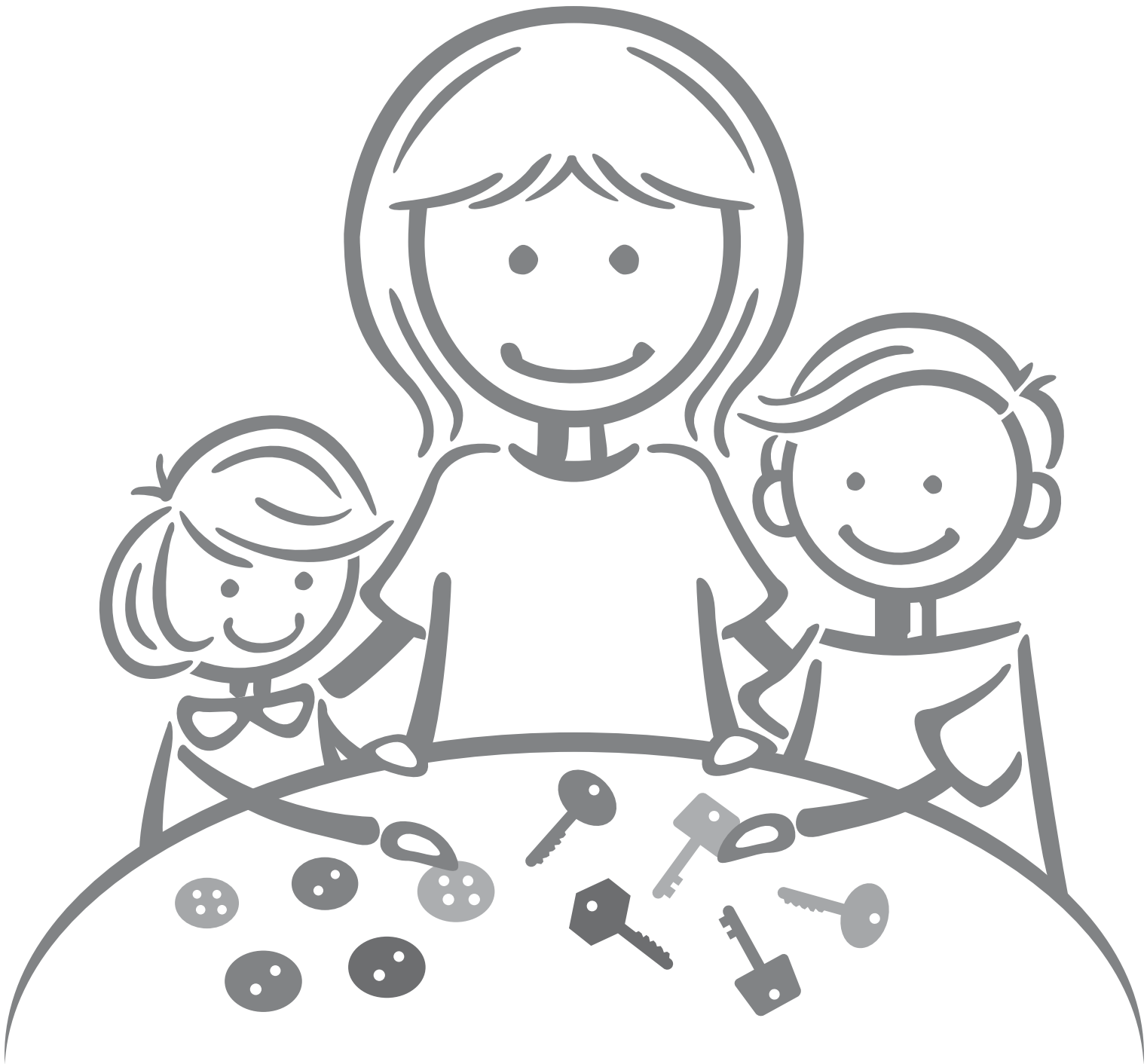


DEVELOPING MATH CONCEPTS in PRE-KINDERGARTEN



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Introduction

The most important skill we take from the study of mathematics [is] a need to explore, describe and define the universe around us.

—Richard Schaar, Professor of Mathematics, USC

The mathematician and the preschooler have much in common. They approach the study of mathematics in much the same way. Like mathematicians, young children are intrigued by the mathematics in their world. Consider the intensity and determination children bring to their investigations of the mathematics that surrounds them. This comes not from a need to complete an assignment or because it will be useful to them. It comes from a need to know.

Guided only by their feeling for symmetry, simplicity, and generality, and an indefinable sense of the fitness of things, creative mathematicians now, as in the past, are inspired by the art of mathematics rather than by any prospect of ultimate usefulness.

—Eric Temple Bell, Mathematician

Young children are naturally interested in numbers and spontaneously ask: *How many are there? How many do we need? Do we have enough? Who has the most? Are there any extras?*

They are interested in geometry and explore to find out: *How are these shapes alike? How are they different? Which ones fit together? Which ones leave spaces? What can we build with these? What other shapes can we make using these shapes?*

They want to measure and compare, and they wonder: *Which is bigger? More? Heavier? Longer? Shorter? How can we find out?*

They experience the connections between math and music when exploring rhythm and patterns, and between math and art when working with symmetry and design.

The mathematics that engages the mathematician and the preschooler is the study of patterns and relationships, order and predictability. This is not mathematics the way most of us experienced it in school. Yet it is the mathematics that will serve our children most as they move through school, and, in time, that will await them as they enter the workforce. The mathematician and the preschooler need and want to experience the joy and excitement of the world of mathematics.

Mathematics in Pre-kindergarten: What does it look like?

The learning of mathematics is an active endeavor. Children need to be involved in investigating, comparing, wondering, and checking to see what happens. They need to think about what they are experiencing, to notice what happens, and to begin to talk about what they notice. They need to listen and think about what other people have to say. They need to begin to make connections, to see relationships between mathematical ideas. They need to see how experiences can be recorded: with blocks, with pictures, and sometimes with mathematical symbols. Important mathematical ideas will naturally arise through children's play.

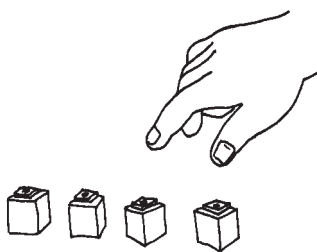
Children's play and interests are the sources of their first mathematical experiences. These experiences become mathematical as the children represent and reflect on them...The most powerful mathematics for a preschooler is usually not acquired while sitting down in a group lesson but is brought forth by the teacher from the child's own self-directed, intrinsically motivated activity.

—Douglas H. Clements, *Mathematics for Young Children*

Children's mathematics includes:

Number experiences ...

that ask them to find out: *How many? How many leaves did you pick up? How many pockets do you have? Can you put three horses in the corral? Can you put four fish in the ocean? Can you give me four of those?*



Acting out counting songs helps bring meaning to the counting sequence they are learning. Children learn to count beyond what they already know by counting along with the teacher.

"How many children are here today?" the teacher asks, and models writing numerals when the children tell how many.

Some children will label what they counted with a numeral and show how many horses they put in the corral, and a few will even try to write the numeral on their own.

Spatial experiences ...

that allow them to see how shapes fit together through experiences with puzzles and blocks and boxes. Children have many experiences working with blocks, and they build structures that begin to look more and more organized and stay balanced a bit longer.

They hear their teacher using language that describes where objects are located:

"Here it is—under the table."

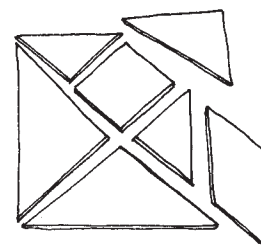
"Let's look behind the cubbies."

"We put the calendar over the bookshelf."

And they begin to use spatial language themselves, like

"The baby is in the baby bed. She is under the covers."

"The cookies are inside our play oven."



Pattern experiences ...

that focus on figuring out what comes next. The teacher begins a rhythmic pattern, and the children join in. "Clap, clap, floor, floor, clap, clap, floor, floor."

The teacher models patterning when she makes a long Unifix train with her students and asks: "Red, red, yellow, red, red, yellow. What color do we think comes next?"

The teacher notices and points out to her students the patterns on their clothing: "Look, I see a pattern on your shirt. Red, green, white, red, green, white."



Children are given opportunities to copy repeating patterns using a variety of materials—for example, block, button, block, button. If they have pattern cards available to copy, some will continue the pattern beyond the card.

Children experience patterns through music and become aware of rhythm and repetition when they march around the room, tap their rhythm sticks, or beat on their drums. They experience patterns through stories when teacher reads predictable books, and they delight in guessing what comes next.

Measurement experiences ...

that focus on making comparisons between objects. "This pumpkin is bigger than this one." "This one is really heavy." Children find things to compare in all their daily activities: "My blocks go higher than yours." "I made a long, long train."



They begin to go beyond describing relationships only as *big* and *little*, and they hear the teacher use terms like *heavy* and *light*, *long* and *short*, *wide* and *narrow*, *thick* and *thin*, *tall* and *short*, *large* and *small*, and *more* and *less*.

Some children will line up blocks to look like stair steps or put yarn lengths in order from the shortest to the longest. It is not always an easy task, and they line them up carefully to see which is longer.











Data collection experiences ...

that focus on sorting and counting as a means of finding answers to everyday questions like:

"How many boys are in school today?"

"How many children had a turn on the slide?"

"Here are all the leaves we collected on our walk. Let's put the green ones here and the brown ones here. Did we find more brown ones or more green ones?"

Shoes		
Laces	Velcro	Other
		
		
		
		
		

How Children Learn Math Concepts

The National Council of Teachers of Mathematics included prekindergarten standards for the first time in its 2000 *Principles and Standards for School Mathematics*. The NCTM document outlines the mathematics that children should learn as they progress through school, identifying both content and process standards.

The content standards are organized into five areas: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The process standards are Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.

The mathematics presented in *Principles and Standards for School Mathematics* provides a broad view of what mathematics is and can be for young children. The NCTM standards show that mathematics can provide children with ways to understand and appreciate the world around them and enrich their experiences.

The following sections highlight what children need to know in each of the math strands and how they learn the mathematics that are and will be important to them.

Learning Number Concepts

In pre-kindergarten through grade 2, all students should

- Count with understanding and recognize “how many” in sets of objects;
- Connect number words and symbols to the quantities they represent.

—NCTM *Principles and Standards*

The complexity of the number concepts young children must encounter and sort out is not always immediately observable to the adults in their world. We sometimes get clues from children, yet do not realize what these clues are telling us. A child brings a package of gum to school, hoping to share it with the entire class. Another child puzzles his teacher by bringing one pencil when the teacher asked for four. When asked to show why he thought he was supposed to bring that pencil, the child counts, “One, two, three, four. See, this is four,” and hands the pencil “named” four to the teacher.

We watch another child counting his toy cars and see him pointing to some more than once and skipping others. “This time I have nine,” he says. Another child, with great enthusiasm, shares, “My dog had a hundred puppies!” These same children may know how to count by rote to 20 or know the numerals to 10. But as we can tell by these examples, there is more to understanding what number is all about than rote counting and numeral recognition.

As useful as these skills may be, we want much more for our children than that. The most important underlying number concept for children to begin to develop in pre-kindergarten

is a sense of quantity. This is not something easily attained and certainly not mastered in pre-kindergarten, but it is the search for the sense of number that is worthy of the child's time.

So, we will not be content even if children can count to 20 or recognize numerals. We want them to be able to use counting when they need to find out how many. We want them to begin to become aware of when an answer is reasonable or not reasonable. We want them to become more consistent and accurate when they count. We want them to begin to see relationships between numbers. And, most of all, we want children to believe that numbers make sense and to be confident about their own abilities to deal with them.

In order to provide the kind of environment that allows young children to develop a sense of number confidently in their own time and way, we must understand something about how this happens for them. Young children do not see the world of number in the same way adults do. They do not yet trust that quantities remain the same when they appear to be different. They are not necessarily aware that they must keep track of what they have already counted so some items won't be counted twice.

It is a natural part of their development for children to believe that quantities are unstable and unpredictable. It is not helpful to them to be told that their incomplete understanding is wrong—that they should know, for example, that the quantity doesn't change just because it is rearranged. If they are told that they should know what they are not yet able to know, they lose faith in their ability to make sense of things and instead begin early to play the game “Don't think—just figure out what the teacher wants you to say.”

True understanding of number must be developed in the child's mind as a result of his or her own experiences and reflections on those experiences. As teachers, we must resist the temptation to tell and explain. Simply provide opportunities and ask questions that get children to look and to think. Showing or telling children will not make them understand. They must figure things out for themselves.

Number sense can only develop if children are given many and varied experiences of finding out “How many?” and are allowed to come to an understanding about number in their own time and way. We provide opportunities to look, to find out, to notice, to ponder, to wonder, and to think about numbers.

Number is an idea that a child must understand about the objects being counted that goes beyond their physical properties. The following basic concepts are those that young children are confronted with as they seek to make sense of numbers: quantification, one-to-one correspondence, conservation of number, relationships between numbers, and symbolization.

Quantification: When children first learn to talk, they are very interested in labeling things. They learn “bottle” and “ball” and “baby.” But numbers are not labels for particular objects. To understand number, children need to understand that the number word they

say includes all the objects previously counted. We see a lack of understanding of this idea when children respond to the request "Give me four" by picking up the one object they were pointing at when they said "four" while counting.

One-to-one correspondence: When children are first learning to count, they have a general notion about pointing while they say the counting sequence "One, two, three, four, five..." and they imitate the counting behavior they have observed. At first, they are neither very precise nor too concerned about making sure each object is counted once and only once. So we see children saying the counting sequence faster or slower than the speed at which they are pointing to the objects being counted. Children do not really understand what counting is all about until they are able to consistently count each object (physically or mentally) once and only once.

Conservation of number: What is often most surprising to adults when they attempt to understand children's development of number concepts is seeing how children are misled by their perceptions. When children have not yet developed an understanding of conservation of number, they believe that the number changes when the appearance of the group of objects changes. They believe that they have more objects if they are spread out and thus fill up more space. They think they have more if their sandwich is cut into four pieces instead of two pieces. They are not sure what happens to the quantity of objects if some are hidden. They may believe there are more chairs than pencils because the pencils are smaller.

Relationships between numbers: Children do not automatically see the relationships between numbers. They don't see that four is contained in and is a part of six. Even when children know how to count, they do not necessarily know what happens when one is added to a set of objects. They often need to count to see. It is especially difficult for them to know, without counting, how many are in a group of objects if one has been removed. Additionally, they do not know that when sets are reorganized in different ways, the quantity stays the same. So they don't realize, for example, that 4 and 2 is equivalent to 1 and 5 or even 2 and 4.

Symbolization: Children will often deal with symbols as though they were physical realities. They see the symbol "3" literally as a squiggle on paper. They don't see it as a mark representing something else. It is important then to help them associate symbols with the quantities they represent. Students will learn much about representing numbers with numerals through the opportunities that naturally arise when teachers find it necessary to use numerals and to model their use in context. When you write the number of days the children have been in school, you will be using numerals. When you record attendance or keep track of the number of children buying milk, record the numerals in a way that children can see them. The main goal is for the children to see numerals as useful tools for writing down information. Many will be interested and will pick up the names of the numerals even from this indirect exposure. Others will be becoming familiar with the peculiar shapes and forms that make up the numerals. And others will benefit from the small group work where symbols are used to write down "How many?"

Learning Geometry Concepts

In pre-kindergarten through grade 2, all students should

- Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
- Describe attributes and parts of two- and three-dimensional shapes;
- Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes;
- Describe, name, and interpret relative positions in space and apply ideas about relative position;
- Recognize and create shapes that have symmetry;
- Recognize geometric shapes and structures in the environment and specify their location.

—NCTM *Principles and Standards*

The study of geometry helps us look more closely at the physical world in which we live. When we observe children who are intrigued and challenged when building with blocks, constructing towers, or creating symmetrical designs, we are seeing them engaged in the study of geometry.

In the study of shape, our goals are not so very different from those of the ancient Greek philosophers: to discover similarities and differences among objects, to analyze the components of form, and to recognize shapes in different representations. Classification, analysis, and representation are our three principal tools.





—Marjorie Senechal, *On the Shoulders of Giants*, p. 140




In their study, they are finding out how two-dimensional and three-dimensional forms of all kinds fill up space, how they stack, and how they fit together. As children work with blocks of various kinds, as they create and copy designs and structures, as they examine and analyze boxes and containers, they become more and more discriminating.

Rather than simply learning the names of the basic shapes, they learn to recognize the attributes of shapes, to notice how shapes are alike and different. They learn to identify and sort by such attributes as the number of sides, the number of corners, the number of faces, the relationship of lengths of sides, whether the figure has straight lines or curves, whether it rolls or doesn't roll, and whether it has symmetry or not. They learn what things change and which stay the same when direction, position, or size is changed.

As they explore their world, children discover forms and structures that occur over and over again. Some of these will be simple forms like circles or rectangles, but they also will become aware of more complex forms, such as pyramids or spirals.

Often the focus of geometry in the early years is on learning the names of shapes. However, it is important to recognize that the name of a geometric figure not only names but defines that figure. The name carries with it the particular attributes of the shape. For example, when we know what a square is, we also know it is a four-sided figure that has four right angles with all four sides the same length. However, when children learn the name of a shape without understanding what attributes define that shape, they can end up with misconceptions.

For example, children who think of  as a triangle often think that  is not a triangle because it is upside down. Some will consider  as half a triangle because it looks like half of the triangle that is familiar to them. If children think a rectangle is the shape that looks like a door, they won't realize that  is also a rectangle. And they won't recognize that a square is a rectangle, too.

Sometimes adults are impressed when children use mathematical language, but they are not always aware that children do not fully understand the words they are using. For example, when children learn to call the yellow pattern block a hexagon, it appears that they are becoming more proficient at using mathematical language. However, children are not usually learning what about that block is or is not relevant to its being called a hexagon but are just naming that particular piece. If the yellow pattern block  is the only hexagon children experience, they won't realize that  is also a hexagon and they may mistakenly think  is a hexagon because it looks like a hexagon.

Before children can make sense of geometric language and apply it appropriately, they must be able to distinguish and label those attributes that define the shape. In the process of learning the language, they will be more apt to focus on the attributes if they are asked to describe the attributes in their own words, using whatever language makes sense to them.

The first step in geometric language development should be learning to see, to notice, to discriminate. The next step should be determining which shapes with similar attributes go together and why. Children should learn the formal labels only when they are ready to apply them to many different versions of a particular shape. Learning the language prematurely can only cause confusion and misconceptions and keep children from looking closely at important attributes. Simply naming geometric shapes is not what is important. Rather, it is important that children look carefully at the properties of various shapes and learn to distinguish among them.

It is very important that children have experiences exploring blocks and the other math materials. It is through exploration that children discover and explore mathematical relationships. Block play is more important than some might realize. According to the research, pre-kindergarten children who are able to build complex structures with blocks have a

better chance of mathematical success in middle and high school, even taking into account students' IQ levels, social class, and gender (Wolfgang, Stannard, and Jones, 2001).

Learning Sorting and Classifying Concepts

In pre-kindergarten through grade 2, all students should

- **Sort, classify, and order objects by size, number, and other properties.**

—NCTM *Principles and Standards*

Sorting and classifying is an important intellectual process that we go through when we seek to organize our world. Classifying requires us to create mental structures based on complex relationships and interrelationships. Children begin the process of identifying classes of things early in life as they learn to identify chairs in all their forms and to distinguish them from the various kinds of couches, or to tell a horse from a cow or a dog from a cat. I remember the first time I realized how complex this could be when my own young daughter pointed out a fluffy little animal and said, "Look, there's a kitty dog."

Even though the process of organizing the world through sorting and classifying begins very early in the child's life, it is a process that takes many years and proceeds through many stages. Children's beginning work toward an understanding of classes and classification includes sorting and forming collections using real objects. In order to sort, children must be able to recognize and identify attributes of the materials being sorted. Next, they determine which attributes are alike (which things go together when sorting) and which are different. Their ability to do this grows with time and experience. Through handling materials, they will come to notice finer and more detailed distinctions. They will learn to attend to one particular attribute. They will learn to ignore the differences and focus on what makes a group of objects alike. As children develop their awareness, their language will also develop as the need to be more precise increases.

Children generally first identify the color of objects and later notice shape, size, and other characteristics. Their first intentional sorting behavior is often one of finding pairs that are just alike. Another step in sorting is finding all the objects in a collection that have a particular attribute, while leaving those that do not. For example, they may pull out all the red ones from a group. All the rest that are not red are left in the original pile.

It isn't until around age six that most children will be able to sort all the objects in a set according to a particular property. For example, they will be able to find all the round ones and then go on to find all the square ones and then continue to sort all the rest of the shapes in their collection. Before this there is a transition stage, where children start with one attribute and move to another in the middle of sorting. For example, a child might start sorting all the red ones and then start making piles of circles. Their focus is more on finding things that are alike than on determining a classification that would encompass all the objects they are working with.

One reason that sorting is so challenging for children is that until they reach a certain stage, they cannot think of more than one attribute at a time. For example, it is difficult for them to consider an object as both red and round. If they are trying to sort objects into particular sets and find something that could go in two different places, they often just ignore it.

Learning Pattern Concepts

In pre-kindergarten through grade 2, all students should

- Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;
- Analyze how both repeating and growing patterns are generated.

—NCTM *Principles and Standards*

Much of our lives is spent in an active search to make sense of things—to organize and sort things out. When we are able to get a sense of the basic order of things, we are able to predict—to count on things happening—and thus to become more secure and confident. We learn that morning always follows night, that second grade comes after first, that a flash of lightning will be followed by a roll of thunder.

Seeing patterns in the way things work is an incredibly powerful learning tool that most of us have developed intuitively to some degree. If we add *s* to words to make them plural, if we know a dog we have never seen before is a dog and not a cat, if we know that red lights always turn green and that green lights turn yellow, we are using our sense of pattern.

Mathematicians say that mathematics is the study of pattern. Pattern is the basis on which our number system was created. For many of us, mathematics has not been about patterns but rather a series of rules and steps to follow so we could get answers to teachers' questions. We did not try to look for the underlying order or sense of things in mathematics. When we have not discovered the pattern in numbers, learning mathematics has been much more difficult than it needs to be, and we missed much of the beauty that is in mathematics.

We can give our students a sense of the beauty and order that is mathematics. We can give them the confidence that comes when things are predictable. The activities in this book are designed to give children such opportunities.

Children move through several stages as they develop an understanding of pattern. At first, many do not know what is being referred to when the term *pattern* is used. They often have misconceptions and incomplete understandings as they seek to figure out what a pattern is.

Once children begin to recognize the underlying order and predictability in the patterns they experience, they begin to create their own very simple patterns. The first patterns children create are frequently simple AB (alternating) patterns, so it might seem logical to

present AB patterns first and make sure children understand these before moving on to more complex patterns. However, when we try to teach children in a step-by-step fashion, we sometimes limit their view. They are likely to end up with a misconception of what a pattern is, falsely concluding that all patterns are AB patterns. This confusion may not show up until children try to apply their notions about pattern to a new situation.

If we want children to fully understand what a pattern is, we must surround them with a variety of patterns of many forms and in many different situations. Children learn to make connections and to see relationships when they are immersed in experiences—when they see an idea portrayed in many different ways. It is important that we present the variety of patterns to children from the very beginning and not try to oversimplify the idea for them.

Learning Measurement Concepts

In pre-kindergarten through grade 2, all students should

- **Recognize the attributes of length, volume, weight, area, and time.**

—NCTM *Principles and Standards*

Young children's interest in measurement is primarily focused on finding out who or what is the biggest. One of the important concepts they need to learn is that "big" can mean many different things depending on the property being considered. A jar may be bigger if we are talking about height but smaller if we are talking about how much it holds. We might consider one child to be bigger than another because he weighs more, or another child might be considered bigger because he is taller.

When we measure, we need to decide what we want to know and then choose the tool that will help us find out. Do we want to know how long something is, how tall it is, how much space it takes up, how much it holds, or how heavy it is? Learning which property is being measured can be challenging to the young child. Young children who are just beginning to work with measurement often mix up the language, revealing the difficulty of the ideas they are exploring. For example, one child lay down next to a long Unifix train he had made and declared, "This is how big I weigh."

One of the underlying concepts that influences the young child's understanding of measurement is the idea of conservation. Conservation is the recognition that a quantity or amount stays the same even though it has been rearranged in some way and appears to be different. This seems obvious to an adult but is not at all obvious to a young learner. For example, at a certain stage of development, a child will say that he has "more cracker" because he has broken it into pieces.

A child once told me, "I can finish my whole lunch. My mom used to cut my sandwich into four pieces, but that made too much. Now she cuts my sandwich in two pieces, and I can eat it all!" Children reach these kinds of conclusions in part because they generally can attend to only one attribute at a time.

For example, if two sticks are lined up as in the following illustration (left), children will see they are the same length.



However, when one is moved (right), children may focus on only one of the end points and thus think the stick in front is now longer.

When children pour water from one short, wide jar into a tall, narrow jar, many think the taller jar holds more. When a ball of clay is rolled into a “snake,” children think the quantity has changed and is either more or less depending on the attribute on which they are focused.

If children are truly at this stage of thinking, they cannot be shown the right answer. What they perceive to be true is more powerful in their thinking than what someone else might try to get them to understand. When we ask children to work with measurement concepts, we must take into consideration the influence their understanding of conservation has on their understanding of measurement and look at their responses in light of this stage of thinking.

As children develop an understanding of measurement, they move through predictable stages of development. Their first understanding of measurement requires them to compare things directly to see what is “bigger” and what is “littler.” Eventually, they will see that, in some situations, it is difficult to line things up. They will then be encouraged to find tools to help them measure. The first tool they experience might be a piece of string they use to measure the length of an object. The idea of units of measure will begin with finding out how many scoops of rice it takes to fill a jar or how many cubes an object weighs. These kinds of experiences along with questions and conversations will prepare children to work with standard units and tools when they are presented in the months or years ahead.

Learning Data Collection Concepts

In pre-kindergarten through grade 2, all students should

- Represent data using concrete objects, pictures, and graphs.

—NCTM *Principles and Standards*

One of the ways we use mathematics to understand our world is through the collection of data and the organization of that data into graphs and charts. When information is organized into graphs, the visual arrangement of the information reveals much about a situation and makes that information more easily accessible to us.

We can help children become familiar with graphs and their purposes if we provide them with opportunities to collect and organize data to answer questions. The questions asked should be those that have some interest to young children and that deal with their immediate world. Mathematics and science, as well as mathematics and social studies, are integrated naturally when we gather and collect data.