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How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

Introduction to Eureka Math (chapter 1): This introduction consists of two sections: “Vision and Storyline” and “Advantages to a Coherent Curriculum.”

Major Mathematical Themes in Each Grade Band (chapter 2): The first section presents year-long curriculum maps for each grade band (with subsections addressing *A Story of Units*, *A Story of Ratios*, and *A Story of Functions*). It is followed by a detailed examination of math concept development for PreK to Grade 5. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

Grade-Level Content Review (chapter 3): The rationale for why topics are grouped and sequenced in the modules as they are is presented in this chapter. The Alignment to the Standards and Placement of the Standards in the Modules chart lists the standards that are addressed in each module of the grade.

Curriculum Design (chapter 4): The approach to modules, lessons, and assessment in *A Story of Units* is detailed in this chapter. It also provides a wealth of information about how to achieve the components of instructional rigor demanded by the new standards: fluency, concept development, and application.

Approach to Differentiated Instruction (chapter 5): This chapter describes the approach to differentiated instruction used in *A Story of Units*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

Grade-Level Module Summary and Unpacking of Standards (chapter 6): This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

Mathematical Models (chapter 7): This chapter presents information on the mathematical models used in *A Story of Units*.

Terminology (chapter 8): The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by grade level and module number where they are introduced in *A Story of Units*. The chapter also offers descriptions, examples, and illustrations associated with the terms.

Grade-Level Content Review

The Grade-Level Content Review begins with a list of modules developed to deliver instruction aligned to the standards at a given grade level. This introductory component is followed by two sections: the Rationale for Module Sequence and the Alignment to the Standards and Placement of Standards in the Modules chart.

The Rationale for Module Sequence portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics. The Alignment chart for each grade lists the standards that are addressed in each module of the grade. Throughout the Alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

The Grade-Level Content Review offers key information about grade-level content and provides a recommended framework for grouping and sequencing topics and standards.

Sequence of PreKindergarten Modules Aligned with the Standards

Module 1: Counting to 5

Module 2: Shapes

Module 3: Counting to 10

Module 4: Comparison of Length, Weight, Capacity, and Numbers to 5

Module 5: Addition and Subtraction Stories and Counting to 20

RATIONALE FOR MODULE SEQUENCE IN PREKINDERGARTEN

A *Story of Units* begins! Beans, pennies, apples, and cubes are some of the units, the objects being counted, that students playfully manipulate during their PreKindergarten year. Lessons are planned from simple to complex so that, by the end of the year, students use their math words to communicate precise observations such as, “3 bananas and 2 bananas make 5 bananas.” Notice how this sets the foundation for later mathematics using, for example, place value and fractional units (e.g., “3 hundreds and 2 hundreds make 5 hundreds” and “3 fourths and 2 fourths make 5 fourths”).

The story kicks off in Module 1 with analyzing, sorting, and making a group sharing an attribute (e.g., things that have wheels). Students count the number of objects in the group in different configurations, match those quantities to numerals, and finally, construct a number stair by placing 1 more cube and verbalizing, “Three. One more is 4.” Module 1 works exclusively with numbers 1, 2, 3, 4, and 5.



In Module 2, students also analyze their world, this time sorting, comparing, repositioning, identifying, and constructing two-dimensional shapes. They then apply their observations to three-dimensional shapes. This module is carefully positioned between Modules 1 and 3 to allow students time to develop their fluency with numbers to 5 before moving on to the larger numbers of Module 3.

Module 3 builds the numbers 6–10 in relationship to 5. Students use their fingers to support the $5 + n$ perspective (e.g., “Six is 5 and 1”; “When I count to 8 on my fingers, I have 5 fingers here and 3 fingers here”). Thus numbers 6–10 are “5 and some more,” making numbers 6–10 manageable. As in Module 1, students get busy manipulating units into different configurations and then counting them in systematic ways. For example, they learn to place eight objects in a line and count from left to right. They then learn to arrange those same objects into an array and then into a circle, addressing the complexities that the new configurations bring. After eight objects have been under study for a few lessons, the next number is introduced by putting 1 more object in place. Notice how this sets the foundation for later mathematics using, for example, place value units and units of 8 (e.g., “236 has 3 tens. One more ten is 246. $236 + 10 = 246$.” “Five eights is 40. One more eight is 48. Six eights is 48”).

In Module 4, students recognize the measureable attributes of length, weight, capacity, and number and learn to use the language of comparison (e.g., *shorter than* and *taller than*, *heavier than* and *lighter than*, *empty* and *full*, *more* and *less*). This rich module develops the foundational concept of two quantities in relationship to each other. Remember, the second “book” in the Eureka trilogy is *A Story of Ratios*. An understanding of ratio begins here at the concrete level as students compare. The critical use of the words *more than* and *less than* is embedded in a lovely, playful sequence of exploratory experiences, also enabling their skills with counting to 10 to develop during fluency practice.

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