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Published by Jossey-Bass

A Wiley Brand

One Montgomery Street, Suite 1000, San Francisco, CA 94104-4594—www.josseybass.com

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Library of Congress Cataloging-in-Publication Data has been applied for and is on file with the Library of Congress.

ISBN 978-1-118-81207-5 (paper); ISBN 978-1-118-81314-0 (ebk.); ISBN 978-1-118-81326-3 (ebk.)

Printed in the United States of America

FIRST EDITION

PB Printing 10 9 8 7 6 5 4 3 2 1

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How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

Introduction to Eureka Math (chapter 1): This introduction consists of two sections: “Vision and Storyline” and “Advantages to a Coherent Curriculum.”

Major Mathematical Themes in Each Grade Band (chapter 2): The first section presents year-long curriculum maps for each grade band (with subsections addressing *A Story of Units*, *A Story of Ratios*, and *A Story of Functions*). It is followed by a detailed examination of math concept development for courses typically taught from Grade 9 to Grade 12. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

Course Content Review (chapter 3): The purpose and recommended fluencies for the course are presented in this chapter, along with a rationale for why topics are grouped and sequenced in the modules as they are. The Alignment to the Standards and Placement of Standards in the Modules chart lists the standards that are addressed in each module of the course.

Curriculum Design (chapter 4): The approach to modules, lessons, and assessment in *A Story of Functions* is detailed in this chapter.

Approach to Differentiated Instruction (chapter 5): This chapter describes the approach to differentiated instruction used in *A Story of Functions*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

Course Module Summary and Unpacking of Standards (chapter 6): This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

Terminology (chapter 7): The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by course and module number where they are introduced in *A Story of Functions*, and definitions for these terms are provided.

Course Content Review

The Course Content Review begins with a list of modules developed to deliver instruction aligned to the standards for a given course. This introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the Alignment to the Standards and Placement of Standards in the Modules chart. The Summary of Year portion of each course includes three pieces of information:

- The purpose of the course
- The Recommended Fluencies for the course
- The Major Emphasis Clusters for the course

The Rationale for Module Sequence portion of each course provides a brief description of the instructional focus of each module for that course and explains the developmental sequence of the mathematics.

The Alignment chart for each course lists the standards that are addressed in each module of the course. Throughout the Alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each course have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

The Course Content Review offers key information about course content and provides a recommended framework for grouping and sequencing topics and standards.

Sequence of Geometry Modules Aligned with the Standards

Module 1: Congruence, Proof, and Constructions

Module 2: Similarity, Proof, and Trigonometry

Module 3: Extending to Three Dimensions

Module 4: Connecting Algebra and Geometry Through Coordinates

Module 5: Circles With and Without Coordinates

Summary of Year

The fundamental purpose of this Geometry course is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving toward formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school standards. The Standards for Mathematical Practice apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that relies on their ability to make sense of problem situations.

Recommended Fluencies for Geometry

- Triangle congruence and similarity criteria
- Using coordinates to establish geometric results
- Calculating length and angle measures
- Using geometric representations as a modeling tool
- Using construction tools, physical and computational, to draft models of geometric phenomena

Major Emphasis Clusters

Congruence

- Understand congruence in terms of rigid motions
- Prove geometric theorems

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems using similarity
- Define trigonometric ratios and solve problems involving right triangles

Expressing Geometric Properties with Equations

- Use coordinates to prove simple geometric theorems algebraically

Modeling with Geometry

- Apply geometric concepts in modeling situations

RATIONALE FOR MODULE SEQUENCE IN GEOMETRY

Module 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions—translations, reflections, and rotations—and have strategically applied a rigid motion to informally show that two triangles are congruent. In this module, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They build on this familiar foundation of triangle congruence to develop formal proof techniques. Students make conjectures and

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