

the Algebra Game

Quadratic Equations



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Introduction

Getting Started with *The Algebra Game: Quadratic Equations*

Congratulations on your purchase of *The Algebra Game: Quadratic Equations*! By matching cards for quadratic equations to cards for corresponding graphs, roots, factors, and coordinate pairs tables, as well as vertex, axis of symmetry, discriminant, and quadratic formulas, your middle school and high school students will be making sense of quadratic equations. The instructional result is that students learn about mathematics of quadratic equations while actively building new knowledge from experience.

The *Quadratic Equations* set has four basic decks plus a 72-page teacher manual describing how to use the decks, including instructions for over 20 activities and games. For some activities and games, a set of four Advanced decks may be purchased separately. Decks are organized to match students' skills level, starting with beginners and progressing to more advanced learners. For example, students can study how points on a parabola graph are represented by coordinate pairs in a table and how both are related to the matching quadratic equation.

The Algebra Game: Quadratic Equations Supports Different Learning Styles

The flexibility of the card decks in *The Algebra Game: Quadratic Equations* allows teachers to enhance the various learning styles and strengths that students bring to the classroom.

- Students are engaged visually, kinesthetically, and logically as they match, sort, and classify cards in activities and games that focus on the relationships among the characteristics of quadratic equations and their graphs.
- Organizational grids provide the opportunity for students to see how all of the cards fit together.
- Discussion Questions and Conjecture worksheets allow students to collaborate and describe the ideas shared in groups, as well as intuit new algebraic relationships not previously noticed.

- Space is provided on all worksheets for students to write about their observations during card activities and games, either independently or as a group.

In addition, teachers can also use the optional Write-Your-Own Activity templates to tailor lesson activities to their individual classroom needs.

The Algebra Game: Quadratic Equations and Math Standards

The Algebra Game: Quadratic Equations supports the Math Standards, focusing on Grade 8 and High School Content Standards involving quadratic equations and related concepts. Each Activity is correlated to one or more Math Content Standards, which are identified at the top of each Activity sheet. Correlation tables for Math Content Standards also appear at the front of the book.

The Algebra Game: Quadratic Equations also supports the Math Process Standards, which are represented as the eight Mathematical Practices listed below. Each Process Standard is aligned with one or more Activities, which are identified in the Process Standards correlation table at the front of the book.

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

(Common Core State Standards for Mathematics, pp. 6–8, © Copyright 2010 National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.)

Working with *The Algebra Game: Quadratic Equations*

How to Use the Decks

Deck Organization *The Algebra Game: Quadratic Equations Basic* contains four decks: A, B, C, and D. Each deck contains 12 matched sets of six types of cards: Equation, Graph, Coordinate Pairs, Roots, Factors, and Standard Form. For example, the set of six matching cards for the quadratic equation $y = x^2$ are shown below. The **Repair Cards** Extra Masters at the back of the book can be copied, cut out, and filled in to replace any misplaced cards or to create a custom set of matching cards.

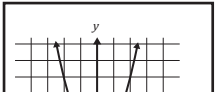
The Algebra Game: Quadratic Equations Advanced also contains 4 decks: E, F, G, and H. Each deck contains 12 matching sets of four types of cards: Vertex, Axis of Symmetry, Quadratic Formula, and Discriminant. (The advanced set can be purchased separately at didax.com/the-algebra-game.) This set

is required to complete the Activities on pages 26 through 39. The **Repair Cards** blackline masters at the back of the book can be copied, cut out, and filled in to replace any misplaced cards or to create a custom set of matching cards.

The four basic decks are organized by level of difficulty:

- Deck A has only integer roots.
- Deck B includes some irrational roots.
- Deck C includes some complex roots.
- Deck D includes x^2 -coefficients not equal to 1.

The table below specifies the number of matching sets in each basic and advanced deck based on characteristics of each equation and its related graph.

$y = x^2$ <div>equation</div> $ax^2 = c$		coordinate pairs <table><tr><th>x</th><th>y</th></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>1</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr></table>	x	y	-2	4	-1	1	0	0	1	1	2	4	$x = 0$ <div>roots</div> $0 = x$	$(x)(x)$ <div>factors</div> $(x)(x)$	$x^2 - y = 0$ <div>standard form</div> $0 = a - cx$
x	y																
-2	4																
-1	1																
0	0																
1	1																
2	4																
equation	graph	coordinate pairs	roots	factors	standard form												

The set of matching cards for $y = x^2$, from left to right, includes the Equation, Graph, Coordinate Pairs, Roots, Factors, and Standard Form cards.

Quadratic Equations Card Deck Specifications (Number of Matching Sets per Deck)								
Feature	Deck A	Deck B	Deck C	Deck D	Deck E	Deck F	Deck G	Deck H
Symmetrical to y-axis	4	5	1	8	4	5	1	8
Non-function	2	2	0	0	2	2	0	0
Vertex at origin	2	1	0	6	2	1	0	6
Opens down	5	5	4	0	5	5	4	0
Opens up	5	5	8	12	5	5	8	12
Difference-between-two-squares equation or factor (functions only)	2	5	1	2	2	5	1	2

Quadratic Equations Card Deck Specifications
(Number of Matching Sets per Deck)

Feature	Deck A	Deck B	Deck C	Deck D	Deck E	Deck F	Deck G	Deck H
Binomial-square equation or factor (functions only)	3	5	0	2	3	5	0	2
One integer root (functions only)	5	5	0	6	5	5	0	6
Two integer roots (functions only)	5	2	9	0	5	2	9	0
Irrational roots (at least one)	0	3	0	1	0	3	0	1
Fractional roots (at least one)	0	0	0	5	0	0	0	5
Complex number roots	0	0	3	0	0	0	3	0
Coefficient of x^2 equal to 1 or -1	10	10	10	0	10	10	10	0
Coefficient of x^2 less than 1, greater than -1 , and not equal to 0	0	0	0	3	0	0	0	3
Coefficient of x^2 greater than 1 or less than -1	0	0	2	9	0	0	2	9
Discriminant = 0 (functions only)	5	5	0	8	5	5	0	8
Discriminant < 0 (functions only)	0	0	3	0	0	0	3	0
Discriminant > 0 (functions only)	5	5	9	4	5	5	9	4

Arranging the Students into Groups By solving problems and discussing solutions within groups, students reinforce their understanding of the subject matter, resulting in greater retention of material for assessments.

When arranging groups for *The Algebra Game: Quadratic Equations*, you can introduce the cards before the Activities and Games start. For example, if four students are in each group, you can arrange groups by using four matching cards such as Equation, Graph, Coordinate Pairs, and Roots. Select as many matching sets as needed and distribute the cards to all students in the class. Then, one by one, students with Equation cards can announce their equations to the class or write them on the board. Students with corresponding Graph, Coordinate Pairs, and Roots cards can then self-identify, with students with matching cards being seated together.

Similar strategies can be followed using two matching cards for two partners or use three or more matching cards for making larger groups. You may want to use matching cards that

represent an earlier lesson so that students can have a quick review of what happened earlier or absent students can have a chance to catch up.

Determining the Number of Decks and Levels

Boxed sets of *The Algebra Game: Quadratic Equations* include four decks of 72 cards each, for a total of 288 cards. In addition, some Activities and Games require the use of an advanced set of cards, which includes four decks of 48 cards each. Each of the four advanced decks matches one of the four basic decks, expanding the 12 matching sets per deck from 6 to 10 cards each.

The number of decks needed depends on how many decks are used and how they are divided between groups of students.

- Using four decks with the Activities: Separate each deck into two sets so that you have 8 packages for groups of four students each (32 students in class).
- Using three decks: Separate the decks into 6 sets (two from each deck level) resulting in enough cards for 5 students in each group.

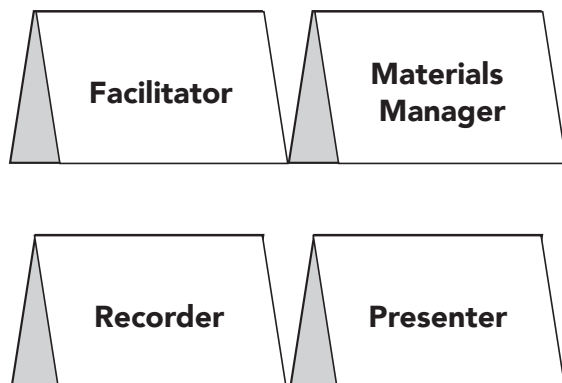
The recommended decks are listed for each Lesson, as well as every Activity and Game. In order for each group to have the exact same cards, you may purchase additional boxed sets as well as advanced sets for the student groups in your classroom by going to didax.com/the-algebra-game.

Before Class Preparation

1. Duplicate Activity Instructions and Worksheets Activity sheets, Discussion Questions, and some worksheets are located in the lesson itself, while generic worksheets, called “Extra Masters,” are located in the back of the book. Make enough copies so that there is one Activity sheet and one Discussion Questions sheet per group for each activity. Requirements for additional worksheets and materials are listed at the beginning of each lesson.

2. Assign Student Roles Arrange the students into groups and assign roles. The Activity instructions describe tasks for each role in the group. For groups with more than four members, additional cards can be created and include Time Keeper, Observer, and/or Encourager.

Make copies of the Role cards in the back of the book and give each student a corresponding card to display in front of themselves. Students should keep track of the roles that they have been assigned so as not to repeat the same role in a given session.



3. Managing the Groups The following management ideas are suggestions by teachers for using the cards with groups of students.

- Designate group member roles.
- Assemble the materials for each group (decks, Activity directions, and so on) in baskets or other similar-size containers. Distribute the baskets, one per group, after general instructions have been presented to the class. Color-code the containers for easy identification.
- Work within specific time limits. Use a kitchen timer or a clock. If increased time is needed for any given task, use 5-minute increments.
- If anyone in the group has a question, encourage the member to ask all group members first before the group asks the teacher or outside resource.
- Predetermine signals for students to stop talking and refocus back to the teacher—for example, dim the lights, clap your hands three times, or raise your hand and students raise theirs.

4. Collect Any Extra Materials You May Need

Every lesson has a list of materials needed for the specific activities plus some optional items. Some collaborative activities or games may require extra sheets for recording information or keeping score. The Extra Masters at the end of this book include many additional sheets for collaborative activities.

To organize and display materials, a pocket chart can be useful. Pocket charts can be found in most educational catalogs. The pockets should be at least 3 inches wide and allow for at least six matching cards or more in a display.

Many teachers have incorporated graphing calculators and computer software when using these decks. Some teachers provide one graphing calculator for each group, and other teachers allow access to several graphing calculators or computers for the whole class to use as needed. Because the decks provide the opportunity for students to make initial decisions about the

graph and algebraic relationships, using simple text editing or slide show software can be helpful for extending any conjectures or exploring new assumptions in mathematics.

Lesson QuickStarter

The steps below provide an outline of how a lesson can be organized. Start the class with an activity Launch in which students match cards to make groups with 4 or 5 members each.

Launch the Activity Give each student one card. Tell the students to locate the other members of their team for today by finding the matching cards. For start-up activities, use cards that match the current day's lesson. For reinforcement, use cards that represent skills learned in an earlier lesson. Some suggested cards for a launch, based on selected lesson topics, are listed in the chart on the next page.

Suggested Cards for the Launch
Factors and Equations: Use Factors and Equation cards from Decks A, B, and C.
Roots and Factors or Roots, Factors, Coordinate Pairs, and Graphs: Use just Roots and Factor cards selected from Decks A and B. If students are familiar with complex numbers, then include Deck C. Or, use Roots, Factors, and Coordinate Pairs cards, including the Graph card for a fourth student in the group.
Quadratic Formula, Roots, and Equation*: Use Quadratic Formula Advanced cards selected from E, F, G, and H decks and Roots cards from the matching basic deck. Or, use Equation cards selected from Quadratic Equations Basic deck and the matching Quadratic Formula cards selected from the advanced deck. You may also match Quadratic Formula cards to Vertex and Axis of Symmetry cards.
<small>*To purchase the Quadratic Equations Advanced cards, please visit didax.com/the-algebra-game.</small>

As an alternative to a Launch with cards, each lesson includes various worksheets that you

can copy and distribute at the beginning of the lesson, allowing students to write their observations as an initial start-up or warm-up class activity. Students can then bring these sheets with their own observations to the group for discussion.

If you have arranged the materials in baskets or other containers before class for each group, it is appropriate at this time for the Materials Manager in each group to go get the container for their group. You also have the option of distributing the baskets yourself or allowing one student to distribute all of the baskets to the different groups.

Complete the Activity Once the groups have been formed and roles have been assigned, students complete the activity by following the instructions on the Activity sheet. Each student takes on their assigned role for that Activity and performs their assigned tasks. Note that if students are working in pairs, you will need to make some minor adjustments to the assigned student roles.

You may notice that all of the Activities start with students dividing the decks of cards and sorting them into stacks by type. One reason for this practice is so that students can have an overview of which cards they will be working with on any given activity. Also, sorting by the type of card allows students to practice their linear equation vocabulary.

Set Time Limits Many or most of the Activities, along with their discussion worksheets, can be completed in 15 to 30 minutes. Designate a specific time frame for students to work within so that there is enough time for students to report their findings to the rest of the class. Time intervals can vary depending on the difficulty of the Activity. For the easier tasks, allow 15 minutes for task and discussion worksheet completion.

Review Discussion Worksheets Always allow students to discuss the group's responses to the Discussion Questions in a public classroom forum either during the same day or on the following

day. All groups should have the opportunity to present their findings and to ask their questions. You may want to appoint a general Classroom Recorder to record all of the groups' findings or collect the group s before the students leave.

Extension Activities As an extension of each lesson topic, students may complete an Extension Activity using the Activity sheet on page 44 and one of the Organizational Grid Descriptions on pages 45 through 46, with Discussion Questions on page 47. By organizing all cards in one or more of the decks to create each grid according to its description, students can explore algebraic relationships between quadratic equations and graphs as they identify patterns in the rows and columns.

Conclusion By using the different decks of cards with the activities and games in *The Algebra Game: Quadratic Equations*, students have the opportunity to discover how graphs, factors, roots, and coordinate pairs tables all connect to unique quadratic equations. Whether you use the guidelines in this manual or develop your own structures for using the Quadratic Equations decks, your students will develop a deeper understanding of basic and advanced algebraic concepts by actively engaging with *The Algebra Game: Quadratic Equations*!

Parabola Graphs

Recognizing Parabola Graphs

To recognize parabola graphs, students will find it helpful to look for patterns in the curves of these graphs:

- Is the graph symmetrical?
- As the x -coordinates of points on the graph increase or decrease, how do the ratios of the y -coordinates to the x -coordinates change?

The Activity directions and the Discussion Questions prompt students to identify the patterns in the coordinate pairs tables and associate the patterns with the related graphs.

Make sufficient copies of all worksheets for students.

Extension Activity: Use the Activity on page 44 and choose one Organization Grid (or use several grids with different groups).

Recommended decks: Use Deck A, Deck B, Deck C, and/or Deck D. Use Deck D only if students are familiar with x^2 -coefficients not equal to 1.

Materials

- Decks A, B, C, and/or D
- One Activity sheet per group for each activity
- Discussion Questions sheets (one of each per group); Parabola Coordinates sheet (one per group, back of book); Coordinate Pairs, Graphs, Parabola, and Roots Sorting Mats (one of each per group) as the activity requires
- **Optional:** Conjecture Worksheet (one per group, in back of book), graphing calculators

Matching Graphs to Coordinate Pairs

Activity

1.1

Materials Manager: Divide the decks between group members for sorting.

All: Sort cards into 6 separate stacks: Equation, Graph, Coordinate Pairs, Factors, Roots, and Standard Form. Pool the stacks so that there are 6 stacks for the whole group.

Facilitator: Put aside the Equation, Factors, Roots, and Standard Form cards. Deal the Graph and Coordinate Pairs cards to all group members. Place the Coordinate Pairs Sorting Mat and the Graphs Sorting Mat in the center of the table.

All: Work together to place the cards in the correct location on the Coordinate Pairs Sorting Mat and the Graph Sorting Mat. Discuss strategies for placing the cards on the mats. Then, discuss strategies for matching the cards on one mat with the cards on the other mat. Record all graphs and coordinate pairs on the Parabola Coordinates Worksheet.

Recorder: After all cards are sorted and matched, read the discussion questions to the group. Write the responses that the whole group agrees on. Be sure to include responses for the two new questions posed by the group.

Presenter: Report the group's responses to the class.

HSF-IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Also **HSA-REI.D.10** and **HSF-IF.C.7a**.

Group Members: _____

Discussion Questions for Activity 1.1

1. Describe all of the patterns that you see on the Coordinate Pairs cards.

2. Explain how you decided how to match the Graph cards to the Coordinate Pairs cards.

3. What is the significance of the coordinate pair in the middle of a table on a Coordinate Pairs card?

4. Describe the connections that you notice between the Coordinate Pairs cards and the x-axis on the Graph cards.

As a group, write two questions to ask the rest of the class. Before asking the questions, decide as a group on acceptable responses.

Question 1. _____

Question 2. _____

Coordinate Pairs Sorting Mat

Opens down		Opens down
Opens up		Opens up
Opens sideways		Opens sideways

Graph Sorting Mat

Opens down		Opens down
Opens up		Opens up
Opens sideways		Opens sideways

Matching Roots to Graphs

Activity

1.2

Materials Manager: Divide the decks between group members for sorting.

All: Sort cards into 6 separate stacks: Equation, Graph, Coordinate Pairs, Factors, Roots, and Standard Form. Pool the stacks so that there are 6 stacks for the whole group.

Facilitator: Put aside the Equation, Factors, Coordinate Pairs, and Standard Form cards. Place the Parabola Sorting Mat and the Roots Sorting Mat in the center of the table.

Materials Manager: Deal the Graph and Roots cards to all group members.

All: Work together to place the cards in the correct locations on the Parabola Sorting Mat and the Roots Sorting Mat. Then, match Graph cards and Roots cards. Group members may discuss strategies for placing cards on the mats and matching cards on one mat with cards on the other mat.

Recorder: After all Graph and Roots cards are matched, read the discussion questions to the group and write the responses that the whole group agrees on. All questions should have a response including the new questions to ask the rest of the class.

Presenter: Report the group's responses to the class.

HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Also **HSF-IF.B.4** and **HSF-IF.C.7.a**

Matching Roots, Graphs, and Factors

Activity

1.3

(Use after Activity 1.2. Keep the Graph and Roots cards matches together.)

Materials Manager: Divide the rest of the cards between group members for sorting.

All: Sort cards into 4 separate stacks: Equation, Coordinate Pairs, Factors, and Standard Form. Pool the stacks so that there are 4 stacks for the whole group.

Facilitator: Put aside the Equation, Coordinate Pairs, and Standard Form cards. Place the matching pairs of Graph and Roots cards from Activity 1.2 in the center of the table.

Materials Manager: Deal the Factors cards to all group members.

All: Work together to match the Factors cards to the correct pairs of Graph and Roots cards. Group members may discuss strategies for matching the cards.

Recorder: After all cards are matched, read the discussion questions to the group and write the responses that the whole group agrees on. All questions should have a response including the new questions to ask the rest of the class.

Presenter: Report the group's responses to the class.

HSF-IF.C.8a: Use the process of factoring to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Also **HSA-N-CN.C.7**, **HSA-SSE.B.3a**, and **HSA-REI.B.4b**.

Group Members: _____

Discussion Questions for Activity 1.2

1. Explain how you decided how to match the Graph cards to the Coordinate Pairs cards.

2. Can one Roots card match more than one Graph card? Explain.

3. Match the Coordinate Pairs cards to the Roots cards. Describe the reasons that you think they might be connected.

4. Compare the Graph, Roots, and Coordinate Pairs cards. Do you think there might be a connection between the Graph, Roots, and Coordinate Pairs cards? Explain.

As a group, write two questions to ask the rest of the class. Before asking the questions, decide as a group on acceptable responses.

Question 1. _____

Question 2. _____

Group Members: _____

Discussion Questions for Activity 1.3

1. How do you think the Graph cards are linked with the Factors cards?

2. Use the Coordinate Pairs cards. Match a Coordinate Pairs card to each matched set of Graph, Roots, and Factors cards. What connections do you think there might be between the Coordinate Pairs and the Roots cards?

3. What other connections do you think there might be between the Graph, Roots, Factors, and Coordinate Pairs cards? Describe two connections (in addition to the connection in your answer to Question 2).

As a group, write two questions to ask the rest of the class. Before asking the questions, decide as a group on acceptable responses.

Question 1. _____

Question 2. _____

Parabola Sorting Mat

Intersects x-axis once	Intersects x-axis twice	Does not intersect x-axis
Intersects x-axis once	Intersects x-axis twice	Does not intersect x-axis

Roots Sorting Mat

Has two roots		Has two roots
Has one root		Has one root
Has an <i>i</i>		Has an <i>i</i>