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How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

Introduction to Eureka Math (chapter 1): This introduction consists of two sections: “Vision and Storyline” and “Advantages to a Coherent Curriculum.”

Major Mathematical Themes in Each Grade Band (chapter 2): The first section presents year-long curriculum maps for each grade band (with subsections addressing *A Story of Units*, *A Story of Ratios*, and *A Story of Functions*). It is followed by a detailed examination of math concept development for Grade 6 to Grade 8. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

Grade-Level Content Review (chapter 3): The key areas of focus and required fluencies for a given grade level are presented in this chapter, along with a rationale for why topics are grouped and sequenced in the modules as they are. The Alignment to the Standards and Placement of Standards in the Modules chart lists the standards that are addressed in each module of the grade.

Curriculum Design (chapter 4): The approach to modules, lessons, and assessment in *A Story of Ratios* is detailed in this chapter.

Approach to Differentiated Instruction (chapter 5): This chapter describes the approach to differentiated instruction used in *A Story of Ratios*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

Grade-Level Module Summary and Unpacking of Standards (chapter 6): This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

Terminology (chapter 7): The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by grade level and module number where they are introduced in *A Story of Ratios*. The chapter also offers definitions, descriptions, examples, and illustrations associated with the terms.

Grade-Level Content Review

The Grade-Level Content Review begins with a list of modules developed to deliver instruction aligned to the standards at a given grade level. This introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the Alignment to the Standards and Placement of Standards in the Modules chart. The Summary of Year portion of each grade level includes three pieces of information:

- The critical instructional areas for the grade
- The Key Areas of Focus for the grade
- The Major Emphasis Clusters for the grade

The Rationale for Module Sequence portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The Alignment Chart for each grade lists the standards that are addressed in each module of the grade. Throughout the Alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

The Grade-Level Content Review offers key information about grade-level content and provides a recommended framework for grouping and sequencing topics and standards.

Sequence of Grade 7 Modules Aligned with the Standards

Module 1: Ratios and Proportional Relationships

Module 2: Rational Numbers

Module 3: Expressions and Equations

Module 4: Percent and Proportional Relationships

Module 5: Statistics and Probability

Module 6: Geometry

Summary of Year

Grade 7 mathematics is about (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Key Areas of Focus for Grade 7: Ratios and proportional reasoning; arithmetic of rational numbers

Major Emphasis Clusters

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

RATIONALE FOR MODULE SEQUENCE IN GRADE 7

In Module 1, students build on their Grade 6 experiences with ratios, unit rates, and fraction division to analyze proportional relationships. They decide whether two quantities are in a proportional relationship, identify constants of proportionality, and represent the relationship using equations. These skills are then applied to real-world problems, including scale drawings.

Students continue to build an understanding of the number line in Module 2 from their work in Grade 6. They learn to add, subtract, multiply, and divide rational numbers. Module 2 includes rational numbers as they appear in expressions and equations—work that is continued in Module 3.

Module 3 consolidates and expands students' previous work with generating equivalent expressions and solving equations. Students solve real-life and mathematical problems using numerical and algebraic expressions and equations. Their work with expressions and equations is applied to finding unknown angles and to problems involving area, volume, and surface area.

Module 4 parallels Module 1's coverage of ratio and proportion, but this time with a concentration on percent. Problems in this module include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. In addition, this module includes percent problems about populations, which prepare students for probability models about populations covered in the next module.

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