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How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

Introduction to Eureka Math (chapter 1): This introduction consists of two sections: "Vision and Storyline" and "Advantages to a Coherent Curriculum."

Major Mathematical Themes in Each Grade Band (chapter 2): The first section presents year-long curriculum maps for each grade band (with subsections addressing A *Story of Units*, A *Story of Ratios*, and A *Story of Functions*). It is followed by a detailed examination of math concept development for PreK to Grade 5. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

Grade-Level Content Review (chapter 3): The key areas of focus and required fluencies for a given grade level are presented in this chapter, along with a rationale for why topics are grouped and sequenced in the modules as they are. The Alignment Chart lists the standards that are addressed in each module of the grade.

Curriculum Design (chapter 4): The approach to modules, lessons, and assessment in *A Story of Units* is detailed in this chapter. It also provides a wealth of information about how to achieve the components of instructional rigor demanded by the new standards: fluency, concept development, and application.

Approach to Differentiated Instruction (chapter 5): This chapter describes the approach to differentiated instruction used in *A Story of Units*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

Grade-Level Module Summary and Unpacking of Standards (chapter 6): This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

Mathematical Models (chapter 7): This chapter presents information on the mathematical models used in *A Story of Units*.

Terminology (chapter 8): The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by grade level and module number where they are introduced in *A Story of Units*. The chapter also offers descriptions, examples, and illustrations associated with the terms.

CHAPTER 3

Grade-Level Content Review

The Grade-Level Content Review begins with a list of modules developed to deliver instruction aligned to the standards at a given grade level. This introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the Alignment Chart with the grade-level standards. The Summary of Year portion of each grade level contains four pieces of information:

- The critical instructional areas for the grade.
- The Key Areas of Focus for the grade band.
- The Required Fluencies for the grade.
- The Major Emphasis Clusters for the grade.

The Rationale for Module Sequence portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The Alignment Chart for each grade lists the standards that are addressed in each module of the grade. Throughout the alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module, so that a strong foundation can be built over time.

The Grade-Level Content Review offers key information about grade-level content and provides a recommended framework for grouping and sequencing topics and standards.

Sequence of Grade 5 Modules Aligned with the Standards

Module 1: Place Value and Decimal Fractions

Module 2: Multi-Digit Whole Number and Decimal Fraction Operations

Module 3: Addition and Subtraction of Fractions

Module 4: Multiplication and Division of Fractions and Decimal Fractions

Module 5: Addition and Multiplication with Volume and Area

Module 6: Problem Solving with the Coordinate Plane

Summary of Year

Fifth-grade mathematics is about (1) developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Key Areas of Focus for Grades 3–5: Multiplication and division of whole numbers and fractions–concepts, skills, and problem solving.

Required Fluency: 5.NBT.5 Multi-digit multiplication.

Major Standard Emphasis Clusters

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations–Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

RATIONALE FOR MODULE SEQUENCE IN GRADE 5

Students' experiences with the algorithms as ways to manipulate place value units in Grades 2 to 4 really begin to pay dividends in Grade 5. In Module 1, whole number patterns with number disks on the place value table are easily generalized to decimal numbers. As students work word problems with measurements in the metric system, where the same patterns occur, they begin to appreciate the value and the meaning of decimals. They apply their work with place value to adding, subtracting, multiplying, and dividing decimal numbers with tenths and hundredths.

Module 2 begins by using place value patterns and the distributive and associative properties to multiply multi-digit numbers by multiples of 10 and leads to fluency with multi-digit whole number multiplication.[1] For multiplication, students must grapple with and fully understand the distributive property (one of the key reasons for teaching the multi-digit algorithm). Whereas the multi-digit multiplication algorithm is a straightforward generalization of the one-digit multiplication algorithm, the division algorithm with two-digit divisors requires far more care in teaching because students also have to learn estimation strategies, error correction strategies, and the idea of successive approximation (all of which are central concepts in math, science, and engineering).

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