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# Contents

<i>Introduction by Lynne Munson</i>	vii
<i>From the Writers by Cristina Metcalf</i>	ix
<i>Foreword by Scott Baldridge</i>	xi
<i>How to Use This Book</i>	xiii
<b>Chapter 1 Introduction to Eureka Math</b>	<b>1</b>
Vision and Storyline	1
Advantages to a Coherent Curriculum	2
<b>Chapter 2 Major Mathematical Themes in Each Grade Band</b>	<b>5</b>
Year-Long Curriculum Maps for Each Grade Band	5
Math Content Development for PreK–5: A Story of Units	5
How A Story of Units Aligns with the Instructional Shifts	10
How A Story of Units Aligns with the Standards for Mathematical Practice	14
<b>Chapter 3 Grade-Level Content Review</b>	<b>17</b>
Rationale for Module Sequence in Grade 3	18
<b>Chapter 4 Curriculum Design</b>	<b>25</b>
Approach to Module Structure	25
Approach to Lesson Structure	26
Approach to Assessment	36
<b>Chapter 5 Approach to Differentiated Instruction</b>	<b>39</b>
Scaffolds for English Language Learners	40
Scaffolds for Students with Disabilities	41
Scaffolds for Students Performing below Grade Level	43
Scaffolds for Students Performing above Grade Level	44
<b>Chapter 6 Grade-Level Module Summary and Unpacking of Standards</b>	<b>45</b>
Module 1: Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10	46
Module 2: Place Value and Problem Solving with Units of Measure	54
Module 3: Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10	61
Module 4: Multiplication and Area	72
Module 5: Fractions as Numbers on the Number Line	79
Module 6: Collecting and Displaying Data	86
Module 7: Geometry and Measurement Word Problems	90
<b>Chapter 7 Mathematical Models</b>	<b>103</b>
Array and Area Models	104
Base-Ten Blocks	105
Bundles	106

Money	107
Number Bond	108
Number Disks	110
Number Line	111
Number Path	113
Number Towers	114
Place Value Chart	115
Rekenrek	117
Tape Diagram	118
Ten-Frame	120
<b>Chapter 8 Terminology</b>	<b>123</b>
Grade PreK	123
Grade K	126
Grade 1	128
Grade 2	130
Grade 3	133
Grade 4	135
Grade 5	138
Notes	141
Board of Trustees	143
Eureka Math Study Guide: A Story of Units Contributors	145
Index	147

# How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

**Introduction to Eureka Math (chapter 1):** This introduction consists of two sections: “Vision and Storyline” and “Advantages to a Coherent Curriculum.”

**Major Mathematical Themes in Each Grade Band (chapter 2):** The first section presents year-long curriculum maps for each grade band (with subsections addressing *A Story of Units*, *A Story of Ratios*, and *A Story of Functions*). It is followed by a detailed examination of math concept development for PreK to Grade 5. The chapter closes with an in-depth description of how alignment to the instructional shifts and the standards of mathematical practice is achieved.

**Grade-Level Content Review (chapter 3):** The key areas of focus and required fluencies for a given grade level are presented in this chapter, along with a rationale for why topics are grouped and sequenced in the modules as they are. The Alignment Chart lists the standards that are addressed in each module of the grade.

**Curriculum Design (chapter 4):** The approach to modules, lessons, and assessment in *A Story of Units* is detailed in this chapter. It also provides a wealth of information about how to achieve the components of instructional rigor demanded by the new standards: fluency, concept development, and application.

**Approach to Differentiated Instruction (chapter 5):** This chapter describes the approach to differentiated instruction used in *A Story of Units*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

**Grade-Level Module Summary and Unpacking of Standards (chapter 6):** This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

**Mathematical Models (chapter 7):** This chapter presents information on the mathematical models used in *A Story of Units*.

**Terminology (chapter 8):** The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by grade level and module number where they are introduced in *A Story of Units*. The chapter also offers descriptions, examples, and illustrations associated with the terms.



# Grade-Level Content Review

The Grade-Level Content Review begins with a list of modules developed to deliver instruction aligned to the standards at a given grade level. This introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the Alignment Chart with the grade-level standards. The Summary of Year portion of each grade level contains four pieces of information:

- The critical instructional areas for the grade
- The Key Areas of Focus for the grade band.
- The Required Fluencies for the grade.
- The Major Standard Emphasis Clusters for the grade.

The Rationale for Module Sequence portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The Alignment Chart for each grade lists the standards addressed in each module of the grade. Throughout the alignment charts, when a cluster is included without a note, it is taught in its entirety; there are also times when notes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

The Grade-Level Content Review offers key information about grade-level content and provides a recommended framework for grouping and sequencing topics and standards.

## ***Sequence of Grade 3 Modules Aligned with the Standards***

Module 1: Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

Module 2: Place Value and Problem Solving with Units of Measure

Module 3: Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

Module 4: Multiplication and Area

Module 5: Fractions as Numbers on the Number Line

Module 6: Collecting and Displaying Data

Module 7: Geometry and Measurement Word Problems

## Summary of Year

Third-grade mathematics is about (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Key Areas of Focus for Grades 3–5:  
Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving.

Required Fluency:

3.OA.7 Multiply and divide within 100.

3.NBT.2 Add and subtract within 1,000.

## Major Standard Emphasis Clusters

### *Operations and Algebraic Thinking*

- Represent and solve problems involving multiplication and division.
- Understand the properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations and identify and explain patterns in arithmetic.

### *Number and Operations—Fractions*

- Develop understanding of fractions as numbers.

### *Measurement and Data*

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

## RATIONALE FOR MODULE SEQUENCE IN GRADE 3

Module 1 builds on the foundation of multiplicative thinking with units started in Grade 2. First, students concentrate on the meaning of multiplication and division and begin developing fluency for learning products involving factors of 2, 3, 4, 5, and 10 (see Key Areas of Focus and Required Fluency above). The restricted set of facts keeps learning manageable and provides enough examples to do one- and two-step word problems and to start measurement problems involving weight, capacity, and time in the second module.

Module 2 focuses on measurement of time and metric weight and capacity. In exploratory lessons, students decompose a kilogram into 100 gram, 10 gram, and 1 gram



# Index

Page references followed by *fig* indicate an illustrated figure.

- Accommodations: for English language learners (ELLs), 40–41; *Eureka Math* use of scaffolding and, 2; for students performing above grade level, 44; for students performing below grade level, 43–44; for students with disabilities, 41–43
- Action and expression: provide English language learners (ELLs) means of, 41; provide students performing above grade level, 44; provide students performing below grade level with means of, 43; provide students with disabilities means of, 42
- Addition: overview of the *A Story of Units* (PreK–5 grades) curriculum on, 9; standard algorithm used for two- and three-digit measurement, 54–55, 59–60; ten-frame module used for, 103, 120–121
- Algebraic thinking, 18
- Alignment Chart: description and function of, 17; Module 1: Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10, 20; Module 2: Place Value and Problem Solving with Units of Measure, 21; Module 3: Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10, 22; Module 4: Multiplication and Area, 22; Module 5: Fractions as Numbers on the Number Line, 22; Module 6: Collecting and Displaying Data, 23; Module 7: Geometry and Measurement Word Problems, 23
- Application Problem (lesson component): description and function of, 29; lesson order of the, 25, 26, 29; sample Lesson 1 (Grade 3 Module 2) on, 31*fig*, 32*fig*
- Application problems: description and strategies for, 29; Distribution of Instructional Minutes on, 27*fig*; sample Lesson 1 (Grade 3 Module 2), 31*fig*, 32*fig*
- Application rigor: description of, 13–14, 26, 27; dual intensity of, 13–14; goals of, 13
- Area measurement: applications of area using side lengths of figures, 73, 78; arithmetic properties using area models, 73, 76–77; concepts of, 72–73, 75–76; foundations for understanding, 72, 74–75; problem solving with perimeter and, 91, 100–101
- Area models, 103, 104–105
- Arithmetic patterns, 48
- Array model, 103, 104–105
- Assessment Summary, 26
- Assessments: Assessment Summary information on the, 26; Daily Formative, 36, 40; End-of-Module, 26, 37, 40; Mid-Module, 26, 37, 40; rigor in the, 37
- Bar graphs, 86, 87, 88
- Base-ten blocks model, 103, 105–106
- Bundles model, 103, 106–107
- Categorical data, 86, 88
- Clock (telling time), 31*fig*–32*fig*
- Coherence Links: Module 1: Properties of Multiplication and Division, 49, 50, 51, 52, 53; Module 2: Place Value and Problem Solving with Units of Measure, 57, 58, 59, 60, 61; Module 3: Multiplication and Division, 66, 67, 69, 70, 71, 72; Module 4: Multiplication and Area, 75, 76, 77, 78; Module 5: Fractions as Numbers on the Number Line, 81, 82, 83, 84, 85; Module 6: Collecting and Displaying Data, 88, 89; *A Story of Units* (PreK–5 grades), 6*fig*–7*fig*; Topic Overview on the, 26, 45
- Coherent curriculum: advantages of, 2–3; supported by use of concrete and pictorial models, 12, 103
- Commutative property, 47
- Concept Development (or Lesson): as component of instructional rigor, 12–13, 26; description and function of, 28–29; Distribution of Instructional Minutes on, 27*fig*; importance of, 27; sample Lesson 1 (Grade 3 Module 2), 31*fig*, 33*fig*–34*fig*
- Counting: group, 31*fig*, 32*fig*; minute, 32*fig*
- Curriculum: *A Story of Functions* (9–12 grades), 1, 2, 5, 9*fig*; *A Story of Ratios* (6–8 grades), 1, 2, 5, 8*fig*. *See also* *A Story of Units* (PreK–5 grades)
- Curriculum design: approach to assessment, 36–37; approach to lesson structure, 26–35*fig*; approach to module structure, 25–26
- Curriculum maps: *A Story of Functions* (9–12 grades), 5, 9*fig*; *A Story of Ratios* (6–8 grades), 5, 8*fig*; *A Story of Units* (PreK–5 grades), 5