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How to Use This Book

As a self-study resource, these *Eureka Math* Study Guides are beneficial for teachers in a variety of situations. They introduce teachers who are brand new to either the classroom or the *Eureka Math* curriculum not only to *Eureka Math* but also to the content in a way they will find manageable and useful. Teachers already familiar with the curriculum will also find this resource valuable as it allows a meaningful study of the content in a way that highlights the connections between modules and topics. The guidebooks help teachers obtain a firm grasp on what it is that students should master during a given year. The structure of the book provides a focus on the connections between the standards and the descriptions of mathematical progressions through the grades, topic by topic. Teachers therefore develop a multifaceted view of the standards from a thorough analysis of the guide.

The *Eureka Math* Study Guides can also serve as a means to familiarize teachers with adjacent grade levels. It is helpful for teachers to know what students learned in the grade level below the one they are currently teaching as well as the one that follows. Having an understanding of the mathematical progression across grades enhances the teacher's ability to reach students at their level and ensure they are prepared for the next grade.

For teachers, schools, and districts that have not adopted *Eureka Math*, but are instead creating or adjusting their own curricular frameworks, the study guides offer support in making critical decisions about how to group and sequence the standards for maximal coherence within and across grades. *Eureka Math* serves as a blueprint for these educators; in turn, the study guides present not only this blueprint but a rationale for the selected organization.

The *Eureka Math* model provides a starting point from which educators can build their own curricular plan if they so choose. Unpacking the new standards to determine what skills students should master at each grade level is a necessary exercise to ensure appropriate choices are made during curriculum development. The *Eureka Math* Study Guides include lists of student outcomes mapped to the standards and are key to the unpacking process. The overviews of the modules and topics offer narratives rich with detailed descriptions of how to teach specific skills needed at each grade level. Users can have confidence in the interpretations of the standards presented, as well as the sequencing selected, due to the rigorous review process that occurred during the development of the content included in *Eureka Math*.

This *Eureka Math* Study Guide contains the following:

Introduction to Eureka Math (chapter 1): This introduction consists of two sections: "Vision and Storyline" and "Advantages to a Coherent Curriculum."

Major Mathematical Themes in Each Grade Band (chapter 2): The first section presents year-long curriculum maps for each grade band (with subsections addressing A *Story of Units*, A *Story of Ratios*, and A *Story of Functions*). It is followed by a detailed examination of math concept development for statistics and probability from Grade 6 to Precalculus. The chapter closes with an in-depth description of how alignment to the Instructional Shifts and the Standards of Mathematical Practice is achieved.

Statistics and Probability Content Review (chapter 3): The rationale for why topics are grouped and sequenced in the modules as they are is presented in this chapter. The Alignment to the Standards and Placement of the Standards in the Modules chart lists the standards that are addressed in each module that covers statistics and probability.

Curriculum Design (chapter 4): The approach to modules, lessons, and assessment in A *Story of Ratios* and A *Story of Functions* is detailed in this chapter. It also provides a wealth of information about how to achieve the components of instructional rigor demanded by the new standards: fluency, concept development, and application.

Approach to Differentiated Instruction (chapter 5): This chapter describes the approach to differentiated instruction used in *Eureka Math*. Special populations such as English language learners, students with disabilities, students performing above grade level, and students performing below grade level are addressed.

Statistics and Probability Module Summary and Unpacking of Standards (chapter 6): This chapter presents information from the modules to provide an overview of the content of each and explain the mathematical progression. The standards are translated for teachers, and a fuller picture is drawn of the teaching and learning that should take place through both middle and high school on the topics of statistics and probability.

Terminology (chapter 7): The terms included in this list were compiled from the New or Recently Introduced Terms portion of the Terminology section of the Module Overviews. Terms are listed by grade level and module number where they are introduced in A *Story of Ratios* and A *Story of Functions*, and definitions for these terms are provided.

CHAPTER 3

Statistics and Probability Content Review

The Statistics and Probability Content Review begins with a list of modules developed to deliver instruction aligned to the statistics and probability standards. This introductory component is followed by the Rationale for Module Sequence in Statistics and Probability, which provides a brief description of the instructional focus of each module and outlines the developmental sequence.

The Alignment to the Standards and Placement of Standards in the Modules chart lists the standards that are addressed in each module. All statistics and probability standards have been carefully included in the module sequence. The Statistics and Probability Content Review offers key information about content and provides a recommended framework for grouping and sequencing topics and standards.

Sequence of Modules Aligned with the Statistics and Probability Standards

Grade 6 Module 6: Statistics

Grade 7 Module 5: Statistics and Probability

Grade 8 Module 6: Linear Functions

Algebra I Module 2: Descriptive Statistics

Algebra II Module 4: Inferences and Conclusions from Data

Precalculus and Advanced Topics Module 5: Probability and Statistics

RATIONALE FOR MODULE SEQUENCE IN STATISTICS AND PROBABILITY

Grade 6 Module 6: Students develop an understanding of statistical variability and apply that understanding as they summarize, describe, and display distributions. In particular, careful attention is given to measures of center and variability.

Grade 7 Module 5: Students learn to draw inferences about populations based on random samples. Through the study of chance processes, students learn to develop, use, and evaluate probability models.

Grade 8 Module 6: Students return to linear functions in the context of statistics and probability as bivariate data provide support in the use of linear functions.

Algebra I Module 2: This module builds on students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students display and interpret graphical representations of data and, if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities. They analyze their knowledge of the context of a situation to justify their choice of a linear model. With linear models, they plot and analyze residuals to informally assess the goodness of fit.

Algebra II Module 4: Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data, including sample surveys, experiments, and simulations, and the role that randomness and careful design play in the conclusions that can be drawn. Students create theoretical and experimental probability models following the modeling cycle. They compute and interpret probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

Precalculus and Advanced Topics Module 5: This is a capstone module in the Precalculus course on modeling with probability and statistics in which students consolidate their study of statistics as they analyze decisions and strategies using newly refined skills in calculating expected values.

ALIGNMENT TO THE STANDARDS AND PLACEMENT OF STANDARDS IN THE MODULES

Module and Approximate Number of Instructional Days	Standards Addressed in Statistics and Probability Modules
Grade 6 Module 6: Statistics (25 days)	**Develop understanding of statistical variability.** 6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* 6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **Summarize and describe distributions.** 6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 6.SP.B.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

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