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Chapter 2

Selecting Virtual Manipulatives for Classroom Use

Ms. Smith's third-grade class has been exploring equivalent fractions. The children have examined fractional amounts represented by shading a portion of a rectangular region drawn on grid paper and have found different names for the region by dividing the area into smaller parts. Ms. Smith wants her students to look for patterns in the fraction names they find in order to help them discover a computational procedure for finding equivalent fractions. Today, the students are working with a virtual manipulative tool designed to help them create equivalent fractions. Students are shown a fractional amount represented by the shaded region of a rectangle and the name for that amount. The instructions ask the students to divide the rectangle into different numbers of pieces in order to find equivalent fractions. Once they find a fraction name, they can enter it into the computer and check their answer. Ms. Smith has asked them to record the picture of the original fraction, the name of the original fraction, and any equivalent fractions they find.

One of the third-graders begins with the fraction $\frac{5}{6}$. He begins to manipulate the computer applet to divide the region into more pieces. (See Figure 1.) As he increases the number of divisions, black lines appear on the rectangle, reflecting the new divisions. When the student gets to 12 pieces, the black lines turn red as they match up exactly to the lines demarcating the shaded region. He has found an equivalent fraction. He counts the number of shaded pieces and enters $\frac{10}{12}$ into the computer. The computer tells him that he is correct; $\frac{10}{12}$ is another name for $\frac{5}{6}$. The student records this on his sheet and goes back to work. In a short time, he has found sev-

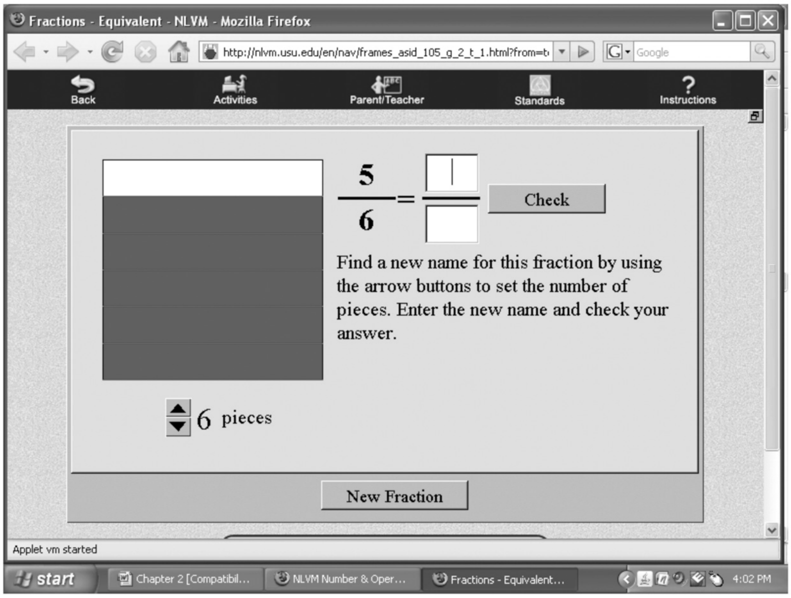

FIGURE 1 *NLVM applet Fractions – Equivalent*

eral other names for the region. As he begins to examine his recordings, he believes he has discovered a pattern in the numbers. When he presents his conjecture to Ms. Smith, she suggests that he test his ideas on the next fraction. The virtual manipulative will help him verify his thinking.

What ideas is this student exploring with this virtual manipulative? What features of the virtual manipulative might support his exploration? How did Ms. Smith decide which virtual manipulative to use for this task? How might her choice impact the learning that takes place? These are the questions that will be addressed in this chapter.

How Do Virtual Manipulatives Support Learning?

As described in Chapter 1, a virtual manipulative is "an interactive, Web-based, visual representation of a dynamic object that provides opportunities for constructing mathematical knowledge" (Moyer, Bolyard, and Spikell 2002, 373). How might virtual manipulatives support the construction of new knowledge and what features of these tools might impact learning? Research and theories of learning in the areas of cognitive science, multimedia learning, and the use of representation and technology in mathematics education contribute ideas for consideration. The sections below provide general background information on these ideas and then suggest guidelines based on these ideas to use when selecting virtual manipulatives for mathematics instruction.

The Role of Representation in Mathematics and Learning through Multimedia

As stated in the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards* (2000), the representation of mathematical concepts and ideas is fundamental to understanding. Representational systems are both internal and external in nature (Goldin and Shteingold 2001). Internal representation systems are those that exist within the mind of the individual, that is, the way the individual makes sense, sees, or understands ideas. External representations can be used to communicate an individual's mathematical ideas and understandings to others. This may be done through multiple representational forms, including manipulative models, pictures, written symbols, real-world situations, and oral language (Lesh, Post, and Behr 1987). Each of these forms has the potential to highlight different features of a mathematical idea or relationship (NCTM 2000). Translating among and between these representational forms builds understanding of mathematical ideas (Hiebert 1990).

Research on multimedia learning and instructional design provides theoretical models for understanding how the use of different representational forms may impact learning. Multimedia is generally defined as combining words (including written and spoken) and pictures (including static or dynamic objects and graphs) in an instructional presentation (Reed 2006). Learning theories from the field of cognitive science have influenced research in multimedia learning through three basic ideas: (1) humans process information through two channels, visual-pictorial and auditory-verbal; (2) these channels can hold and process a limited amount of knowledge at a given time; and (3) learning occurs as pieces of information in the channels interact and integrate with each other and with prior knowledge (Mayer 2002). These three assumptions have informed several principles in the design of multimedia learning instruction with the goal of better learning and retention (Mayer 2002). The main principle states that it is better to present an explanation in words and pictures than solely in words. The presentation should locate related words and pictures near each other, should present words and pictures simultaneously, and should eliminate extraneous words, pictures, and sounds. The presentation should provide signals for key ideas or steps and should allow the user some control over the words, pictures, and pace of the presentation. This interaction may reduce the cognitive load on working memory, enabling the user to pause, organize, and reflect on new information, allowing for building a mental model (Mayer and Chandler 2001).

The Use of Technology to Support Mathematics Learning

Although the multimedia principles discussed above apply to instruction across all types of media (text, illustrations, etc.), there are unique features of some media that may allow instructional design opportunities that are not readily available in other forms. Technology is one example. According to NCTM (2000), the use of technology in mathematics teaching and learning is essential due to its capabilities to (a) produce detailed visual images of mathematics concepts, (b) facilitate the organization and analysis of data, and (c) support the investigation of new concepts and ideas. Technology makes it possible for virtual and other computer-based manipulative programs to build in instructional supports that must otherwise be incorporated into instruction from an outside source. Such features include multiple, linked representations, interactivity, and immediate feedback.

Virtual manipulatives are examples of *externalized representations* (Zbiek, Heid, Blume, and Dick 2007) through which students can communicate their internal representations of mathematical ideas. They can act as a visual model of a mathematical process or idea (Kurz, Middleton, and Yanik 2005). In addition, virtual manipulatives are *cognitive technical tools* (Zbiek, Heid, Blume, and Dick 2007) which enable users to perform an action on a representation of a mathematical object or concept and see the result of that action. In the example described at the beginning of the chapter, the student clicked on a command to increase the number of divisions in the rectangular region. As he did, he could see the area being divided into increasingly smaller pieces. If he instructed the computer to lower the number of divisions, the pieces would become fewer and larger. The tool served as a visual model of the relationship between the number of pieces into which a whole is divided and the size of those pieces.

Furthermore, unlike other external representations, these computerized versions have the ability to link multiple representational forms of mathematical concepts in dynamic ways so that action on one form (such as a manipulative or object) is reflected not just in that form, but in other forms as well (for example, the symbolic notation). These links help students see connections among representations of a concept, while highlighting the unique features of the concept evident in each representational form.

Technology tools, like virtual manipulatives, also provide constraints that highlight or signal important ideas to ensure that students are engaging with the relevant mathematics. For example, the tool used in Ms. Smith's classroom provided a pictorial representation that illustrated when the number of divisions created by the student aligned with the original amount. Originally, the lines bounding the fractional region were red. As the additional divisions are created, the lines representing them are black. Once an equivalent amount is found, all lines turn red, signaling that an equivalent fraction has been found.

Constraints and other features available in technology environments can support actions on objects that are more closely representative of the mathematical relationship being explored than can other types of medium (such as physical tools) (Zbiek, Heid, Blume, and Dick 2007). For example, if students were exploring equivalent fractions on paper and creating the divisions by hand, it might be expected that they would have difficulty accurately dividing the region into equivalent and increasingly smaller pieces. It is likely that students would find one or two equivalent amounts and then be stalled due to the difficulty of creating accurate divisions. Using the virtual tool, the students can create divisions up to 99 pieces. This more closely represents the behavior of rational numbers in the sense that the same number can be expressed in infinite ways.

Finally, virtual manipulatives support the exploration of rich mathematical ideas, such as pursuing hypotheses. In the case of the equivalent fraction tool, the student felt he had discovered a pattern in the equivalent fraction names. Rather than having to repeatedly draw new fractions and create multiple divisions by hand, the student could use the virtual tool to assist him in these processes. The capabilities of the computer to accurately and efficiently create new divisions and provide feedback on the accuracy of his entries removed these responsibilities from the student and allowed him to focus on testing and refining his emerging ideas.

How to Select a Virtual Manipulative

Virtual manipulatives can represent mathematical ideas while incorporating multimedia design principles and taking advantage of the supports and features made possible through technology. Considering the potential of these tools, how does one select a virtual manipulative for classroom use? The next section provides some suggestions, guidelines, and questions to consider in this process.

Guidelines for Selecting a Virtual Manipulative

1. The first and most important consideration is, what is the mathematics that you wish to teach? While this may seem obvious, clearly defining your goals and objectives is the first and most important step in selecting a virtual manipulative or any other instructional tool. Is your mathematical goal conceptual (i.e., developing an understanding of equivalent fractions, or understanding the relationship between the number of parts in a whole and the size of the parts)? Is your goal to develop or practice a skill (i.e., developing a computational procedure for finding equivalent fractions)? Being clear about the purpose of the lesson or unit will help you determine the most effective instructional approach.
2. Once you have determined your instructional goals and objectives, ask yourself, is a virtual manipulative an appropriate way to reach these goals? What can the virtual manipulative add to instruction that might not be available or possible in another medium? Can it extend the mathematical possibilities that could be explored beyond what is possible using another tool? Would the use of a combination of virtual and physical manipulatives enhance students' experiences? Identifying the unique features of a virtual manipulative that could enhance and support student learning will help you create instructional experiences that will utilize these features more effectively.
3. Is the virtual manipulative an appropriate mathematical model for the concept or skill you want the user to explore? Does the model have the potential to accurately represent the user's developing understanding of mathematical ideas? Will the actions performed with the virtual manipulative appropriately reflect the mathematical ideas being explored? In other words, how well does the behavior of the virtual manipulative reflect the ideas the task is designed to address?
4. Is the virtual manipulative applet interactive? Are images dynamic and interesting (but not too "busy")? What is required of the user? Can users control on-screen objects and representations or are they simply observing actions on the screen? Are the user's actions accurately reflected in the tool? Are

they allowed control over the pace of the exploration or task to allow time for mental processing?

5. Does the website include multiple representations of the concept? Are words, numerals, and pictures on the site used simultaneously to connect representations? Are these representations linked so that the user's actions on one representation are reflected in other representational forms?
6. What type of feedback does the site provide? Is constructive feedback provided? Are correct responses confirmed? Are incorrect responses signaled? If so, how? The type of feedback desired may vary with your goal. For example, if the goal is to practice or develop a specific procedure, feedback that guides the learner through each step would be appropriate. If the goal is to provide students opportunities to develop concepts or test conjectures, more open-ended feedback may be more appropriate.
7. What constraints, supports, or scaffolding does the site provide? Are there constraints that may help students focus on the relevant mathematical ideas? Are there constraints built in that highlight ideas and that may not be available in another medium? Are there hints or suggestions that will help students continue to work on the ideas if they get stuck? Is there a way to make modifications or accommodations in order to meet the needs of diverse learners?
8. Is the site user friendly, with clear instructions for manipulating dynamic features? Would students be able to engage with the site on their own, or would you need to provide instruction for your students? If so, what kind of instruction would be most effective?
9. Other considerations: Can the tools on the site can be altered and used to teach several different concepts (when appropriate)? How can you record or keep a record of what students have done? Are there activities, lessons, or other teacher resources available?

The questions in the above list may be numerous, but they are not intended to form an exhaustive list. Rather, they are offered as suggestions for consideration. It may be unlikely that any tool, virtual manipulative or otherwise, will have all of the features suggested above. However, with an understanding of how these tools may support learning to guide your selection, you, as the teacher, can better determine which features might be most important for the learning goal and your students.

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