Grades
6–12

WORKING WITH ALGEBRA TILES

2nd Edition

Integer Operations • Algebraic Expressions • Equation Solving

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CONTENTS

Welcome to
Working with Algebra Tiles, 2nd Edition

Algebra Tiles are a versatile manipulative that can be used by students to represent algebraic concepts beginning with integer arithmetic, continuing with activities involving linear expressions and equations, and ending with factoring and equation-solving for quadratics.

What's New in the Second Edition?

Building on the success of *Working with Algebra Tiles*, this newly revised edition has been updated to reflect the current mathematics standards for Pre-Algebra and Algebra I. Several activities have been added to focus on areas that often cause difficulty for students.

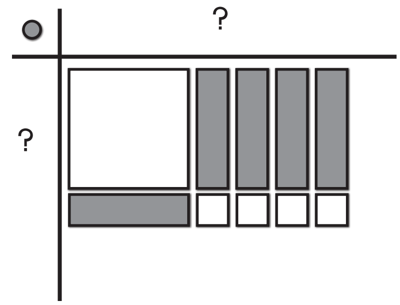
For example, solving linear inequalities when the numerical coefficient of the variable term is negative creates confusion for many:

- Why does the sign change?
- Why does the direction of the inequality change?

When students create visual models of these processes with Algebra Tiles, they acquire a deep understanding of algebraic concepts.

Unit 5: Quadratic Expressions has been expanded to cover special products of binomials; $(a - b)^2$, $(a + b)^2$, and $(a + b)(a - b)$. Also new are activities involving factoring of $ax^2 + bx + c$ and $ax^2 - c$. As students progress towards discovering and using the quadratic equation, they encounter the topic of completing the square of a quadratic. Again, Algebra Tiles lead students to a deeper understanding of this process.

A new Corner Frame mat has been added to assist students as they use Algebra Tiles to model multiplication of binomials and factor quadratic expressions.



In the illustration above, students create a rectangle with the tiles and the Corner Frame to represent the expression $x^2 - 5x + 4$. They then use the tiles to determine the factors. (See Activity 5.7: Factoring $x^2 + bx + c$.)

Using Visual Models for Greater Understanding

The activities and games in *Working with Algebra Tiles, 2nd Edition* offer teachers additional strategies to be used in conjunction with the curriculum. Students develop algebraic thinking and attach visual meanings to the concepts by modeling them with tiles.

Proficiency in algebra is necessary for success in all higher-level mathematics and for many career paths, regardless of whether a student chooses to move on to college.

By using *Working with Algebra Tiles, 2nd Edition,* your students will gain a much deeper understanding of algebraic concepts and will be better prepared for success in their careers and in life!

Correlation to the Math Standards

CCSS Standard	Description	Activities
6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	1.1, 2.3, 2.4
6.NS.6a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.	1.1, 2.3, 2.4
6.EE.2a	Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as $5 - y$.	3.1, 3.2
6.EE.2c	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	3.4, 3.5, 5.2
6.EE.3	Apply the properties of operations to generate equivalent expressions.	5.3
6.EE.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).	3.3, 5.4, 5.5
6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	5.6
7.NS.1a	Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.	1.3
7.NS.1b	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	2.2
7.NS.1c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	2.5, 2.6, 2.7
7.NS.1d	Apply properties of operations as strategies to add and subtract rational numbers.	2.7, 2.8, 2.9, 2.10, 6.2
7.NS.2a	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	3.6

CCSS Standard	Description	Activities
7.NS.2c	Apply properties of operations as strategies to multiply and divide rational numbers.	6.2
7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	4.1, 5.7
7.EE.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	4.2
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	4.4, 4.5
8.EE.7b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	3.7, 3.8, 3.9, 4.3, 5.3
A.REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
A.REI.4	Solve quadratic equations in one variable.	5.1, 5.2, 5.3, 5.4, 5.5, 5.6
A.REI.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	5.7, 5.8, 5.9, 5.10, 6.1, 6.3

UNIT 1

Introduction to Algebra Tiles

Unit Overview

Areas of squares and rectangles are key to naming and working with Algebra Tile pieces. Beginning with the small yellow/red squares, it is common to denote the length of each side as 1. Therefore, the area is 1 square unit. The value of a yellow square is 1, whereas the value of a red square is –1. The blue/red square customarily has sides of length x. Its area is x^2 square units. The value of a blue square is x^2, whereas the value of a red square is $-x^2$. The dimensions of a green/red rectangle are 1 unit by x units, and its area is x square units. Activities in the unit first involve representing integers using the small squares, and then proceed to representing and writing algebraic expressions using the Algebra Tiles. The Zero Principle is introduced and used throughout this unit and future units.

Glossary of Terms

Integers: The set of numbers {. . . –3, –2, –1, 0, 1, 2, 3, . . .}

Opposites: Two numbers that have the same absolute value but opposite signs.

Zero Pair: Two tiles, one of each color. A zero pair sums to zero.

Variable: A symbol, usually a letter, that is used to represent one or more numbers in an algebraic expression. For example, x is a variable in the expression $4x + 3$.

Polynomial: An expression that has one or more terms of the form ax^n where a is any real number and n is a whole number.

Binomial: A polynomial that has two terms.

Trinomial: A polynomial that has three terms.

Term: A part of an expression that is separated by an addition or subtraction sign. In the expression $2x - 4$, the terms are $2x$ and -4.

Activity 1.1 Integer Pieces and the Zero Principle

Objectives: To represent positive and negative integers using Algebra Tiles. To represent the opposite of an integer using Algebra Tiles. To represent zero pairs using Algebra Tiles. To model integers using zero pairs.

Prerequisites:

A. Students need to know that the area of a square is given by $A = s^2$, where s is the length of a side.

B. Students need to be able to represent positive and negative integers using Algebra Tiles.

Getting Started:

A. Introduce the unit squares. Discuss the idea of unit length, so that the area of the square is 1 square unit. Discuss the idea of a negative integer. Note that we can use the yellow unit squares to represent positive integers and the red unit squares to represent negative integers.

B. Show students how two small squares of opposite colors (yellow and red) neutralize each other, so that the net result of such a pair is zero.

C. A positive integer can be represented by other positive integers in many ways.

For example, 6 = 3 + 3 = 5 + 1 = 4 + 2.

Similarly, positive and negative integers can be represented by other positive and negative integers in many ways. Start with a positive integer such as 2. Add zero pairs and discuss the new name for 2. Continue this process several times. Repeat the activity; however, begin with a negative integer and add zero pairs.

Closing the Activity:

A. Present verbal descriptions to students and have them model the result with Algebra Tiles.

B. Present various collections of unit squares, positive and negative, and have students use the Zero Principle to simplify the collection. Given that there are an odd number of unit squares, have students determine possible outcomes: all yellow, positive integer; all red, negative integer; more yellow than red, positive integer; more red than yellow, negative integer.

C. Have students sketch more than one model for a given positive or negative integer. Give students an illustration of several positive and negative integers using Algebra Tiles, and have them name the integer represented in the model.

Answers:

1. 6 yellow squares
2. 4 red squares
3. Answers vary.
4. Answers vary.
5. 8 red squares
6. 10 red squares
7. 4 + (–4); 7 + (–7)
8. (a) 2; (b) –6; (c) 4; (d) 3; (e) –4; (f) 1..........
9. (a) – (f) Teacher check
10. Infinite. Starting with 4 yellow tiles, as many zero pairs can be added as you wish.
11. Infinite. Starting with 3 red tiles, as many zero pairs can be added as you wish.

Activity 1.2 Naming Algebra Tile Pieces

Objective: To name the remaining Algebra Tile pieces (x, x^2).

Prerequisites: An area model is used in naming Algebra Tile pieces. Students must know that the area of a square is given by $A = s^2$, where s is the length of a side. Students must know that the area of a rectangle is given by $A = lw$, where l is the length and w is the width of the rectangle.

Getting Started: Using a document camera, show the other Algebra Tile pieces. Show that the length of a side of the blue/red square is not an integer value. We will say that the length is x. Therefore, the area is x^2 square units. The blue square has a value of x^2, whereas the red square has a value of $-x^2$. The green Algebra Tile piece has a value of x, and the red Algebra Tile piece has a value of $-x$. Using a document camera and a collection of Algebra Tiles, write an algebraic expression for the collection. Then write an algebraic expression and have students model it with the tiles.

Closing the Activity: The activity concludes with change in the length of a side of the small square. With a side of length y, the area now becomes y^2 square units. The value of a yellow square is y^2, and the value of a small red square is $-y^2$. Expressions involving two variables can be written.

Answers:

1.–4. Teacher check

5. $x^2 + 3xy + 4y^2$
6. $2x^2 + (-2xy) + y^2$
7. $-3x^2 + 4xy + (-3y^2)$
8. 4 blue a^2 pieces, 2 red ab pieces, 2 yellow b^2 pieces

Activity 1.3 Expression Grab

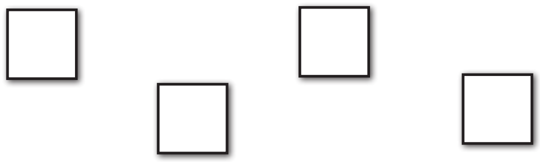
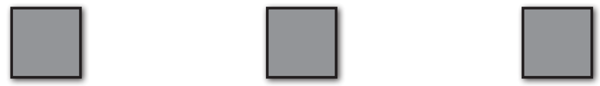
Objective: To practice writing and simplifying algebraic expressions, using the Zero Principle.

Prerequisites: Students must know how to apply the Zero Principle in order to simplify algebraic expressions.

Getting Started: Using Algebra Tiles with a document camera, illustrate how to simplify an algebraic expression, using the Zero Principle where necessary.

Closing the Activity: Show illustrations of Algebra Tile pieces and have students write a simplified expression. Write algebraic expressions and have students write simplified expressions.

To represent integer values, use the small yellow and red square tiles.

 **Each yellow square represents +1.**	 **Each red square represents –1.**

Use the small square tiles to represent each number. Make a sketch of your model.

1. 6	2. –4
3. A single digit odd number	4. An even prime number
5. 10 feet below sea level	6. 5 degrees below zero

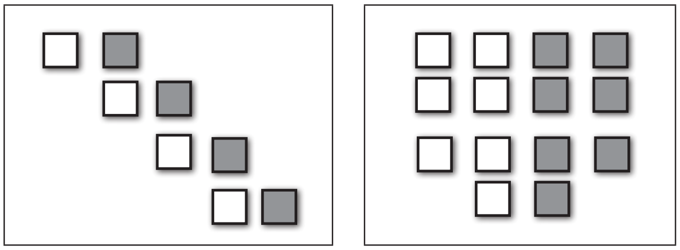
The numbers +1 and –1 *are called* ***opposites****. They sum to* 0*. That is* 1 + (–1) = 0*. We refer to* 1 *and* –1 *as a* ***zero pair****. In an elevator, for example, if you go up one floor* (+1) *and then down one floor* (–1)*, you are back to where you started.*

When we add positive and negative integers, we often use what is called the ***Zero Principle*** *to perform the addition operation.*

The mats below show two basic examples of the *Zero Principle*. We make pairs of yellow and red tiles.

 **1 + (–1) = 0**	 **2 + (–2) = 0**

7. Write an equation for each diagram.



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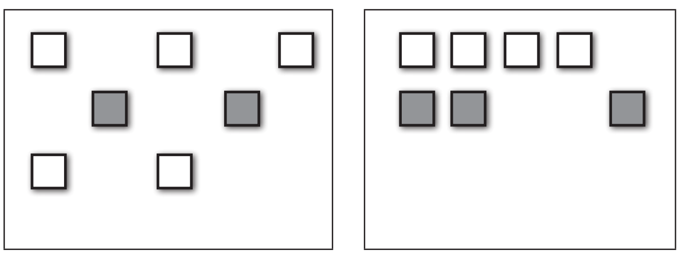
We can use the Zero Principle when we model addition of integers with the tiles. We make zero pairs when possible, and then determine the final sum.

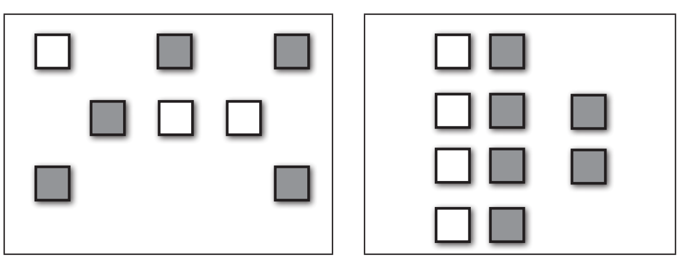
The integer 3 is modeled in each diagram below.



If the two zero pairs are removed, three yellow tiles remain, or positive 3.

We could write an equation:
$5 + (-2) = 3$ or $(-2) + 5 = 3$.

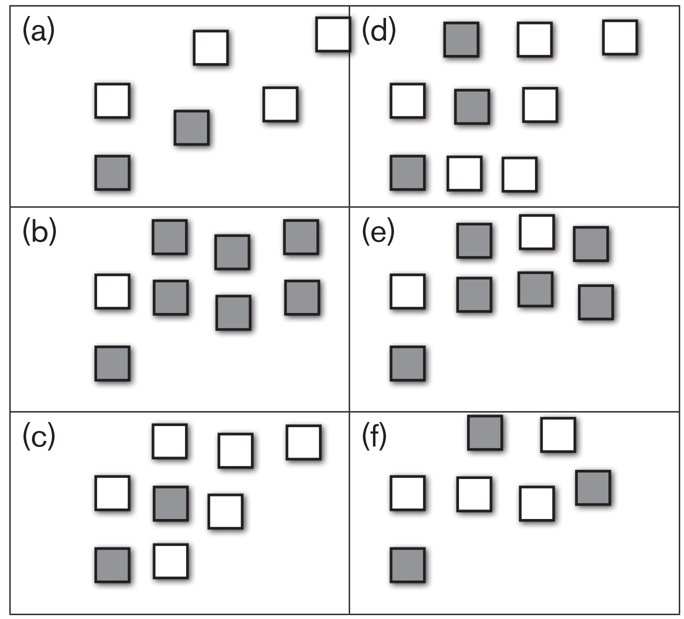
The integer −2 is modeled in the diagrams below.



If the three zero pairs are removed, two red tiles remain, or negative 2.

We could write an equation:
$(-5) + 3 = -2$ or $3 + (-5) = -2$

8. Name the integer modeled in each of the following diagrams. Write an equation for each.



9. Create a model for each integer using yellow and red tiles. Write an equation for each.

(a) 4	(d) 5
(b) −3	(e) −6
(c) −1	(f) 2

10. How many different models are there for the integer 4? Explain.

____

11. How many different models are there for the integer −3? Explain.

____

Integer Operations

Unit Overview:

In Unit 2, Algebra Tiles are used to model integer arithmetic for addition, subtraction, and multiplication. Ideas about zero pairs from Unit 1 are incorporated in **Activities 2.1–2.10.** Subtraction of integers is related to addition of integers. Games are included that provide practice on integer addition and subtraction. Patterns or rules for each operation are highlighted. For multiplication of positive and negative integers, the generalized form $a \times b$ in terms of "*a*" groups of "*b*" squares is presented. Note: Work mat is on page 9.

Glossary of Terms:

Commutative Property of Addition: For any integers *a* and *b*, $a + b = b + a$.

Activity 2.1 Modeling Addition

Objective: To model addition of positive and negative integers using Algebra Tiles.

Prerequisites: Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: Pose basic addition problems such as 2 + 3 = ? and use Algebra Tiles to model it. Then pose addition problems involving negative integers and explain how Algebra Tiles can be used to model and solve these problems using zero pairs.

Closing the Activity: Pose addition problems different than those on the sheet. Have individual students model and solve the problems with Algebra Tiles on their mats, or by using a document camera if available.

Answers:

1. 3 + (–2) = 1
2. (–4) + (–1) = (–5)
3. (–5) + 4 = (–1)

4.–6. Teacher check

Activity 2.2 Addition of Integers

Objective: To add positive and negative integers. To use additive inverses and the additive identity to find sums of positive and negative integers.

Prerequisites: Students must be able to model addition of positive and negative integers using Algebra Tiles.

Getting Started: Review the additive identity property for addition of whole numbers. Review the idea of the opposite or additive inverse of an integer.

Closing the Activity: Review the additive identity property for addition of integers. Discuss the sign patterns involved in adding integers:

> The sum is positive if the signs of both integers are positive.
>
> The sum is negative if the signs of both integers are negative.
>
> The sum is positive if the positive integer is greater than the absolute value of the negative integer.
>
> The sum is negative if the absolute value of the negative integer is greater than the positive integer.

Present problems illustrating each case. Have students model them with Algebra Tiles and determine the sums.

Answers:

1. 0
2. 0
3. 0
4. (–5)
5. (–3)
6. 8
7. opposites
8. the number
9. 9
10. (–8)
11. 12
12. (–10)
13. 12
14. (–13)
15. Suggested Answer: Use the sign of the two numbers and add the absolute values.

16. 633
17. (–655)
18. (–365)
19. 3
20. 4
21. 2
22. 4
23. 3
24. 3
25. Answers vary – When there are more yellow pieces than red pieces the answer will be positive.
26. (–5)
27. (–8)
28. (–3)
29. (–4)
30. (–5)
31. (–3)
32. Answers vary – When there are more red pieces than yellow pieces, the answer will be negative.
33. N
34. P
35. N
36. Answers vary – If there are more yellow pieces, the sum is positive. If there are more red pieces, the sum is negative. In each case, remove the zero pairs and find the number of pieces that remain.
37. 35
38. 133
39. (–6)

40.–43. Teacher check

Activity 2.3 Three Addends

Objective: To find the sum of three or more integers.

Prerequisites: Students must be able to find the sum of two integers.

Getting Started: Provide an example for addition with three positive integers, showing that we can add any two at a time to find the sum. Then, provide examples with three addends where one or more of the addends are negative integers.

Closing the Activity: When adding three or more integers, we can add two at a time in any order. The patterns or rules established in Activity 2.2 then apply.

Answers:

1. 3
2. (–11)
3. (–1)
4. 11
5. (–1)
6. (–2)
7. Answers vary.
8. 188
9. (–143)

Activity 2.4 The Game of One

Objective: To provide practice on adding positive and negative integers in a game format.

Prerequisites: Students must know how to add positive and negative integers. Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: To play the Game of One each student needs 11 unit squares, the game mat, and a Game of One Record Sheet.

Closing the Activity: Have students play several games, closest to 1. Change rules so that the player with the greatest sum (or least sum) is the winner.

Activity 2.5 Modeling Subtraction

Objective: To model subtraction of two integers using Algebra Tiles.

Prerequisites: Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: Model basic subtraction problems with Algebra Tiles. Then model a subtraction problem where the known subtrahend is a negative integer, such as 2 – (–3).

Discuss adding zero pairs to the model.

Closing the Activity: Subtraction of two integers using Algebra Tiles employs the "take–away" language from primary grades. If it is impossible to "take–away" a positive integer or a negative integer, then zero pairs must used.

Answers:

1. 6 – 3 = 3
2. (–8) – (–5) = (–3)
3. (–5) – (–4) = (–1)
4. 6 red squares with 4 circled to take away; (–2)
5. 3 red squares with 2 circled to take away; (–1)
6. 5 yellow squares with 3 circled to take away; 2

7. (–2) – 4 = (–6)
8. 5 – (–6) = 11
9. (–3) – (–5) = 2
10. 4 – (–2) = 6

Activity 2.6 Subtraction and Addition Patterns

Objective: To discover the patterns relating subtraction of integers to addition of integers.

Prerequisites: Students must understand how to add and subtract integers. Students must understand how to model addition and subtraction of integers using Algebra Tiles.

Getting Started: Have students model an addition problem such as –3 + 1 and a related subtraction problem –3 – (–1) to compare answers.

Closing the Activity: Discuss how subtracting a negative integer is the same as adding a positive integer. Provide closure examples such as:
5 – (–3) = 5 + 3 or –2 – (–4) = –2 + 4.
Discuss the various patterns that occur in subtraction of integers.

Answers:

1. 4 red squares with 2 circled to take away;
   (–4) – (–2) = (–2)
2. 4 red squares plus 2 yellow squares;
   (–4) + 2 = (–2)
3. 6 yellow squares, 5 zero pairs added, and 5 red squares circled to take away; 6 – (–5) = 11
4. 6 yellow squares plus 5 yellow squares;
   6 + 5 = 11
5. 3 red squares, 6 zero pairs added, and 6 red squares circled to take away;
   (–3) – (–6) = 3
6. 3 red squares plus 6 yellow squares;
   (–3) + 6 = 3
7. 2 yellow squares, with 7 zero pairs added, and 7 yellow squares circled to take away;
   2 – 7 = (–5)
8. 2 yellow squares plus 7 red squares;
   2 + (–7) = (–5)
9. The even-numbered problems are the addition form for the odd-numbered subtraction problems; (–5) – 4 = (–5) + (–4) = (–9)
10. Subtracting two integers is the same as adding the opposite of the second integer: $a - b = a + (-b)$

Activity 2.7 Subtraction of Integers

Objective: To use the patterns discovered in Activity 2.6 to rewrite subtraction problems as addition problems.

Prerequisites: Students must be able to add and subtract positive and negative integers.

Getting Started: Review the patterns discovered in Activity 2.6.

Closing the Activity: Have students describe the relationship between addition and subtraction problems. Discuss commutativity of integers under addition and under subtraction.

Answers:

1. –6 + (–7) = (–13)
2. 4 + 9 = 13
3. –7 + 3 = –4
4. –8 + (–7) = (–15)
5. 8 + (–11) = (–3)
6. 3 + (–8) = (–5)
7. 5 + 8 = 13
8. 13 + 5 = 18
9. –5 + 9 = 4
10. –12 + 4 = (–8)
11. (a) 12
    (b) (–12)
12. (a) (–4)
    (b) 4
13. (a) (–12)
    (b) 12
14. The numbers for each problem are the same, but the order in which the numbers are subtracted is changed. The absolute value of the answers are equal, but the signs are opposite.
15. No. The order in which subtraction is performed does matter. If $a - b$ is positive, then $b - a$ is negative.

Activity 2.8 Least Is Best

Objective: To provide practice on subtracting positive and negative integers.

Prerequisites: Students must be able to model subtraction of integers and relate subtraction to addition.

Getting Started: Each group of players needs several yellow/red squares, a work mat, a Least Is Best spinner sheet, a spinner, and a score sheet. Read the directions to all students. Either model the example given in the directions or create a different example.

Closing the Activity: When all groups have finished a game, record student scores on the board. Discuss least and greatest scores for each group. Have students determine a class winner.

Activity 2.9 Modeling Multiplication

Objective: To model multiplication of positive and negative integers using Algebra Tiles.

Prerequisites: Students must understand that multiplication is repeated addition and that $a \times b$ means a groups of b squares.

Getting Started: Have students begin with a blank work mat. Pose a multiplication problem such as 2×3 (add in 2 groups of yellow tiles). Have students "add in" the squares. When the first factor is negative, the key to modeling multiplication is to remove the indicated number of groups. However, there are no squares on the work mat. Therefore, the appropriate number of zero pairs must be added to the work mat before any squares can be removed.

Closing the Activity: Pose a problem for each of the four cases for multiplication of integers. Have students describe the process used to multiply integers for each case.

Answers:

1. 3 × 3 = 9
2. 5 × (–2) = (–10)
3. 2 × (–6) = (–12)
4. 4 groups of 3 red squares; 4 × (–3) = (–12)
5. 2 groups of 6 yellow squares; 2 × 6 = 12
6. 3 groups of 5 red squares;
   3 × (–5) = (–15)
7. (–3) × (–3) = 9
8. (–2) × 2 = (–4)
9. (–1) × (–5) = 5

Activity 2.10 Multiplication of Integers

Objective: To develop the patterns or rules for multiplying positive and negative integers.

Prerequisites: Students should be able to model multiplication of integers using Algebra Tiles.

Getting Started: Review how to model multiplication of positive and negative integers using Algebra Tiles.

Closing the Activity: Discuss the rules for multiplication of signed numbers developed in the activity:

> The product of two integers with the same signs is positive.
> The product of two integers with different signs is negative.

Answers:

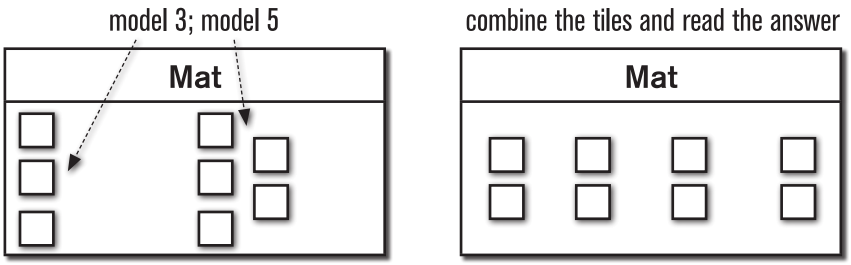
1. 18
2. 8
3. 7
4. (–15)
5. (–24)
6. (–12)
7. positive
8. negative
9. 6615
10. (–1500)
11. (–6106)
12. 20
13. 8
14. 9
15. (–24)
16. (–14)
17. (–8)
18. positive
19. negative
20. 322
21. 945
22. (–3026)
23. Teacher check
24. (–100)
25. 72
26. (–27)
27. 1000
28. (–8)
29. 16
30. Yes, if there is an odd number of negative signs, the product is negative. If there is an even number of negative signs, the product is positive.

Modeling Addition

Name:

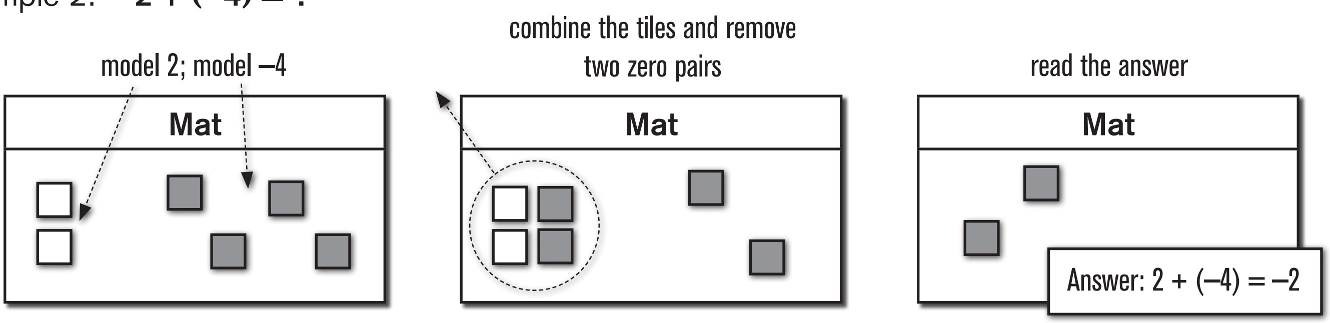
To add two integers, model each quantity on the work mat, combine the Algebra Tiles, remove any zero pairs, and read the answer.

Example 1: **3 + 5 = ?**



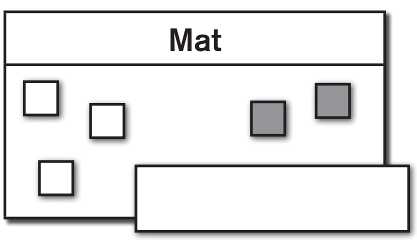
Answer: 3 + 5 = 8

Example 2: **2 + (–4) = ?**

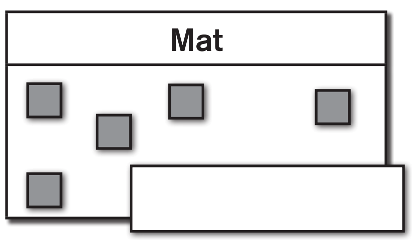


Write the addition problem modeled in each diagram. Use Algebra Tiles to determine the sum.

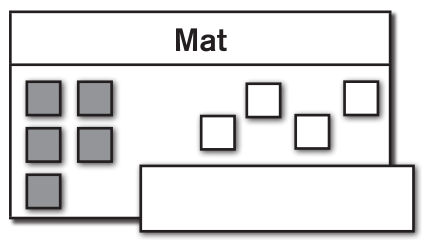
1.



2.

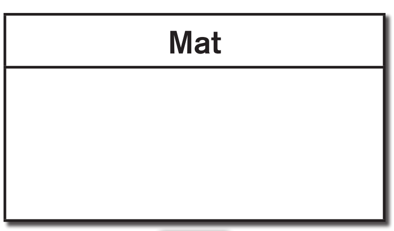


3.

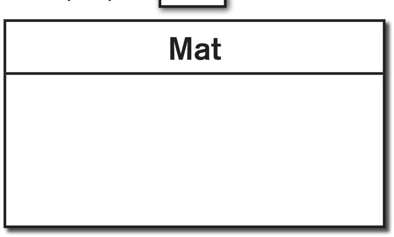


Use Algebra Tiles to determine the sum. Draw a diagram to model each addition problem. Circle any zero pairs. Use the second work mat to draw a diagram of the sum and record your answer.

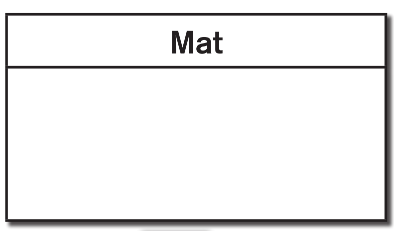
4. 4 + (–3)



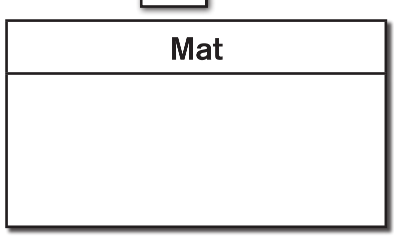
4 + (–3) = ☐



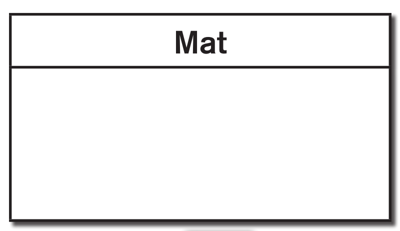
5. –6 + 3



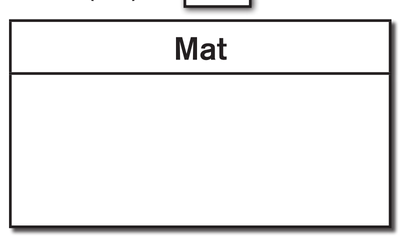
–6 + 3 = ☐



6. –3 + (–5)



–3 + (–5) = ☐



Addition of Integers

Name: ____________________

Use the Algebra Tiles and work mat to find the following sums.

1. $4 + (-4) =$ ______ 2. $-2 + 2 =$ ______ 3. $6 + (-6) =$ ______
4. $0 + (-5) =$ ______ 5. $-3 + 0 =$ ______ 6. $8 + 0 =$ ______

Concept Summary

7. If the sum of two integers is zero, then the two numbers must be ____________________.
8. The sum of zero and any number is ____________________.

Use the Algebra Tiles and work mat to find the following sums.

9. $6 + 3 =$ ______ 10. $-5 + (-3) =$ ______ 11. $7 + 5 =$ ______
12. $-6 + (-4) =$ ______ 13. $8 + 4 =$ ______ 14. $-4 + (-9) =$ ______

Concept Summary

15. To add two integers with the same signs, ____________________
____________________.

Use the rule you have written to find the following sums.

16. $415 + 218 =$ ______ 17. $-395 + (-260) =$ ______ 18. $-247 + (-118) =$ ______

Now consider the sum of two integers with different signs. Use the Algebra Tiles and work mat to find the following sums.

19. $5 + (-2) =$ ______ 20. $-5 + 9 =$ ______ 21. $6 + (-4) =$ ______
22. $-6 + 10 =$ ______ 23. $-2 + 5 =$ ______ 24. $8 + (-5) =$ ______
25. Describe any patterns that you observed in answering Questions 19 to 24. ____________________
____
____

Try a few more addition problems involving the sum of two integers with different signs. Use the Algebra Tiles and work mat to find the following sums.

26. $4 + (-9) =$ ______ 27. $-11 + 3 =$ ______ 28. $5 + (-8) =$ ______
29. $-10 + 6 =$ ______ 30. $-9 + 4 =$ ______ 31. $7 + (-10) =$ ______

UNIT 3

Linear Expressions

Unit Overview:

Activities in the first two units have focused on introducing Algebra Tile pieces and modeling integer operations with the tiles. Unit 3 is an extension of Unit 2; it provides activities that involve algebraic expressions–those that use the Algebra Tiles *x*-pieces.

Algebra vocabulary/terminology is highlighted in **Activity 3.1.** Students continue work with terminology in **Activity 3.2**, first spinning two monomials, then writing a symbolic and written expression for that result. Making matches between models and algebraic expressions is the focus of bingo and concentration games in **Activity 3.3.** Additionally, students find the additive inverses of algebraic expressions. To evaluate a linear expression using Algebra Tiles, the expression is modeled and then each *x*–tile is replaced with its value.

Activity 3.4 leads students through cases where *x* is positive and *x* is negative. Closest to Zero in **Activity 3.5** is a game for students to practice evaluating linear expressions. Students reach into a bag and grab Algebra Tiles, and then evaluate the corresponding expression with a given *x*-value. The distributive property developed in elementary and middle schools is extended to algebraic expressions in **Activity 3.6.** The focus in **Activity 3.7** is on adding like or similar terms, using the Zero Principle to simplify sums.

Activity 3.8 involves using Binomial Expression Cards, where students draw cards from a deck, model the linear expressions with Algebra Tiles, and simplify to find the sum. Finally, in **Activity 3.9,** students use Algebra Tiles to subtract linear expressions. The lesson contains two methods. In the first, students use the "take–away" model from elementary school, and in the second, they use the idea presented in Unit 2 that a subtraction problem can be rewritten as an addition problem. Note: Work mat is on page 9.

Glossary of Terms:

Distributive Property: For all integers *a*, *b*, and *c*, $a(b + c) = ab + ac$.

Like or Similar Terms: Terms with identical variable parts raised to the same power

Activity 3.1 Translating Algebraic Expressions

Objective: To translate from a word description of an algebraic expression to a model using Algebra Tiles.

Prerequisites: Students must know the names of the Algebra Tiles pieces. Students must be able to write an algebraic expression.

Getting Started: Provide an algebraic expression, such as $3x - 2$, for students to discuss. Write various word descriptions for the expression:

Three times a number *x* decreased by 2
A multiple of 3 minus 2
The difference between three times a number and 2
Three times a number less 2

Closing the Activity: Write an algebraic expression on the board or use a document camera. First, have a student model the expression with Algebra Tiles. Have students provide verbal descriptions of the expression. Second, write a verbal description on the board or use a document camera, then have students at their seats model the expression with tiles and write the corresponding algebraic expression.

Answers:

1. $2x - 1$
2. $-5 + 4x$
3. $2x + 3$
4. $-x + 4$
5. $-3x + 5$
6. $3x - 1$

Activity 3.2 Spin an Expression

Objective: To practice writing a word description for an algebraic expression expressed in symbols.

Prerequisites: Students must be able to model addition of positive and negative integers using Algebra Tiles.

Getting Started: Provide each student with a copy of the Algebra Tiles Spinner Sheet, a spinner, and Activity 3.2. Discuss the directions with students and provide an example such as $2x - 3$. Have students express word descriptions for the expression.

Closing the Activity: Review the additive identity property for addition of integers.

- Discuss the sign patterns involved in adding integers.
- The sum is positive if the signs of both integers are positive.
- The sum is negative if the signs of both integers are negative.

- The sum is positive if the positive integer is greater than the absolute value of the negative integer.
- The sum is negative if the absolute value of the negative integer is greater than the positive integer.

Activity 3.3 Expression Match

Objective: To practice matching an algebraic expression to its corresponding Algebra Tiles model.

Prerequisites: Students must be able to find the sum of two integers.

Getting Started: Provide an example of an algebraic expression such as $2x - 3$. Have students model the expression with Algebra Tiles, or have one student show the model with a document camera. Then reverse the process by showing a model of an expression using Algebra Tiles and have students write the corresponding expression. Designate one of the games to play in small groups, either a concentration game, a bingo game, or Zero.

Closing the Activity: Discuss the idea of additive inverse. Provide an example and have students write and model the inverse.

Activity 3.4 Evaluating Linear Expressions

Objective: To evaluate linear expressions using Algebra Tiles.

Prerequisites: Students must know how to add positive and negative integers. Students must understand the Zero Principle and using Algebra Tiles to form zero pairs. Students must be able to model algebraic expressions.

Getting Started: Model a particular linear expression with Algebra Tiles. State a positive value for x. Then show how to evaluate the expression using the tiles. In a similar fashion, show how to evaluate an expression with a negative x-value.

Closing the Activity: Have students use the tiles to evaluate an expression where the x-value is positive and where it is negative. Discuss how they might evaluate an expression without using Algebra Tiles. Discuss how to evaluate an expression when the x-value is a fraction or decimal.

Answers:

1. $3x - 3$; 9
2. $x + 5$; 4
3. $4x - 2$; -10
4. 0
5. -7
6. 9
7. -9
8. 9
9. -3
10. $-2x + 4$; -4
11. $-3x + 3$; 9
12. $-x - 3$; -1
13. -7
14. 6
15. 2
16. 6
17. 11
18. 4

Activity 3.5 Closest to Zero

Objective: To practice evaluating algebraic expressions.

Prerequisites: Students must understand the Zero Principle and using Algebra Tiles to form zero pairs. Students must be able to write an algebraic expression for a given Algebra Tiles model. Students must be able to evaluate an algebraic expression for a given x-value.

Getting Started: Using a document camera, have a student grab a handful of Algebra Tiles, and arrange them in place. Give a value for x. Have a different student evaluate the expression, either modeling with tiles or by substituting the value into the expression.

Closing the Activity: Evaluating an algebraic expression involves substituting a given x-value in the expression for x and then simplifying the result. Students must be able to complete this task with positive and negative numbers.

Activity 3.6 Distributive Property

Objective: To model the distributive property with Algebra Tiles.

Prerequisites: Students must be able to model algebraic expressions with Algebra Tiles.

Getting Started: Review the distributive property using counting numbers with examples such as $2(3 + 5) = 2(3) + 2(5)$. Show an algebraic expression such as $2(x + 2)$ and model it with tiles. Form the rectangular arrangement and then break into the two parts, $2x$ and $2(2)$.

Closing the Activity: Present various products of the forms $a(b + c)$ or $(a + b)c$. Have students model them with Algebra Tiles to find the product. Have students complete several without the tiles. Present forms involving subtraction, $a(b - c) = ab - ac$.

Answers:

1. $3(x + 2) = 3x + 6$
2. $2(2x + 1) = 4x + 2$
3. $3(x + 4) = 3x + 12$
4. $3x + 12$
5. $5x + 10$
6. $4x + 2$
7. $8x + 12$
8. $ab + ac$
9. $ab + ca$; multiplication is commutative, so $a(b + c) = (b + c)a$.

10. $2(x - 3) = 2x - 6$; there are two groups of the expression $x - 3$.

Activity 3.7 Adding Linear Expressions

Objective: To add linear expressions with like terms using Algebra Tiles.

Prerequisites: Students must be able to model algebraic expressions with Algebra Tiles. Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: Present an addition problem with like terms such as $(2x + 3) + (x - 2)$. Model the two expressions with Algebra Tiles and show how to add like terms, $(2x + x) + (3 + (-2))$.

Closing the Activity: Discuss how subtracting a negative integer or monomial expression is the same as adding a positive integer or monomial expression. Provide closure examples where students must work from a given algebraic expression or a model of one to construct addition problems with two or more addends, such as $(-5x - 4) + (3x + 7) = -2x + 3$.

Answers:

1. $6x + 2$
2. $2x - 1$
3. $-2x + 1$
4. $4x$
5. $(3x + 3) + (2x) = 5x + 3$
6. $(2x - 3) + (-4x) = -2x - 3$
7. $(-2x + 2) + (x - 4) = -x - 2$
8. $3x + 6$
9. $5x + 1$
10. $-x + 9$
11. $2x + 1$
12. 0
13. $-2x$
14. Possible answers: $(-x + 3) + (-x + 2)$; $(-3x + 4) + (x + 1)$

Activity 3.8 Find the Sum: Adding Linear Expressions

Objective: To practice adding linear expressions that have like terms.

Prerequisites: Students must be able to model algebraic expressions with Algebra Tiles. Students must understand the Zero Principle and using Algebra Tiles to form zero pairs. Students must be able to add linear expressions with like terms using Algebra Tiles.

Getting Started: Review the procedures from Activity 3.7 for adding linear expressions containing like terms. Have students work in groups of four. Give each group a set of Binomial Expression Cards. Have students shuffle the deck and place them facedown on a desk. On a turn, a student draws two cards, represents each binomial with Algebra Tiles, and finds the sum of the two expressions.

Closing the Activity: Have students describe how to add linear expressions with like terms. Extend the activity by using three or more expressions and finding the sum.

Activity 3.9 Subtracting Linear Expressions

Objective: To subtract linear expressions using Algebra Tiles.

Prerequisites: Students must be able to model algebraic expressions with Algebra Tiles. Students must understand the Zero Principle and using Algebra Tiles to form zero pairs. Students must be able to add linear expressions with like terms using Algebra Tiles.

Getting Started: Review the rules for subtracting positive and negative integers, relating those rules to subtracting expressions involving the Algebra Tiles x-pieces. Include examples that include terms such as $3x - 2x$, $-4x - x$, $x - 3x$, $-2x - (-x)$, and $-3x - (-4x)$. Extend the ideas to expressions involving constants, such as $(2x - 1) - (3x + 2)$.

Closing the Activity: Provide an example for students to model and solve with Algebra Tiles. Provide closure examples where students must work from a given algebraic expression or a model of one to construct subtraction problems with two or more terms, such as $-2x + 3 = (-5x - 4) - (-3x - 7)$. Have students write two expressions involving subtraction that models the difference that is given.

In groups of four, distribute the Binomial Expression Cards. Following the rules from Activity 3.8, have students participate in a similar activity, except they must subtract one expression from the other.

Discuss also commutativity. Are differences the same or different, depending on which binomial is subtracted?

Answers:

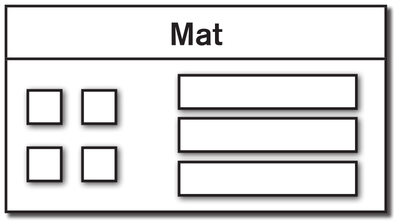
1. $3x - 4$
2. $-x + 8$
3. -8
4. $-5x + 3$
5. $5x - 7$
6. $-7x - 3$
7. Possible answers: $(3x + 1) - (2x + 4)$; $(2x - 4) - (x - 1)$

Translating Algebraic Expressions

Name:

Below are two different word descriptions of algebraic expressions and a representation of each using Algebra Tiles.

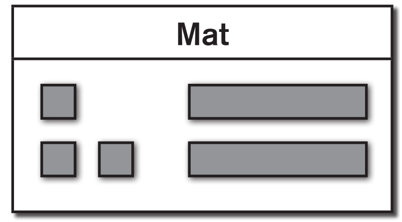
Three times a number increased by 4



What is an algebraic expression for this model?

____

Two times a number decreased by 3



What is an algebraic expression for this model?

____

Below are several word descriptions of algebraic expressions. On your work mat, use Algebra Tiles to model each expression. Sketch your model in the corresponding mat. Then write an algebraic expression for your model.

1. Two times a number x decreased by 1

   Mat

2. Opposite of 5, increased by four times a number x

   Mat

3. Three added to the double of a number x

   Mat

4. The opposite of x increased by 4

   Mat

5. Five added to the opposite of three times a number x

   Mat

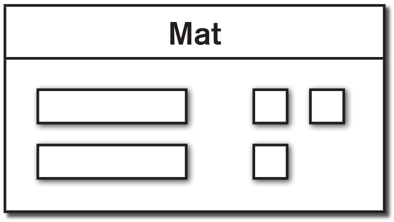
6. Triple of a number x decreased by 1

   Mat

ACTIVITY 3.2 **Spin an Expression**

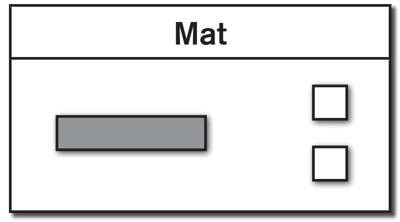
Name:

Understanding basic terminology of algebra is important as you learn more concepts. Look at the Algebra Tiles work mats illustrated below.



The work mat above contains a model of $2x + 3$ that we could describe by any of these expressions:

- 2 times x increased by 3
- $2x$ plus 3
- 3 more than 2 times x
- $2x$ increased by 3
- 2 times some number x added to 3



The work mat above contains a model of $2 - x$ or $-x + 2$ that we could describe by any of these expressions:

- 2 decreased by x
- the opposite of x plus 2
- negative x increased by 2
- 2 minus x
- 2 more than the opposite of x

Get a copy of the **Algebra Tiles Spinner Sheet**. Place your spinner on Dial 1 and spin. Record the term below. Place your spinner on Dial 2 and spin. Record the term. Write two different word descriptions of the binomial expression. Complete the table.

Symbolic Expression	Word Expression
	1. 2.
	1. 2.
	1. 2.
	1. 2.