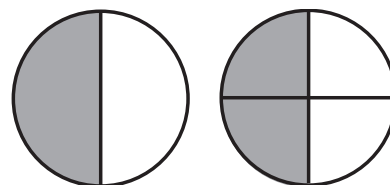
Unpacking the Standard

The first part of this standard appears to be out of sequence, as it suggests using the Identity Property of Multiplication— $(n \times a)/(n \times b)$ —for discovering equivalent fractions. In this book, we will focus exclusively on the use of visual fraction models to discover and record equivalent fractions. Since fraction multiplication is not covered by the standards until fifth grade, we will save the use of the Identity Property of Multiplication to generate equivalent fractions for the forthcoming fifth-grade book.



Equal or Equivalent?

When we describe fractions, we typically use the term *equivalent*. In area models such as fraction circles and fraction rectangles, the pieces we are comparing are technically not “equal” (identical), but “equivalent.” They have the same value for one of their attributes (in this case the same area).



On the number line, however, we are looking at line segments with the same origin and endpoint. The fractions $1/2$ and $2/4$, for example, both begin at 0 and end at the same (identical) point. Because they end at the exact same point on the number line, these two fractions are identical (except for their names). Therefore, we can truthfully say they are “equal.”



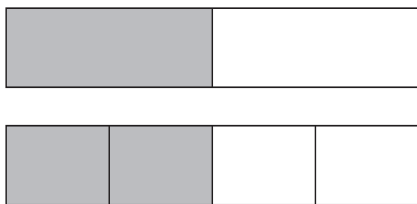
Section 1: Fraction Equivalence (cont.)

The distinction between “equal” and “equivalent” is abstract and confusing for most of us. To simplify these ideas for students in fourth grade, the two terms “equal” and “equivalent” will be considered synonyms and used interchangeably in this book. At a later point in mathematical development, it may become important to distinguish between equal and equivalent.

“Same Size”

The idea of “same size” using visual models will vary depending on the model.

On a two-dimensional rectangular model, “same size” means same distance and same area. These fraction models can be shown as “same size” when they are aligned one above the other.



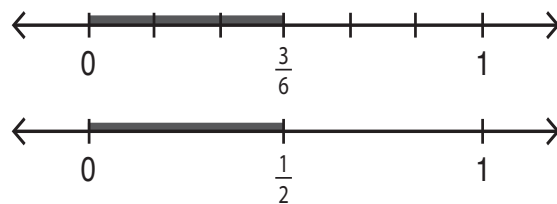
With area models such as fractions circles, “same size” means “same area.” These fraction pieces can be shown to be the “same size” by physically placing one on top of the other and comparing. Any difference in their areas is immediately visible.

For instance, $\frac{2}{4}$ can be placed on top of $\frac{1}{2}$ with no part of the half showing. As further proof, the half can be placed on top of the $\frac{2}{4}$ with no part of the fourths showing. These fractions have the same area, thus they are equivalent.

If $\frac{2}{6}$ is placed on top of $\frac{3}{8}$, part of the $\frac{3}{8}$ remains visible, thus these are not equivalent. Differences and similarities are easily experienced this way.



On a number line model, “same size” means the same distance or interval from the origin (0) to the endpoint (fraction). When shown on the same number line, these are located at the same place.



As students learn to recognize equivalent fractions using these visual models, they will show their understanding by generating new examples and explaining why two fractions might or might not be equivalent.

The Equivalent Fractions Truth Table

The discovery of equivalent fractions, both as a concept and as a skill, is an essential part of learning about fractions. As students work through the activities in this section, they will record the equivalent fractions they discover in the Equivalent Fractions Truth Table on pages 14–15. The tables have been created so that students can quickly locate this information as they tackle the other fourth-grade fraction standards.

Materials for Teaching Fraction Equivalence

The fraction rectangle blackline masters on pages 84–92 are designed for use with the activities covering standards 4.NF.1 and 4.NF.2. As students move into adding and subtracting fractions and exploring mixed numbers (4.NF.3 and 4.NF.4), it will be necessary to replace these two-dimensional materials with commercially available three-dimensional fraction models with discrete (movable) parts.



Discovering Equivalent Fractions with Visual Fraction Models

Standard Addressed

Extend understanding of fraction equivalence and ordering: Explain why a fraction a/b is equivalent to a fraction ... by using visual fraction models ... (4.NF.1)

Learning Outcomes

- Students will recognize fractions with equivalent areas using visual fraction materials with denominators 1, 2, 3, 4, 5, 6, 8, 10, and 12.
- Students will record all equivalent fractions with the denominators listed above in the Equivalent Fractions Truth Table.

Materials

- *Fractions Made Easy, Grade 4*, pages 10–13
- Equivalent Fractions Truth Tables, pages 14–15
- Fraction Rectangles BLM, pages 84–92
- Optional: Commercially available fraction materials with movable parts

Suggested Teaching Sequence

1. Hand out Guided Practice page 10, which shows six pairs of fraction expressions.
2. Ask students to look at the two fraction expressions in the first example and then locate the visual materials that represent these two fractions.
3. Once found, have students compare $1/3$ to $2/6$ to see if the two fractions have the same area.

4. If the two fractions have the same area, have students put an equal sign between the two fractions, creating an equation.
5. Tell students that the fractions $1/3$ and $2/6$ are called “equivalent” fractions and that they will be recording these two fractions in the Equivalent Fractions Truth Table in a little while.

Equivalent Fractions Truth Table	
$\frac{1}{2} = \frac{2}{4}$	$\frac{1}{2} = \frac{3}{6}$

6. Say: *Let's do the next one together.*
7. When you have completed the second example with the students, say: *I would like you to try the third one on your own. When you're done, let's talk about what you discovered.*





Discovering Equivalent Fractions - 1

NAME: _____ DATE: _____

Use your fraction materials. For each example, compare the two fractions to see if they have the same area.

If the two fractions have the same area, put an equal sign (=) between them. If the two fractions do not have the same area, use the greater than (>) or less than (<) symbol to make the expression true.

$$\frac{1}{3} \square \frac{2}{6}$$

$$\frac{2}{3} \square \frac{4}{6}$$

$$\frac{1}{2} \square \frac{4}{6}$$

$$\frac{1}{6} \square \frac{2}{12}$$

$$\frac{3}{4} \square \frac{8}{12}$$

$$\frac{1}{5} \square \frac{2}{10}$$



Discovering Equivalent Fractions - 2

NAME: _____ DATE: _____

Use your fraction materials. Decide which symbol goes in the box to make each statement true. Record your findings in the Equivalent Fractions Truth Table.

 $=, >, <$

$\frac{1}{2} \square \frac{2}{4}$	$\frac{1}{4} \square \frac{3}{8}$	$\frac{1}{3} \square \frac{3}{6}$
$\frac{3}{4} \square \frac{7}{8}$	$\frac{3}{3} \square \frac{4}{6}$	$\frac{2}{2} \square \frac{10}{10}$
$\frac{5}{6} \square \frac{2}{4}$	$\frac{2}{12} \square \frac{1}{6}$	$\frac{2}{3} \square \frac{3}{6}$
$\frac{1}{8} \square \frac{1}{12}$	$\frac{2}{3} \square \frac{4}{6}$	$\frac{3}{6} \square \frac{2}{5}$
$\frac{8}{10} \square \frac{4}{5}$	$\frac{4}{4} \square \frac{6}{8}$	$\frac{3}{8} \square \frac{8}{12}$
$\frac{1}{5} \square \frac{2}{10}$	$\frac{4}{6} \square \frac{8}{12}$	$\frac{3}{8} \square \frac{11}{12}$



Equivalent Fractions Truth Table - 1

NAME: _____ DATE: _____

Record the equivalent fractions you discovered here. Some are recorded for you.

Fractions equivalent to 1 (whole)

$$\frac{1}{1} = \frac{2}{2} = \text{---} = \frac{4}{4} = \text{---} = \frac{6}{6} = \text{---} = \text{---} = \frac{12}{12}$$

Fractions equivalent to $\frac{1}{2}$

$$\frac{1}{2} = \frac{2}{4} = \text{---} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$$

Fractions equivalent to $\frac{1}{3}$

$$\frac{1}{3} = \frac{2}{6} = \text{---}$$

Fractions equivalent to $\frac{2}{3}$

$$\frac{2}{3} = \frac{4}{6} = \text{---}$$

Fractions equivalent to $\frac{1}{4}$

$$\frac{1}{4} = \frac{2}{8} = \text{---}$$

Fractions equivalent to $\frac{3}{4}$

$$\frac{3}{4} = \text{---} = \text{---}$$



Equivalent Fractions Truth Table - 2

NAME: _____ DATE: _____

Record the equivalent fractions you discovered here.

Fractions equivalent to $\frac{1}{5}$

$$\frac{1}{5} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{2}{5}$

$$\frac{2}{5} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{3}{5}$

$$\frac{3}{5} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{4}{5}$

$$\frac{4}{5} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{1}{6}$

$$\frac{1}{6} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{2}{6}$

$$\frac{2}{6} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{4}{6}$

$$\frac{4}{6} = \underline{\hspace{2cm}}$$

Fractions equivalent to $\frac{5}{6}$

$$\frac{5}{6} = \underline{\hspace{2cm}}$$



Discovering Equivalent Fractions

NAME: _____ DATE: _____

Use your fraction materials to prove whether or not the two fractions are equivalent.

Are they equivalent?

$$\frac{1}{3} \text{ and } \frac{2}{12}$$

YES NO

$$\frac{1}{2} \text{ and } \frac{5}{10}$$

YES NO

$$\frac{2}{4} \text{ and } \frac{7}{12}$$

YES NO

$$\frac{5}{5} \text{ and } \frac{8}{8}$$

YES NO

$$\frac{2}{10} \text{ and } \frac{3}{12}$$

YES NO

$$\frac{3}{4} \text{ and } \frac{8}{12}$$

YES NO

$$\frac{3}{4} \text{ and } \frac{9}{12}$$

YES NO

$$\frac{2}{5} \text{ and } \frac{4}{12}$$

YES NO

$$\frac{3}{8} \text{ and } \frac{1}{4}$$

YES NO

$$\frac{5}{8} \text{ and } \frac{5}{12}$$

YES NO

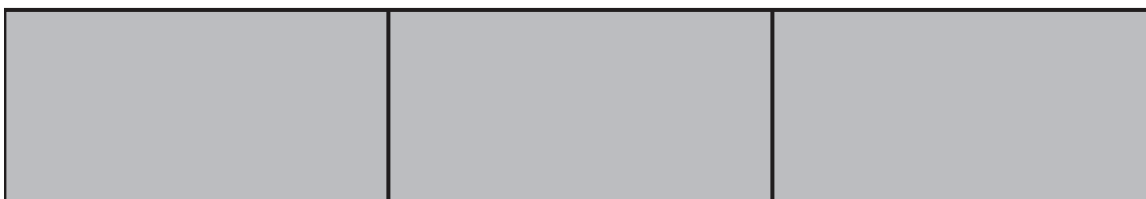
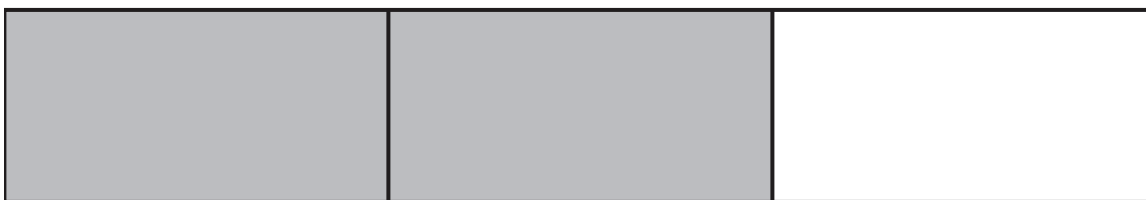
$$\frac{1}{2} \text{ and } \frac{5}{10}$$

YES NO

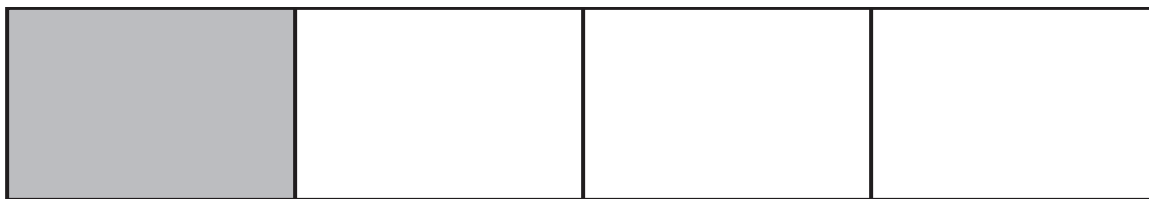
$$\frac{12}{12} \text{ and } \frac{10}{10}$$

YES NO

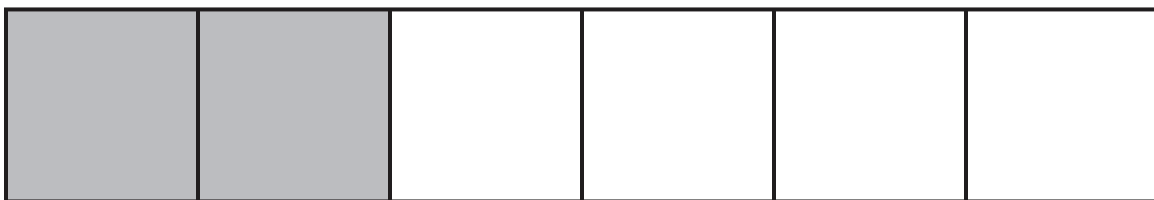
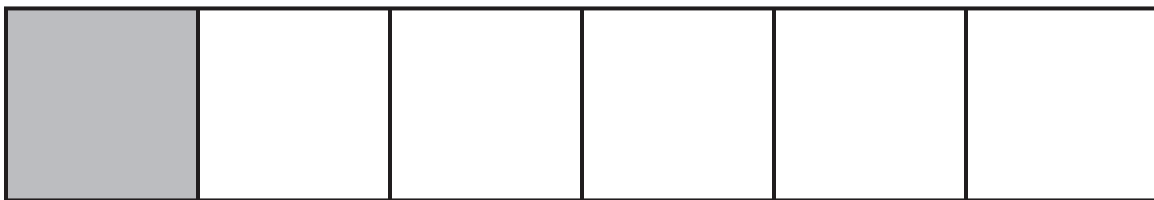
Fraction Rectangles (halves, 3rds)



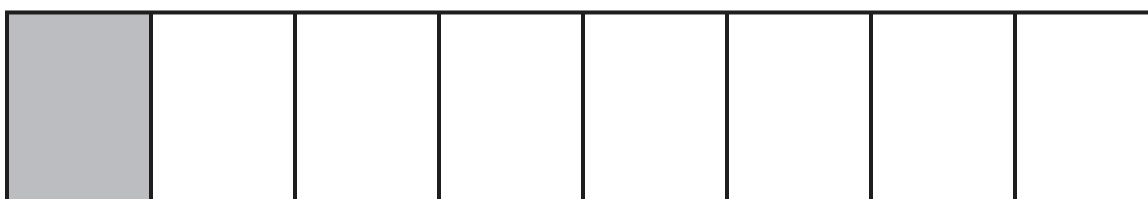
Fraction Rectangles (4ths, 5ths)



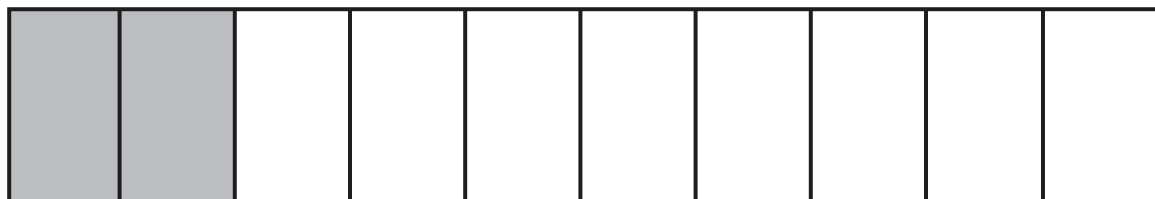
Fraction Rectangles (5ths, 6ths)



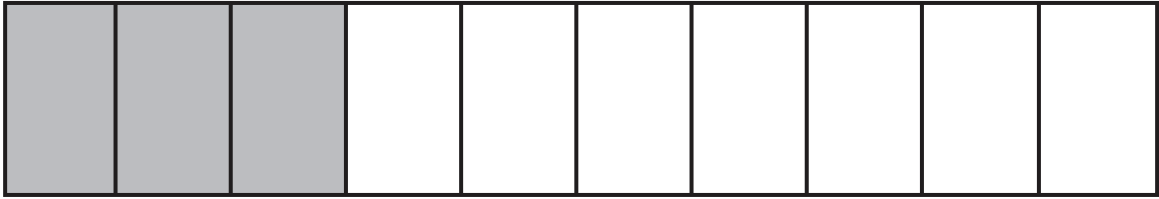
Fraction Rectangles (6ths, 8ths)



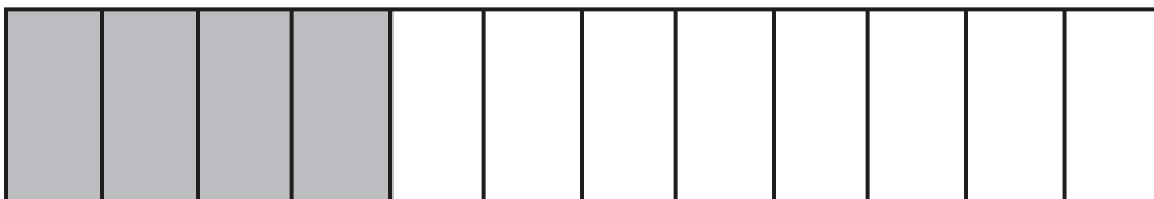
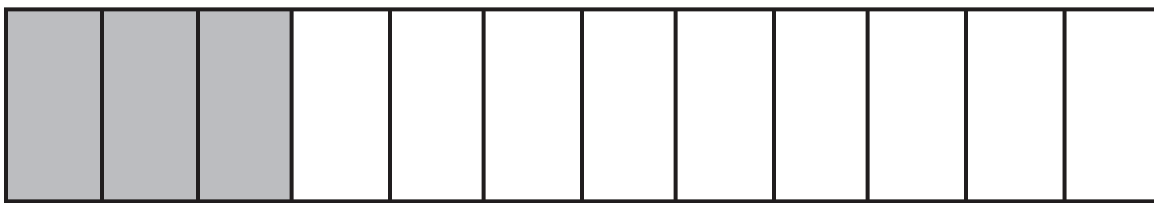
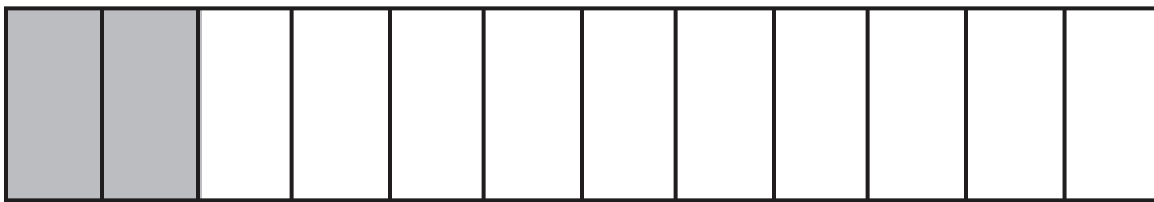
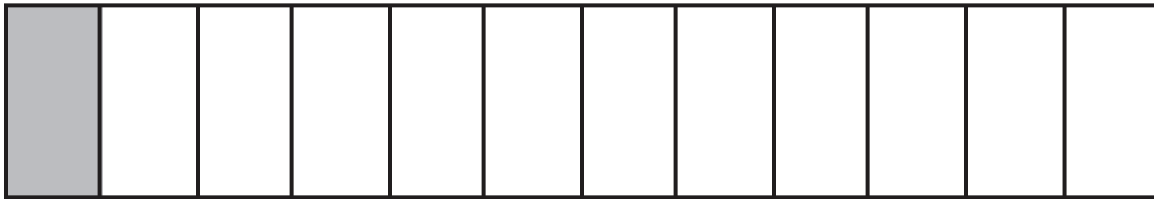
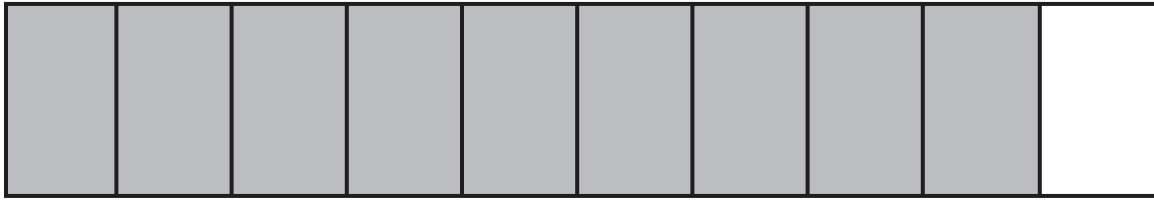
Fraction Rectangles (8ths, 10ths)



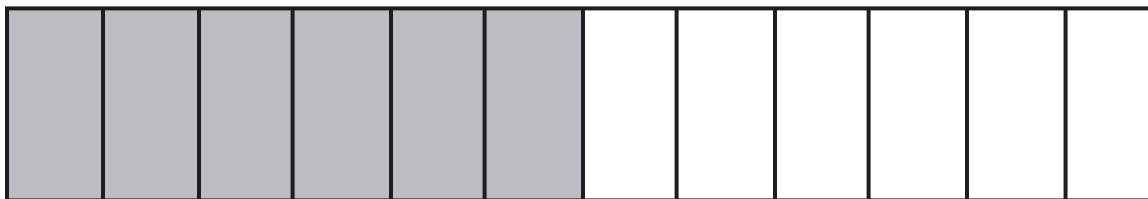
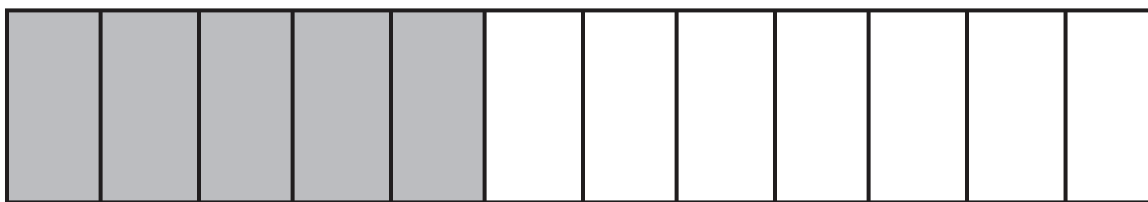
Fraction Rectangles (10ths)



Fraction Rectangles (10ths, 12ths)



Fraction Rectangles (12ths)



Fraction Rectangles (12ths)

