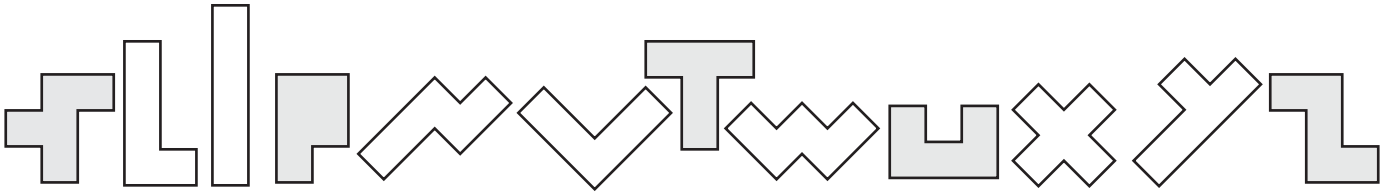

Working with Pentominoes

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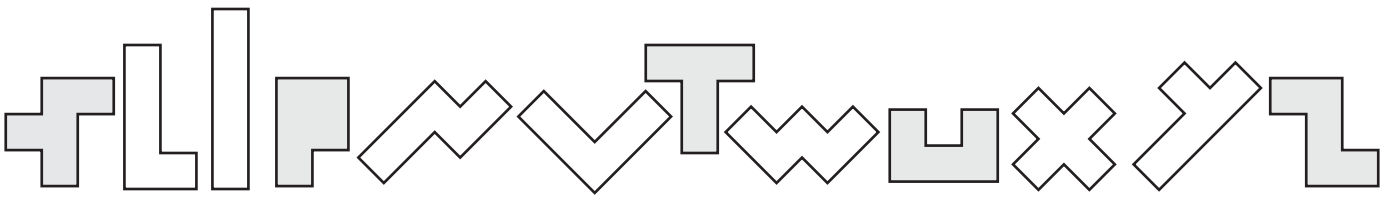


Introduction

Pentominoes are the twelve shapes one can make by joining five squares edge-to-edge.

The name comes from the word domino, a shape made by joining two squares edge-to-edge. Though the first pentomino puzzle was published in 1907, pentominoes were rediscovered and named by mathematician Solomon W. Golomb at Harvard University in 1953. Since then, pentominoes have become one of the most popular branches of recreational mathematics. Mathematicians have created and solved hundreds of pentomino problems. They have proved others to be insoluble. (This branch of math is called combinatorial geometry.) Computer programmers have used computers to solve some of the tougher puzzles. Commercial manufacturers have marketed variations of pentomino games. Teachers at all levels have found pentominoes a valuable addition to their curriculum. Sixty years after their introduction, there is no sign of pentomino popularity waning.

Educationally, pentominoes are a gold mine. They allow the introduction of important mathematical concepts and skills in a playful and non-threatening context. These include concepts of measurement (unit, area, perimeter), number (multiples, triangular numbers), geometry (congruence, similarity), and others (such as maximum-minimum investigations and impossibility proofs). Students' intuition, problem-solving ability, strategic thinking, and sense of two-dimensional space are also enhanced.

The difficulty of pentomino puzzles covers a broad spectrum, from preschool to adult levels. Students who differ widely in ability in arithmetic and algebra start out as equals when working on these puzzles. In fact it is not uncommon for a fifth-grader to solve a pentomino puzzle before the teacher does. Needless to say, this does wonders for the student's self-image.

—Henri Picciotto

How to Use This Book

Recommended Grade Level

This book is intended for grades 4 through 8. However, most of the lessons can be adapted for use in the primary grades and high school.

Materials

Each student should have one set of pentominoes and a supply of quarter-inch and one-inch graph paper. (You may duplicate the sheets in the back of the book or purchase commercial graph paper. Stationery stores sell quarter-inch graph paper with heavier lines on the inch.) Avoid graph paper with squares smaller than one-quarter inch, as many students have trouble working on squares that small.

Organizing Students

While each student should have his or her own set of pentominoes, you should let students share the results of individual work with each other. You may use a bulletin board to post solutions to particularly challenging assignments, and to class-wide projects.

Teacher Notes

The Teacher Notes section that follows includes general comments about each topic as well as comments and suggestions specific to certain activities.

Approach to Solving Difficult Problems

Some of the problems are extremely difficult. There are two ways to deal with this. One approach is to let frustrated students move on to another lesson and return to the tough problems later. You may use part of your pentomino bulletin board to display unsolved problems as a reminder and an incitement to keep at it. On the other hand, you may let students look up hints in the back of the book. Every problem or question in the activity pages that has a corresponding hint in the Hint section is marked with a  . That way, you will know which problems have hints, should you want to use them. This is particularly appropriate for younger or less motivated students

Solutions

For most problems in this book, there is not just one correct answer. Often, there are several solutions—sometimes, even hundreds. The Solutions section provides one answer for most problems. However, answers for some problems are not given, as students can readily work them out on their own. Your students will find their own collection of solutions. In minimum-maximum problems, your students need not hold the world record. Let them struggle to get the best answer possible at their level and then strive to beat that.

Other Resources

This book is designed stand on its own. However, it works best if used in conjunction with other polyomino and pentomino resources. For free, downloadable related puzzles and activities, see these pages on the Math Education Page website:

http://www.MathEducationPage.org/puzzles

http://www.MathEducationPage.org/geometry-labs

Keeping Track of Student Work

It is essential that students keep track of their work and results. They may draw answers to puzzles in the spaces provided on each page, or if there is no room, use pieces of graph paper. Make sure each student has a folder for his or her work.

Correlation to Curriculum Standards

Students are expected to engage with geometry throughout their school career. Developing some visual sense is foundational to this work, and geometric puzzles are one excellent way to do that. Language such as the following is ubiquitous in the Common Core Standards for Mathematics: *analyze, compare, create, and compose shapes; partition shapes into parts; combine components; rearrange pieces;* and so on.

The activities in this book meet the following Standards for Mathematical Practice found in the Common Core State Standands:

1. **Make sense of problems and persevere in solving them.**

   This practice is at the heart of *Working with Pentominoes*, as pentominoes provide a mathematical environment that offers challenges at many levels, gently helping students develop greater and greater perseverance.

2. **Construct viable arguments and critique the reasoning of others.**

   A number of activities provide excellent opportunities for this practice—for example, the pentomino rectangle area lesson on pages 14–16.

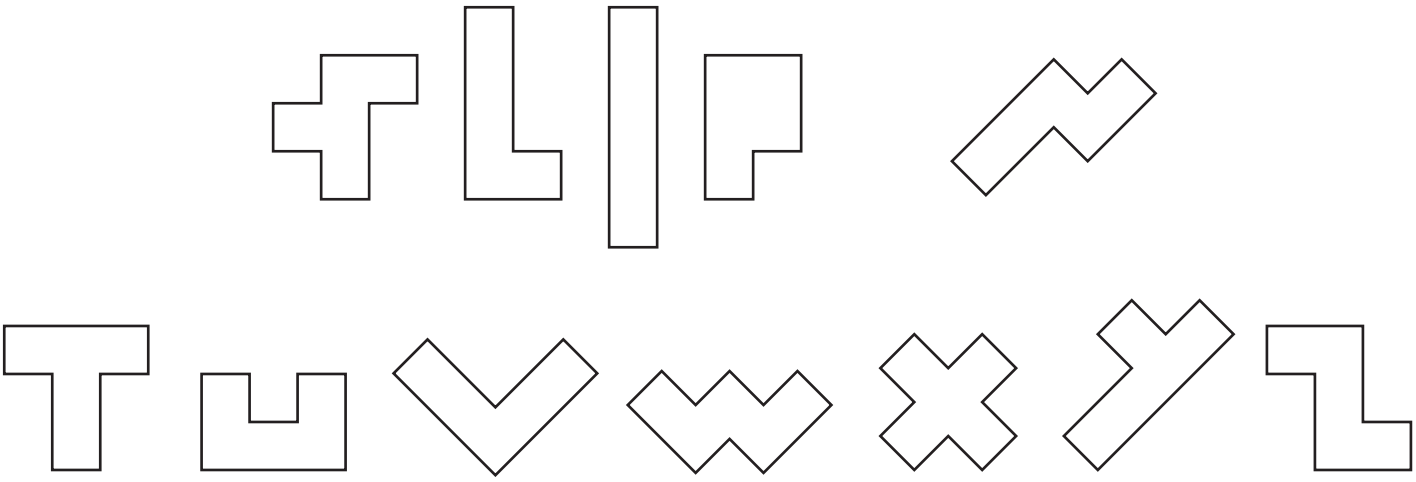
For a complete correlation of the activities in this book to the Common Core State Standards, visit:

www.didax.com/211340

Meet the Pentominoes

These are the 12 pentominoes. In order to be able to discuss pentomino problems, you will need to know the name of each piece. Learn this sequence of letters:

F L I P 'N'



T U V W X Y Z

It will help you check to see whether any pentominoes are missing from your set.

1. Draw the pentominoes on a piece of unlined paper. (Make freehand drawings; no tracing or rulers allowed.)
2. Now draw the pentominoes on the grid below. One has been done for you.

You will record solutions to many pentomino problems on grids like this one.

Measure the Pentominoes

Take out your set of pentominoes. Use a ruler to measure some pentomino sides.

1. Are the sides whole numbers of inches? _____

2. Are the pentominoes whole numbers of centimeters? _____

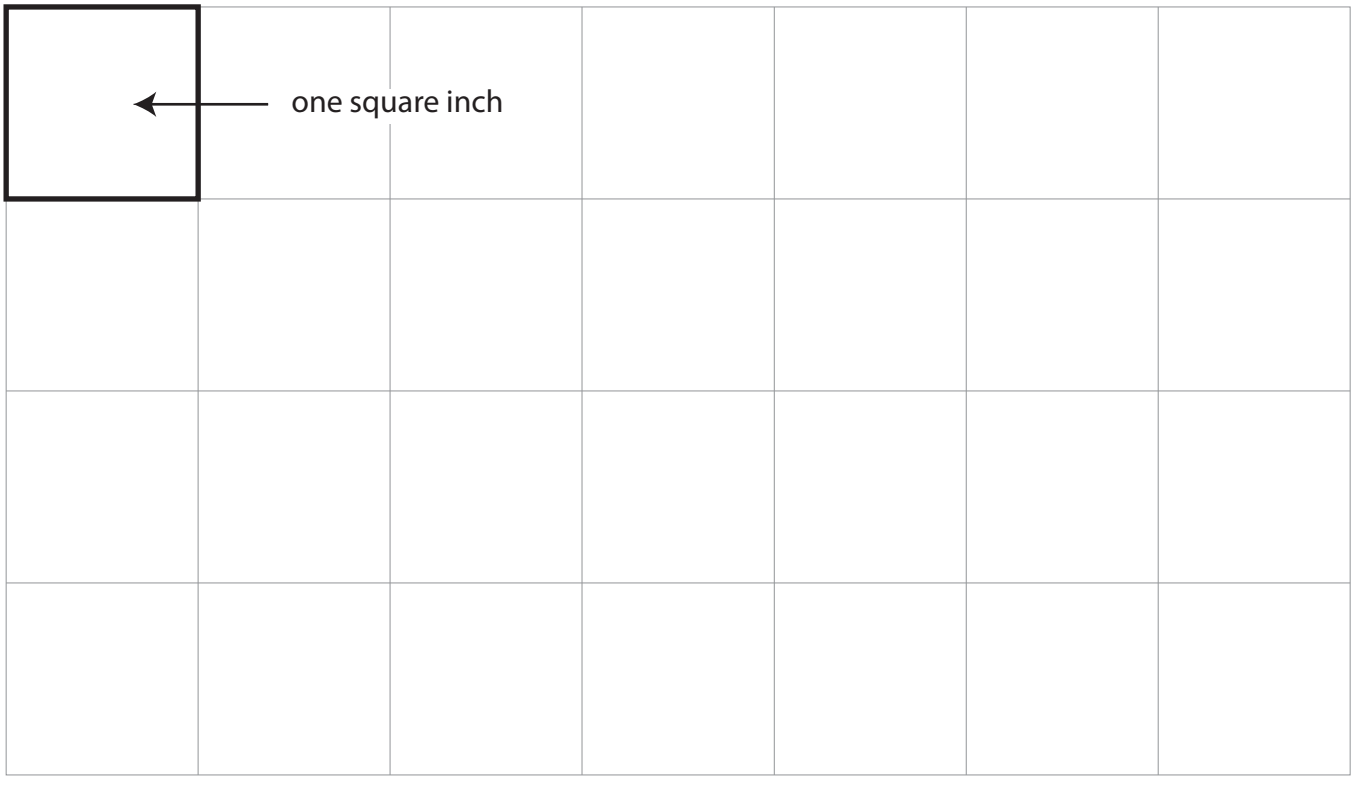
3. Is it easier to measure pentomino sides in inches or centimeters? _____

4. How long is the longest pentomino side? _____

5. How long is the shortest pentomino side? _____

6. What is the area of each pentomino in square inches? In other words, how many one-inch squares does it cover? Use the square-inch grid below to help you figure it out. Write the area of each pentomino in this table:

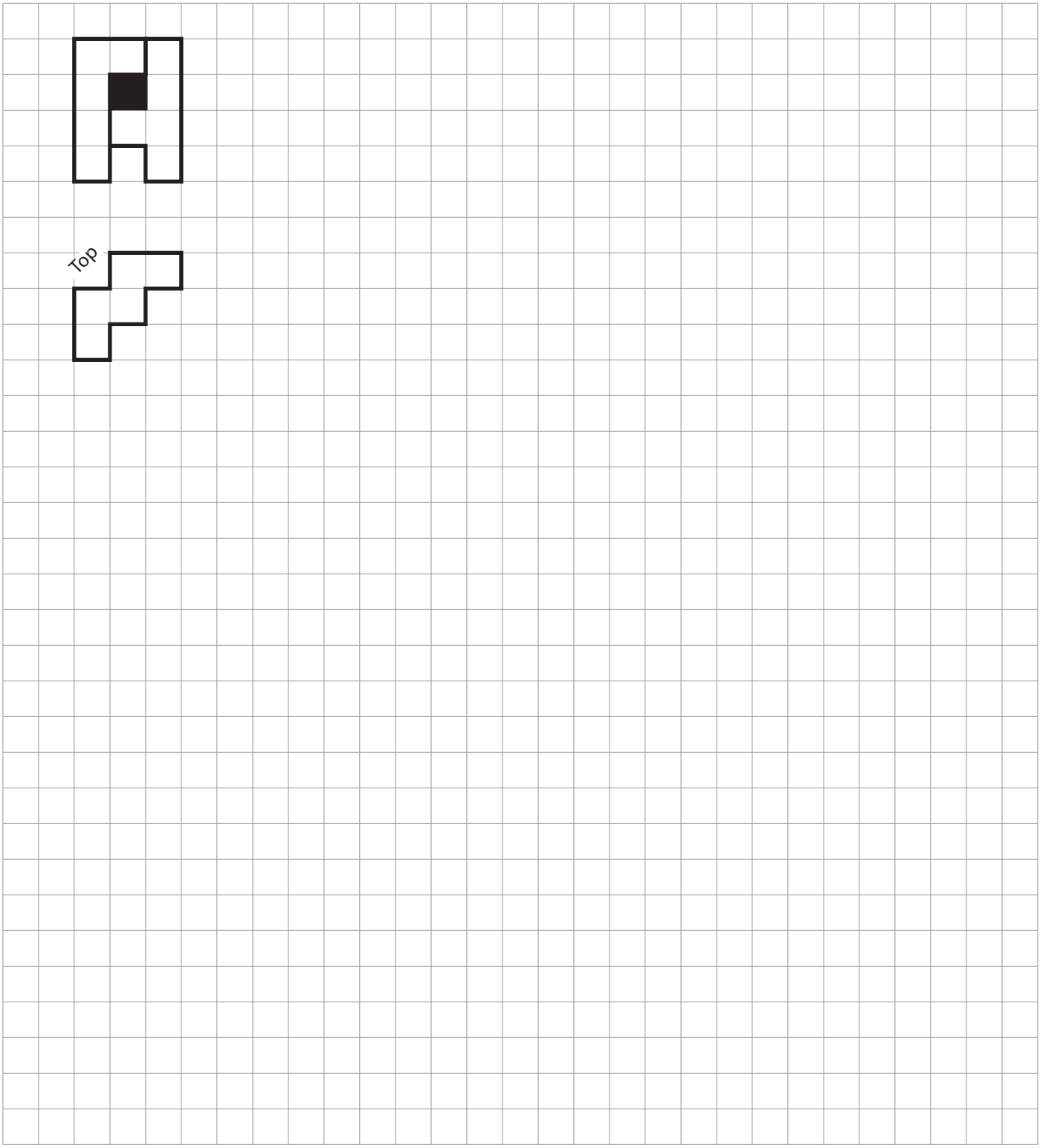
F	L	I	P	N	T	U	V	W	X	Y	Z
____	____	____	____	____	____	____	____	____	____	____	____



7. Which pentomino has the greatest area? _____

Pentomino Characters

Here are the letter A, made from the L and Y pentominoes, and the letter M, made with the W. Use one or more pentominoes to make each letter of the alphabet. Record your solution on the grid.



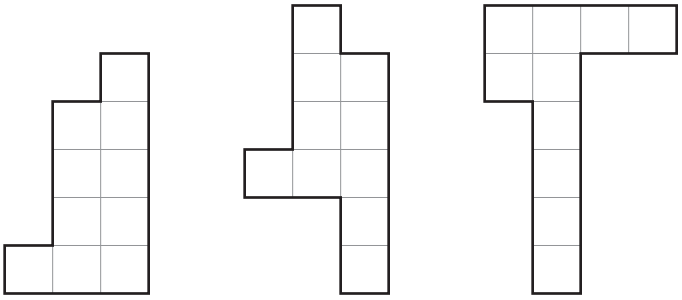
Perimeter

1. Find the perimeter of each pentomino in inches. Write it in this table.

F	L	I	P	N	T	U	V	W	X	Y	Z
____	____	____	____	____	____	____	____	____	____	____	____

2. Which pentomino has the shortest perimeter? _____

3. What is the perimeter of each of these two-pentomino figures?



_____ _____ _____

4. On the grid below make a two-pentomino design with the shortest perimeter possible. Perimeter: _____

5. Now make a two-pentomino design with the longest perimeter possible. The pentominoes must touch on at least one side—that is, share at least one unit of perimeter.

Perimeter: _____

Perimeter (cont.)

6. Find the perimeters of various pentomino figures. Finish filling out this table. Record any solutions you want to on the grid below or on graph paper.

Number of Pentominoes	Minimum Perimeter	Maximum Perimeter
1	10	12
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12