Understanding Numbers

The National Council of Teachers of Mathematics states that in grades 3-5 all students should:

- understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals
- recognize equivalent representations for the same number and generate them by decomposing and composing numbers

Principles and Standards for School Mathematics p. 148
National Council of Teachers of Mathematics

Students who are proficient in mathematics and have the foundational understandings necessary for future success have a deep understanding of place value. They know more than how to tell which digit is in the hundreds place; they know more than how to use an algorithm to get an answer to a computation problem. They understand the structure of the numbers and what happens to the numbers when they add, subtract, multiply, or divide. They understand, for example, that 2,345 is composed of 2 groups of a thousand, 3 groups of one hundred, 4 groups of ten, and 5 ones. They can easily reorganize ones into tens (e.g. 42 ones is 4 tens and 2 ones), tens into hundreds (e.g. 17 tens is 1 hundred and 7 tens), and hundreds into thousands (e.g. 15 hundreds is 1 thousand and 5 hundreds). They use what they know about whole numbers to help them understand decimals. They see that 42 hundredths is worth 4 tenths and 2 hundredths. They know that 7 tens and 5 tens can be combined to form 1 hundred with 2 tens left over and use that information to combine 7 tenths and 5 tenths. They know what quantities are represented by numbers so can easily tell that .30 is smaller than .5.

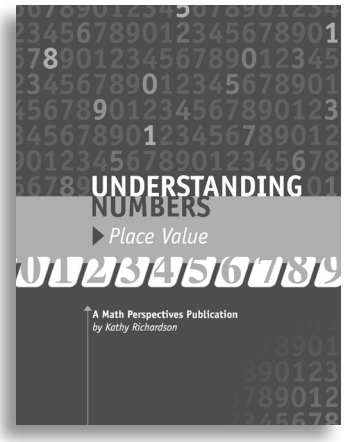
Students who understand the underlying structure of the numbers work with numbers with facility and ease and have the knowledge needed to develop competence with computation.

Developing An Understanding of the Structure of Numbers

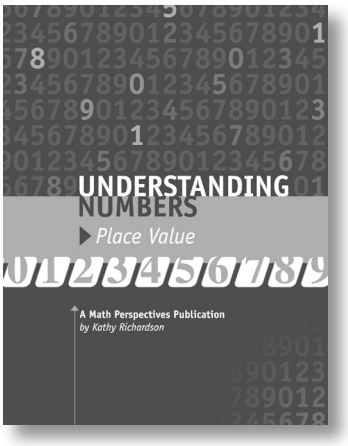
Developing procedures for the addition and subtraction of multi-digit numbers, both whole numbers and decimal numbers, should evolve from the students' understanding of place value.

Principles and Standards for School Mathematics p. 218
National Council of Teachers of Mathematics

Students need many ongoing experiences to develop an understanding of the underlying structure of numbers and the ability to think about numbers flexibly. They need to engage in activities where they group and regroup numbers, break numbers apart, and reorganize them in various ways. They need to find out for themselves that 12 tens, 120 ones, and 1 hundred and 20 ones all describe the same quantities. They need to actually build .12 and 1.2 to perceive the relationship between them.



Understanding Numbers



Understanding Numbers

The Stations

The Understanding Numbers series of stations are sets of 8 tasks that provide students with the meaningful practice necessary for developing an understanding of the underlying structure of numbers, number relationships and operations. The mathematics presented in the stations is foundational and crucial for developing computational fluency. These foundational concepts also provide the basis for students' understanding of the mathematics they will encounter in the future. Each set of stations presents a variety of activities focused on one major concept. The tasks are designed to meet a range of needs allowing all students to work at their own level. These stations should be experienced over and over again until students have developed proficiency with the tasks. Most students will benefit from working with the appropriate set of stations for several weeks.

Using Models

Students develop an understanding of the structure of numbers by working with models that reveal relationships between numbers. They develop proficiency with the numbers represented by the models when their focus is on identifying the relationships the models reveal rather than on manipulating the models to get answers. They focus on these relationships when they organize and reorganize various models into hundreds, tens, and ones or into tenths and hundredths. So, for example, instead of trading 10 tens for 1 hundred (which can cloud the relationship for some students) they reorganize the tens into 1 hundred. Instead of trading 1 hundred for 10 tens so they can take away 4 tens, they mentally take away or cover up the 4 tens to see that 6 tens are left. They don't count by tens to see what 14 tens are worth, but rather they reorganize the tens into 1 hundred and 4 tens to determine that the blocks are worth 140. When combining 8 and 6, they don't trade 10 ones for a ten, but rather they mentally or physically put 2 of the ones with the 8 to make another 10 and see that 4 are left.

Work with models is effective if it leads students to a level of understanding where they can mentally work with the ideas represented by the models and therefore no longer need the models.

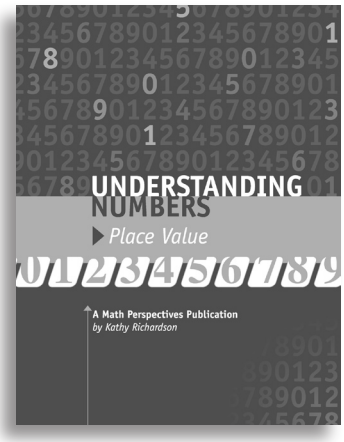
"Research indicates that students' experiences using physical models to represent hundreds, tens, and ones can be effective if the materials help them think about how to combine quantities and, eventually, how these processes connect with written procedures ...The models, however, are not automatically meaningful for students; the meaning must be constructed as they work with the materials."

Adding it Up, p. 198
National Research Council

Differentiating the Tasks to Meet a Range of Needs

... a very important part of the job of a teacher is to guide the child towards tasks where he will be able objectively to do well, but not too easily, not without putting forth some effort, not without difficulties to be mastered, errors to be overcome, creative solutions to be found. This means assessing his skills with sensitivity and accuracy, understanding the levels of his confidence and energy, and responding to his errors in helpful ways."

Margaret Donaldson
Children's Minds p. 120



Understanding Numbers

Students develop a full understanding of important concepts at various times. If each student is to learn all they can, each one must be working at the edge of their own understanding as they work to develop proficiency. The various tasks included in the sets of stations, therefore, are expandable. That is, they can be adapted to meet a range of instructional needs.

Sometimes, having students work with tens and ones rather than with hundreds, tens, and ones, while other students work with numbers in the thousands meets this range of needs. Sometimes the ways that students use models automatically differentiates the task. For example, some students may need to move the blocks or touch the models to solve a problem. Others may need to study the models to aid their thinking but will not need to actually move or touch the model. Others will be able to do the task without the use of a model at all.

Interacting with the Students

Students do not discover or understand mathematical concepts simply by manipulating concrete materials. Mathematics teachers need to intervene frequently as part of the instruction process to help students focus on the underlying mathematical ideas and to help build bridges from the students' work with the manipulatives to their corresponding work with the mathematical ideas or actions.

Jerry Johnson
Teaching and Learning Mathematics, p. 40

Just as books are the tools that students use to learn to read, the sets of stations are tools for learning mathematics. It is not the task itself that determines what students learn, but rather, what the students are focused on while doing the task that influences the learning. Students will be working with the station tasks independently. However, the teacher has an important role in helping students focus on the important mathematics so that the students learn all they can from their work with the various tasks. The teacher needs to interact with the students while they are at work, observing how they solve problems, questioning them as a way of focusing their thinking, and challenging them. Each station includes suggestions for interacting with students at work.

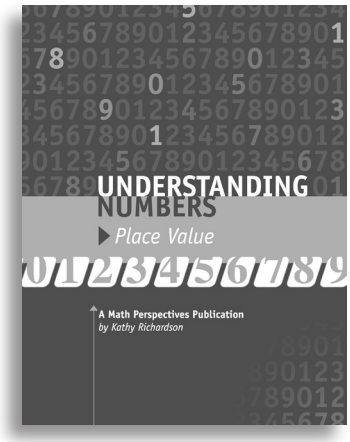
Introducing the Stations

The whole class can work on these stations at the same time. The stations should be introduced to the whole class over a period of 3 or 4 days. The directions for most of the task are simple and easy for the students to understand. After the tasks have been introduced and the students have had some time to work with the stations, the teacher can begin to adapt the task to more appropriately meet the various needs. Suggestions for adapting the tasks are included in the Notes for Teachers for each task.

Readiness to Work with Stations

Students will be more successful working with the tasks if a culture of self-directed learning and hard work has been established. The students should know how to choose a task, work hard at the task, and move to another task without direction from the teacher. It is worth devoting some time to the development of this way of working.

Place Value Stations



Place Value Stations
Introduction

Goals:

When working with groups of hundreds, tens, and ones, the students will:

- Count groups of tens and hundreds
- Learn the combinations of one hundred
- Recognize the structure of numbers as hundreds, tens, and ones
- Know the number of hundreds that can be made from any group of tens
- Combine hundreds, tens, and ones by reorganizing the tens into all the hundreds possible and the ones into all the tens possible
- Combine quantities by reorganizing tens into hundreds and ones into tens without counting to find the totals

The Place Value Stations give students a variety of experiences that help them understand the underlying structure of the base-ten number system. The focus is on hundreds, tens, and ones with variations that focus either on smaller numbers (10s and 1s) or on larger numbers (1000s, 100s, 10s, and 1s.)

Each of the tasks requires the students to determine the total value of a number by reorganizing various groups of hundreds, tens, and ones. They first determine the number of each of the groups. For example, they may have 13 hundreds, 16 tens, and 24 ones. They then reorganize the 13 hundreds into 1 thousand and 3 hundreds, the 16 tens into 1 hundred and 6 tens, and the 24 ones into 2 tens and 4 ones. Next, they combine the hundreds, tens, and ones, forming new groups as needed. Working with the numbers in this way focuses them on the structure and equivalent values of these groups.

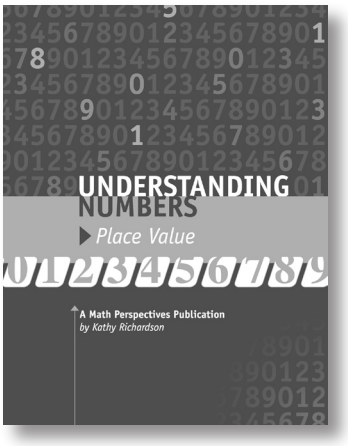
Ongoing Experiences with the Stations

It is through ongoing, repeated experiences with the stations that students develop an understanding of and facility with the base-ten structure. They will move through various stages when working with these tasks. In the beginning some may count to find the total value. For example, they may find out the total value of the 16 tens by counting by 10s, saying “10, 20...” and so forth until they get to “150, 160”. They can get the right answer this way, but they do not learn that 16 tens is equivalent to 1 hundred and 6 tens. With practice, the structure of the numbers will become more evident to the students and eventually they will combine and reorganize the blocks mentally. The students will benefit from ongoing experiences with these tasks until they have reached this stage of thinking.

Interacting with the Students

Students work independently with the various station tasks, choosing the particular task they want to do. The teacher’s role during this time is vital. The teacher observes the students while they are at work and interacts with them, asking questions and focusing and challenging their thinking.

When the teacher sees that a student is counting instead of reorganizing into groups of hundreds and tens, it is more effective to ask a question than to tell the student not to count. The question should engage the student in thinking about the numbers. For example, the teacher might ask, “How many tens do you have? How many



Place Value Stations
Introduction

of these tens do you need to make 1 hundred? How many tens will be left?" Some students will respond by actually rearranging Base-Ten Blocks into a set of 1 hundred in order to see how many leftover tens there are. These same students may still need to count by tens to determine the total. This indicates that the underlying structure of the numbers is not yet obvious to these students. All of the tasks present students with models to aid their thinking and help them develop an understanding of the structure of numbers. The models should not be considered as tools for getting answers to the problems. Rather, they are to help develop an understanding of the structure of numbers and lead students to the point where they no longer need the models. This takes time and lots of experiences.

Several of the tasks require the students to combine 2 items (such as pictures, buildings, or shapes) so they get practice with larger numbers. When adding the two items together, the goal is to learn to combine the tens into hundreds without counting by tens. For example, a student might describe the process this way: "I have 18 tens and 13 tens, I need to put 2 of the tens with the 18 tens to make 2 hundreds. There are 11 tens left so that means I have 1 more hundred and 1 ten so both buildings together are worth 310.

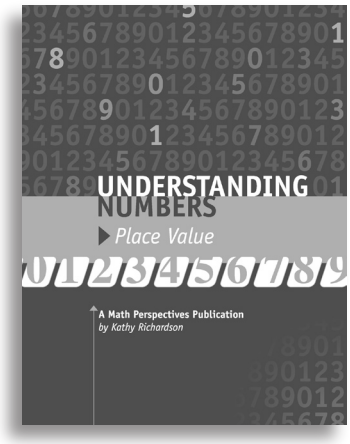
The student who does not yet use the structure of the numbers to solve problems should be asked questions to help him think about more efficient ways. The teacher might ask a series of questions such as: " How many tens do you need to make another hundred? How many tens would be left if you put 2 more tens with the 18 tens? Do you have an idea how many you would have altogether if you added those 11 tens? Can you find out without counting?" One student may realize he can easily add 1 hundred and 1 ten but another student may not be able to do it that way and will need to count to determine the total. The teacher's questions are to help the student notice relationships but should not be used to get the student to do the task in one particular way. Some students will not be able to see what the teacher wants them to see. If a student is unable to do what the teacher expects, it is important to allow them to do it in a way that makes sense to them.

A *Guide to Observations Card* that summarizes what to look for and what questions to ask is included. It is recommended that the teacher keep this guide handy as they begin working with the students at their stations.

Adapting the Task

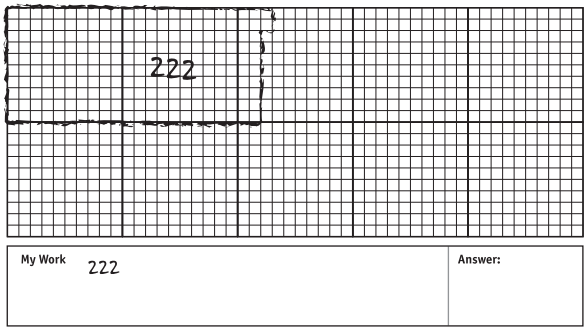
The various tasks included in the sets of stations can be adapted to meet a range of instructional needs.

Changing the Size of the Numbers
When teachers observe students, they will find that some students are still uncertain about how much 2 tens and 5 ones are worth or are unable to add 10 to a number or take 10 away automatically. These students will need to work with tens and ones before they will benefit fully from work with hundreds. The teacher will notice that other students are able to combine hundreds, tens, and ones easily without the use of a model. These students are ready to work with numbers in the thousands. Each of the tasks has variations that allow students to work with either tens and ones or with thousands. Teachers can easily adapt the task by asking students to use the version of the task that is appropriate for them.

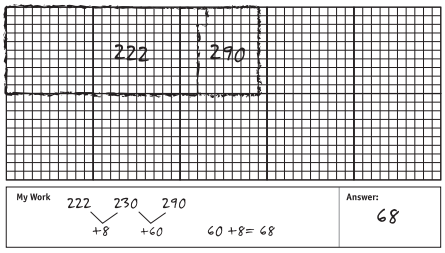


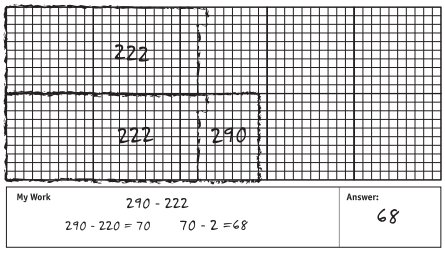
Place Value Stations
Introduction

The Challenge of Comparing Numbers
Teachers can also choose to adapt the tasks for those students who are ready for the challenge of comparing two items. The goal is for students to develop an understanding of what it means to find the difference rather than simply learning to follow the routine procedure of subtracting one number from another when comparing numbers. Students can learn what it means to find the difference through the use of the *Comparing Recording Sheet* and/or the *Thousand Grid Recording Sheet*. When comparing, they first outline the total value of one of the items they worked with (Buildings, Base Ten Pictures, Classroom Objects).



To compare the other item, they can either outline the value on top of the first outline or use the second row. The student should choose the representation that is easiest for them to understand.





Over time the students will learn to compare two items without the aid of a model.

You will find that comparing is a difficult concept for many students. It will be easier for most students to understand what it means to find the difference if they work with smaller numbers than they were using at the stations. For example, students who were working with hundreds, tens, and ones would work with tens and ones when comparing. Students working with thousands would compare numbers less than 1000. Students working with tens and ones would compare numbers less than 20. When they can easily find the differences using the smaller numbers, they will be ready to work with larger numbers.