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How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content of the grade level in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the grade-level content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during the year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grade, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, these grade-level study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

Introduction to Eureka Math (chapter 1): This introduction consists of two sections: "Vision and Storyline" and "Advantages to a Coherent Curriculum."

Major Mathematical Themes in Each Grade Band (chapter 2): The first section presents year-long curriculum maps for each grade band (with subsections addressing A *Story of Units*, A *Story of Ratios*, and A *Story of Functions*). It is followed by a detailed examination of math concept development for courses typically taught from Grade 9 to Grade 12. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

Course Content Review (chapter 3): The purpose and recommended fluencies for the course are presented in this chapter, along with a rationale for why topics are grouped and sequenced in the modules as they are. The Alignment to the Standards and Placement of Standards in the Modules chart lists the standards that are addressed in each module of the course.

Curriculum Design (chapter 4): The approach to modules, lessons, and assessment in *A Story of Functions* is detailed in this chapter.

Approach to Differentiated Instruction (chapter 5): This chapter describes the approach to differentiated instruction used in *A Story of Functions.* Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

Course Module Summary and Unpacking of Standards (chapter 6): This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through the school year.

Terminology (chapter 7): The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by course and module number where they are introduced in *A Story of Functions*, and definitions for these terms are provided.

CHAPTER 3

Course Content Review

The Course Content Review begins with a list of modules developed to deliver instruction aligned to the standards for a given course. This introductory component is followed by the Summary of Year, the Rationale for Module Sequence, and the Alignment Chart with the course standards. The Summary of Year portion describes the purpose of the course. The Rationale for Module Sequence portion provides a brief description of the instructional focus of each module for that course and explains the developmental sequence of the mathematics.

The Alignment Chart lists the standards that are addressed in each module of the course. Throughout the alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each course have been carefully included in the module sequence. Some standards are deliberately included in more than one module, so that a strong foundation can be built over time. The Course Content Review offers key information about course content and provides a recommended framework for grouping and sequencing topics and standards.

Sequence of Precalculus and Advanced Topics Modules Aligned with the Standards

Module 1: Complex Numbers and Transformations

Module 2: Vectors and Matrices

Module 3: Rational and Exponential Functions

Module 4: Trigonometry

Module 5: Probability and Statistics

Summary of Year

Extending their understanding of complex numbers to points in the complex plane, students come to understand that a point in the plane can be identified with a complex number and that multiplying a number $a + bi$ by a complex number z amounts to rotating and dilating the associated point (a, b) in the plane about the origin. Matrices are studied as tools for performing rotations and reflections of the coordinate plane, as well as for solving systems of linear equations. Inverse functions are explored as students further study the relationship between exponential and logarithmic functions and restrict the domain of the trigonometric functions to allow for their inverses. The year concludes with a capstone module on modeling with probability and statistics. The Standards for Mathematical Practice apply throughout each module and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Although a few key (+) standards are included in the Geometry and Algebra II courses to provide coherence to the curriculum, all (+) standards are covered in this course and are also assessed.

RATIONALE FOR MODULE SEQUENCE IN PRECALCULUS AND ADVANCED TOPICS

In Algebra II, students extended their understanding of number to include complex numbers as they studied polynomials with complex zeros. In Module 1 of Precalculus, students graph complex numbers in the complex plane and translate between rectangular and polar forms of complex numbers. In particular, through repeated reasoning, they come to realize that multiplication of complex numbers amounts to rotating and dilating points in the complex plane around the origin. Thinking of a complex number $a+bi$ once again as a point (a, b) in the coordinate plane, students investigate how multiplying by a complex number can be thought of as a map from the coordinate plane to itself. That study, in turn, leads to matrix notation and a natural definition for multiplying a vector by a matrix:

$$(a+bi)(c+di) = (ac-bd)+(ad+bc)i \text{ is equivalent to } \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac-bd \\ ad+bc \end{pmatrix}.$$

Thus students discern structure in the operations with matrices and vectors by comparing them to arithmetic with complex numbers.

Students began the study of transformations in Grade 8 and precisely defined rigid motions in the plane in terms of angles, circles, perpendicular lines, parallel lines, and segments in Geometry. In this module, students precisely define rotations, reflections, and dilations in the coordinate plane using multiplication by 2×2 matrices and define translations by vector addition. These precise definitions of transformations of the coordinate plane shed light on how geometry software and video games efficiently perform rigid motion calculations.

In the first module, students view matrices as tools for performing rotations and reflections of the coordinate plane. In Module 2, they move beyond this viewpoint to study

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