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# How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

**Introduction to Eureka Math (chapter 1):** This introduction consists of two sections: “Vision and Storyline” and “Advantages to a Coherent Curriculum.”

**Major Mathematical Themes in Each Grade Band (chapter 2):** The first section presents year-long curriculum maps for each grade band (with subsections addressing *A Story of Units*, *A Story of Ratios*, and *A Story of Functions*). It is followed by a detailed examination of math concept development for courses typically taught from Grade 9 to Grade 12. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

**Course Content Review (chapter 3):** The purpose and recommended fluencies for the course are presented in this chapter, along with a rationale for why topics are grouped and sequenced in the modules as they are. The Alignment to the Standards and Placement of Standards in the Modules chart lists the standards that are addressed in each module of the course.

**Curriculum Design (chapter 4):** The approach to modules, lessons, and assessment in *A Story of Functions* is detailed in this chapter.

**Approach to Differentiated Instruction (chapter 5):** This chapter describes the approach to differentiated instruction used in *A Story of Functions*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

**Course Module Summary and Unpacking of Standards (chapter 6):** This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

**Terminology (chapter 7):** The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by course and module number where they are introduced in *A Story of Functions*, and definitions for these terms are provided.



# Course Content Review

The Course Content Review begins with a list of modules developed to deliver instruction aligned to the standards for a given course. This introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the Alignment to the Standards and Placement of Standards in the Modules chart. The Summary of Year portion of each course includes three pieces of information:

- The purpose of the course
- The Recommended Fluencies for the course
- The Major Emphasis Clusters for the course

The Rationale for Module Sequence portion of each course provides a brief description of the instructional focus of each module for that course and explains the developmental sequence of the mathematics.

The Alignment chart for each course lists the standards that are addressed in each module of the course. Throughout the Alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each course have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

The Course Content Review offers key information about course content and provides a recommended framework for grouping and sequencing topics and standards.

## ***Sequence of Algebra II Modules Aligned with the Standards***

Module 1: Polynomial, Rational, and Radical Relationships

Module 2: Trigonometric Functions

Module 3: Exponential and Logarithmic Functions

Module 4: Inferences and Conclusions from Data

## Summary of Year

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, trigonometric, and logarithmic functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Standards for Mathematical Practice apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

## Recommended Fluencies for Algebra II

- Divide polynomials with remainders by inspection in simple cases.
- See structure in expressions and use this structure to rewrite expressions (e.g., factoring, grouping).
- Translate between recursive definitions and closed forms for problems involving sequences and series.

## Major Emphasis Clusters

### *The Real Number System*

- Extend the properties of exponents to rational exponents

### *Seeing Structure in Expressions*

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

### *Arithmetic with Polynomials and Rational Expressions*

- Understand the relationship between zeros and factors of polynomials

### *Reasoning with Equations and Inequalities*

- Understand solving equations as a process of reasoning and explain the reasoning
- Represent and solve equations and inequalities graphically

### *Interpreting Functions*

- Interpret functions that arise in applications in terms of the context

### *Building Functions*

- Build a function that models a relationship between two quantities

### *Making Inferences and Justifying Conclusions*

- Make inferences and justify conclusions from sample surveys, experiments, and observational studies

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