

Book One

Developing Number Concepts

Counting, Comparing, and Pattern

..... **Kathy Richardson**

Math Perspectives

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Developing Number Concepts—the Series

Developing Number Concepts is a series of books designed to help young children develop important foundational mathematics concepts.

Each of the three books in the series includes cohesive and organized sets of experiences focused on particular mathematical ideas. Every concept is developed both through teacher-directed and independent activities. Because children learn at different rates the activities are “expandable” and, therefore, meet a range of needs. Questions that guide teachers’ observations of children as they work and learn help in the assessment of children’s ongoing progress.

Book One

Chapter 1: *Beginning Number Concepts*

Chapter 2: *Pattern*

Chapter 3: *The Concepts of More and Less*

Book Two

Chapter 1: *Interpreting and Symbolizing Addition and Subtraction*

Chapter 2: *Internalizing Number Combinations to 10*

Chapter 3: *Developing Strategies for Adding and Subtracting*

Book Three

Chapter 1: *Place Value*

Chapter 2: *Beginning Multiplication*

Chapter 3: *Beginning Division*

The Planning Guide for Developing Number Concepts accompanies the series. It is for the use of teachers of kindergarten through grade three and teachers of multi-grade classes. It includes comprehensive year-long teaching plans along with classroom management ideas.

How the Books Are Organized

Each chapter of *Books One, Two, and Three* includes the following.

■ **What You Need to Know About...**

This section provides the teacher with background information on the featured math concept and a summary of ways in which to teach the concept.

■ **Chapter Overview**

A brief overview of the chapter follows. It offers pertinent information on how the math concept should be taught to children at each grade level, kindergarten through grade three, and to children with special needs.

■ **Goals for Children's Learning**

This section lists the mathematics concepts, ideas, and skills that the children will learn as they work with the activities.

■ **Analyzing and Assessing Children's Needs**

Questions to guide teachers' observations and a discussion of how the activities can be used to meet a range of needs are included. The questions are geared to help teachers determine if the tasks that children are working with are appropriate and are meeting their needs.

■ **Classroom Scenes**

Realistic classroom scenes that deal with the major math concepts covered in the chapter help bring the activities to life as they model ways in which the teacher can work.

■ **About the Activities**

Included here is a brief discussion about the purpose of the activities along with information about materials preparation.

■ **Teacher-Directed Activities and Independent Activities**

A great variety of both teacher-directed and independent and/or partner activities are included for each math concept. This gives teachers many different ways to meet children's needs while it gives children many different ways to learn about a particular concept.

■ **Blackline Masters**

Blackline masters, used both for materials preparation and as children's worksheets, appear at the end of each book.

What You Need to Know About Beginning Number Concepts

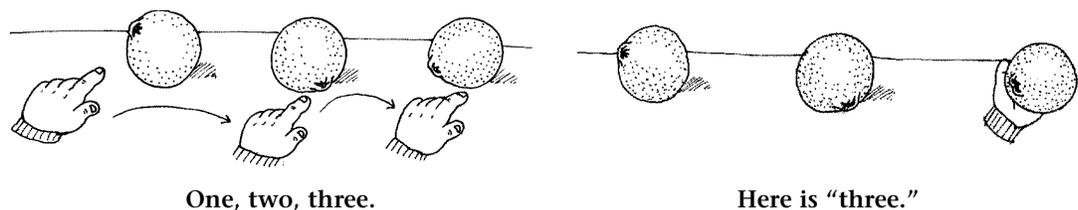
Beginning number concepts are much more complex than most adults realize. It is important to recognize the complexity of those ideas so that we can provide the right kinds of experiences to support children's developing understanding.

When young children can say the counting sequence correctly, and when they can recognize numerals, parents and teachers are pleased. However, beginning number concepts involves much more than rote memorization of words and symbols.

When children develop language, they learn to observe and name the things around them. For example, they learn to use the words *ball* and *water* and *apple*. As they notice how these objects look and feel, they learn to describe them using such words as *round* and *wet* and *red*. Learning about number, however, requires children to think about their world in a different way. When they count, they must ignore the physical properties of what they are counting. Number can't be seen. Number is an idea. We are often unaware of just how complex an idea it is until we see very young children work to figure out number concepts.

Four of the basic understandings that young children need in order to work successfully with number concepts are *inclusion*, *one-to-one correspondence*, *conservation of number*, and *number sense and relationships*. We will be able to work more effectively with children if we are aware of these understandings and how they influence a child's growing internalization of the meaning of number.

Inclusion To understand what it means to count, children must realize that the number they say when they count the last object in a group *includes* all the objects previously counted. We refer to this idea as *inclusion*. A young child who is counting oranges, but who has not developed the idea of inclusion, might respond to the command "Bring me three" by picking up the orange to which she was pointing when she counted "three." She is naming the "three" orange (the third orange) in the same way as she would name any single object such as *book* or *chair*.



What this child must learn is that *three* is not the name of the third orange, but that "three" includes the oranges labeled *one* and *two* as well. She must also recognize that the order in which we count the oranges doesn't matter. The orange she labeled *three* could just as well have been labeled *one* or *two*. Most children will have figured this out by the time they get to kindergarten, but teachers need to watch for clues that indicate that a child has not reached this level of understanding.

One-to-One Correspondence Young children who are just discovering the idea of counting notice that people point to objects while they say number words. Children who are still not secure with the idea may say words faster or slower than they point, having noticed only that they must stop pointing and stop saying words at the same time. In this process, they may skip or even recount an object. It takes a while for children to recognize that we say (or think) one word for each object we count. This matching of one word with one object is referred to as *one-to-one correspondence*. Children develop this skill gradually. A child who has developed an understanding of one-to-one correspondence for numbers through eight or ten may lose track of this idea when working with larger numbers.

Conservation of Number What children *see* plays an important part in their understanding of the world. What to them *appears* to be true is easier for them to believe than what might seem logical from an adult’s point of view. At an early stage of development, a child believes that if something looks different, it is different. A child functioning at this stage who sees eight paper cups lined up close together may think that there are less of them than when the same eight cups are spread farther apart. That child may also think that a single cracker broken into several pieces is *more than* one whole cracker and that five elephants are *more than* five peanuts.

When adults watch a child count out eight objects and then say that there are more than eight when the objects are spread out, it is often difficult to understand how the child is thinking. However, imagine some situations in which we adults are also fooled by our perceptions. Thirty adults in a room may seem, even to us, like more people than if we saw thirty children in that same room. If we don’t actually count, our estimate of the number of people might reflect that general impression. Our experiences over long periods of time have taught us to check our perceptions and trust our logic when perception and logic contradict each other. Children, however, are still tied strongly to their perceptions. They need many different experiences, along with maturation, before they understand what we describe as *conservation of number*—that the number of objects does not change when the objects are moved, rearranged, or hidden.

Number Sense and Relationships Children love to count. They are intrigued with finding the answer to “How many?” As they develop their number sense, they go through different stages of understanding. At first they focus on the task of pointing to each object, saying the numbers in correct sequence, and then reporting the number they “landed on.” They are not as aware of the actual quantity as we might think, but rather, they are focused on accomplishing the counting task itself.

.....
*Counting is a tool
for finding the
answer to
“How many?”*

When children are first learning to count, we see them losing track, forgetting what number they counted to, and getting different answers when they recount the same group over and over again. Developing ease and consistency is a major first task. Children need lots of practice before counting becomes a genuinely meaningful and useful tool.

Children need to move beyond basic counting and develop a sense of quantity and number relationships.

Just as a child learning to read needs repetitive reading tasks in order to develop competence and confidence, a child learning to count needs repetitive counting tasks. Children also need opportunities to count *for a purpose*. For example, they could be asked to count to determine the number of chalkboards needed so that everyone in their group can have one. They could be challenged to count to make sure there are not too many children at a math station. They could be asked to count to make sure all the children going home on the bus are in line.

Counting correctly and telling *how many* is an important part of the young child's accomplishments, but it is just the first step toward an understanding of number. We also want children to develop a sense of different quantities and the relationships between them. We want children to be thinking about and noticing relationships such as *six is one more than five, eight is less than nine, and it takes two more than five to make seven*. We want children to think about and notice what happens when they count. When they are thinking about what they are counting, we hear comments like "I thought the shape puzzle would hold eight cubes, but it only holds six," and "So far I've used four paper clips to measure the yarn. I think I will need three more."

Children need experiences with real objects in order to develop an understanding of counting and number relationships.

In order to see relationships between numbers, to build visual images of quantities, and to develop an understanding of conservation of number, children must have many varied experiences that involve the use of real objects over a long period of time. Given meaningful counting experiences, children will develop a strong sense of number and number relationships as they simultaneously develop facility with counting.

Working with Symbols Just as older students can learn to read and write the symbols πr^2 without really understanding the concepts they represent, young children can learn the names of the numerals without having any idea about what "those squiggles" represent. Recognizing a symbol without knowing the concept it represents is virtually useless. Therefore, numerals should not be taught in isolation but always in association with the quantities they represent. Seeing the symbols should trigger the visual images of what they stand for in the child's mind. Numerical symbols come to have meaning for children only when they are introduced as labels for quantities.

Practice with numerals does not help children develop a sense of number and number relationships. Numerals become meaningful only when they are used as labels for quantities.

Learning to write numerals is a totally separate task from learning to associate numerical symbols with particular quantities. The child who writes the numeral 3 fifteen times will probably become better able to write 3, but will not have learned anything more about the concept of "threeness." We must be very careful not to assume that children are learning anything about quantity and number when they learn merely to write the numerals. Of course, children do need practice writing numerals, but such practice should be considered a handwriting skill rather than a math skill.

T

eaching and Learning

Beginning Number Concepts

Through the activities in this chapter, children develop an understanding of beginning number concepts, including meaningful and purposeful counting. The activities help children develop a sense of conservation of number and a sense of quantity and number relationships while they also develop facility with counting. As children learn to read and write numerals, they discover that they can use these numerals to represent quantities and to keep track of their experiences. Because this chapter connects counting with important beginning number concepts, children who are just learning to count, as well as those who are ready to focus on number relationships, will benefit from these activities.

Using the Chapter

Your use of Chapter One will vary according to the needs of your children. The “Meeting the Needs of Your Children” charts in the introduction to this book and the *Planning Guide* that accompanies this series offer detailed information that can help you plan the use of the chapter’s activities. The following are general suggestions for using the activities with different groups of children.

Kindergarten The Chapter One activities can provide much of the basic number work for the kindergarten year. The children need the ongoing experiences of these activities in order to develop consistency and accuracy with counting skills. They will learn to recognize and write numerals as they develop facility with beginning number concepts.

First Grade As they enter first grade, most children are able to count easily to 20 and beyond. Many are also able to write numerals to ten. We want to make sure, however, that children go beyond simply counting and go on to developing a strong sense of number and number relationships. First-grade children who do the activities in this chapter will solidify their understanding of the concepts they began to develop in kindergarten. They will do many of the same tasks as will kindergartners, but at a higher level, focusing not on the process of counting but on developing a sense of quantity and the relationships that exist between quantities. First-grade children also develop facility in writing numerals as they gain a deeper understanding of number. Use the activities in this chapter at the beginning of the year, when your focus will be on numbers to ten. Use them again later on in the year, when your focus will be on numbers to 20. The activities in this chapter can easily be extended by asking the children to make the kinds of comparisons that are described in Chapter Three, *The Concepts of More and Less*.

Second and Third Grades Unless you have children with special needs, you will not be using activities from this chapter.

Children with Special Needs If you have children who are having difficulty learning to count accurately and consistently, provide them with lots of practice with small numbers until they are confident counting them. Have the children work with the same numbers over and over again, but in a variety of ways. The activities in this chapter provide many ideas for varying this repetitive practice. For specific activities, refer to the “Meeting the Needs of Your Children” chart for Chapter 1 in the introduction to this book. (See “Practice with one-to-one counting to ten.”)

Goals for Children’s Learning*

Goals

When counting objects in a variety of settings, the children will:

- Count to ten with consistency, accuracy, and confidence
- Use numbers to answer the question, “How many?”
- Develop number sense and number relationships by
 - Making reasonable estimates
 - Determining “one more” and “one less” without counting
 - Recognizing small groups of up to five objects without counting
- Recognize the numerals 0 to 10
- Write the numerals 0 to 10 without a model
- Use numerals to record experiences

When working with the activity extensions, the children will:

- Count objects to 20 or 30
- Recognize numerals to 20 and beyond

Analyzing and Assessing Children’s Needs

The goals for children working in this chapter go beyond simply counting to ten and recognizing numerals. The emphasis here is on understanding the usefulness of counting, developing facility with counting, and developing a sense of quantities and relationships.

When we assess children’s understanding of beginning number concepts and make decisions about what they need to learn, it is not enough to know if they can count by rote or recognize numerals. It is also not enough to know if they can match sets to numerals on a workbook page. Instead, we need to know if they can *use* counting and make sense of number in a variety of settings. We can get valuable information about what children understand by paying close attention to the way they respond to tasks. Observing them at work can guide our interactions with them and help us in planning and pacing the activities we present.

* Adapted from *How Do We Know They’re Learning? Assessing Math Concepts*.

The following guide to observations will help you to know what to look for as you assess your children's growth.

Questions to Guide Your Observations*

Always make note of which numbers children are able to work with, as this varies from child to child and changes throughout the year. Are the children able to work with numbers to 6? to 10? to 20? beyond 20?

Questions

Counting Objects

- When the children count, do they use the appropriate counting sequence? Do their errors appear to be random or do they consistently make the same errors?
- Do they count each object once and only once, or do they lose the idea of one-to-one correspondence as they count?
- Do they have a way of keeping track of what they have counted?
- Are they consistent? Are they accurate? If they are inaccurate, are they aware of this? Are they bothered by this?
- Do they check and recheck to make sure they counted correctly?
- Do they remember the number they counted to?
- When asked to get a certain number of objects, do they count correctly or do they count past the number they need?

Number Sense and Relationships

- Can the children instantly identify small groups of up to four or five objects, or do they need to count even when the groups are very small?
- Do they use what they know about one group to help them figure out how many there are in another group?
- Do they make a reasonable estimate about the size of a group? Can they revise their estimate after counting just a few objects in the group?
- When asked to change one number of objects to another, how do they do this? Do they start counting over again? Do they count on? Do they count backward? Do they know how many they need to add or take away without counting?

Working with Symbols

- Can the children say the names of the numerals and build a set that corresponds to each?
- Can they easily write the numerals that name particular sets or do they need models of the numerals in order to write them correctly?

* Adapted from *How Do We Know They're Learning? Assessing Math Concepts*.

Teacher-Directed Activities

1-1 Slide and Check*

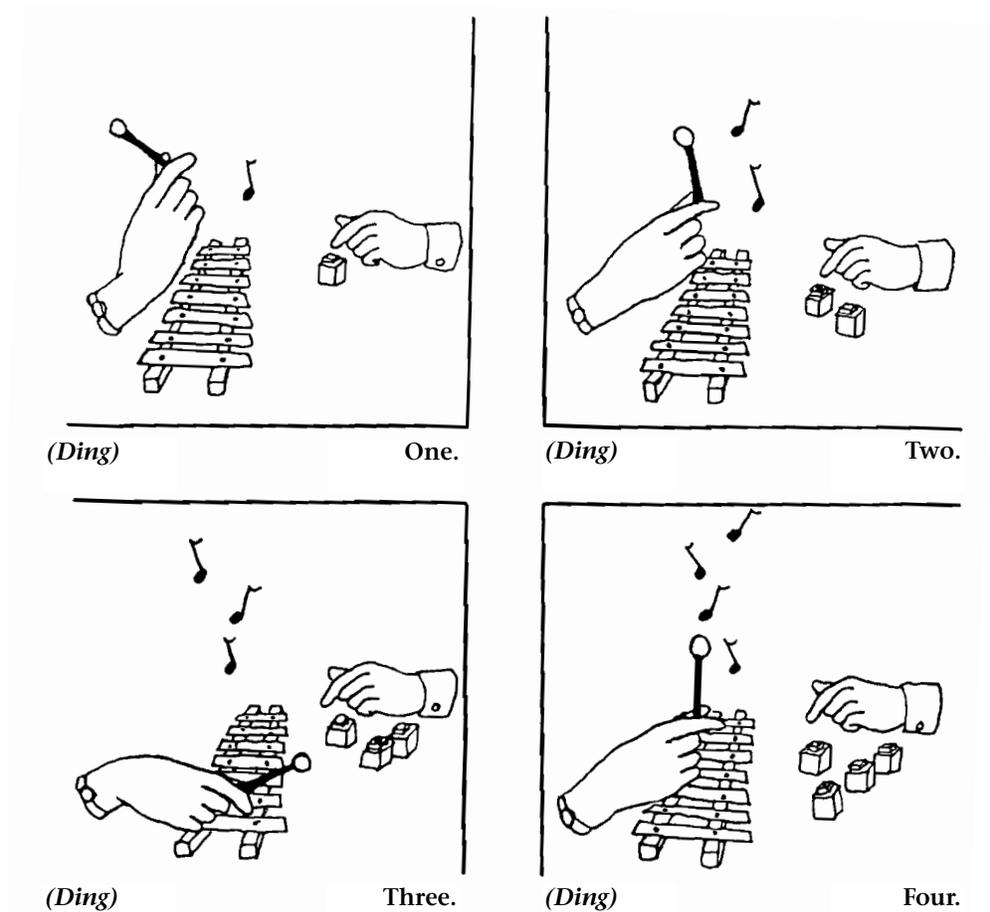
Small-Group Activity

Materials: Counters • Xylophone or other rhythm instrument

The children focus on one-to-one correspondence as you count with them, emphasizing the counting motion by playing a xylophone.

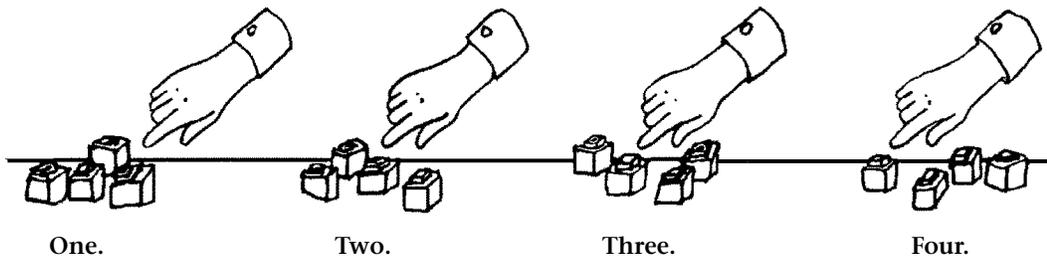
Have the children take counters from a pile and slide them toward themselves one at a time as they practice counting with you over and over again to a designated number. One group of children may need practice in counting to five; another may need practice in counting to nine. On each count strike one note on the xylophone. For example:

We are going to practice counting to four.



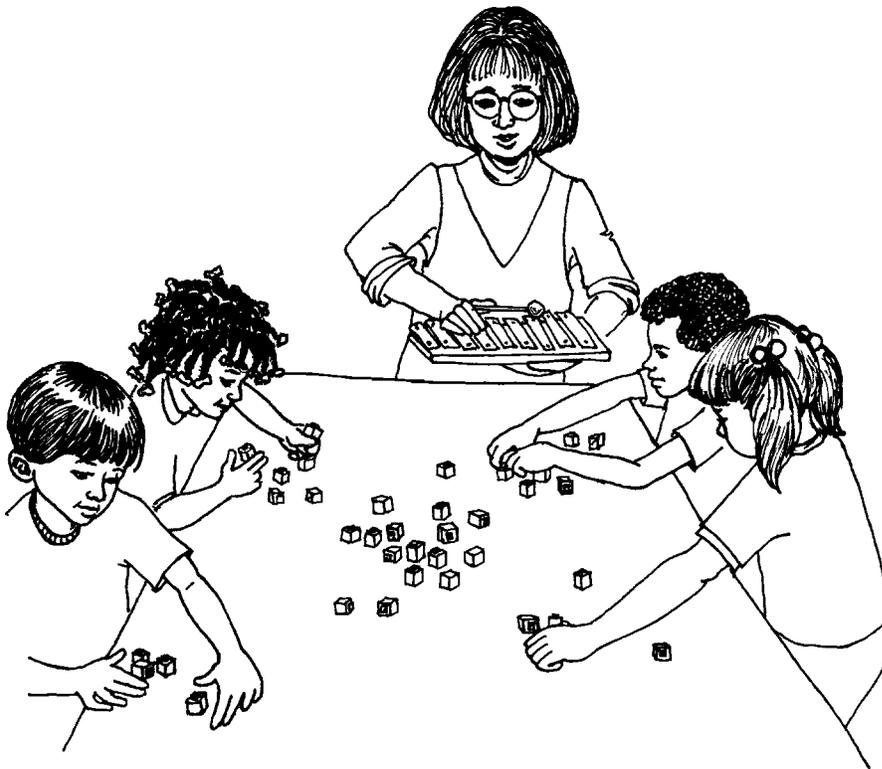
* Based on *Mathematics Their Way*, "The Counting Tape," p. 94.

After the designated number is reached, say “Check,” and have the children recount.



Run the stick across the xylophone as a signal to push the counters back into the pile.

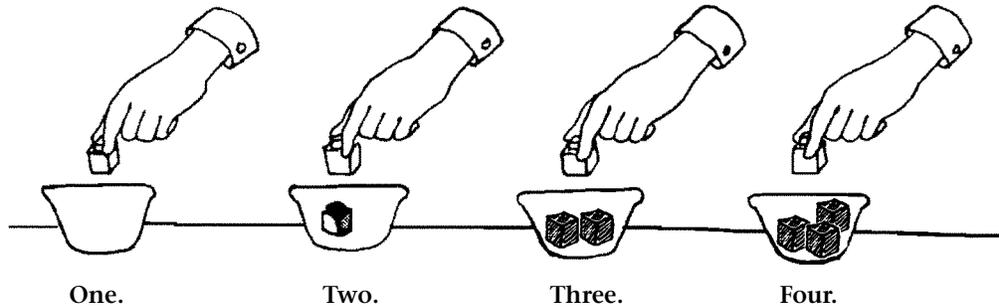
Repeat the activity several times.



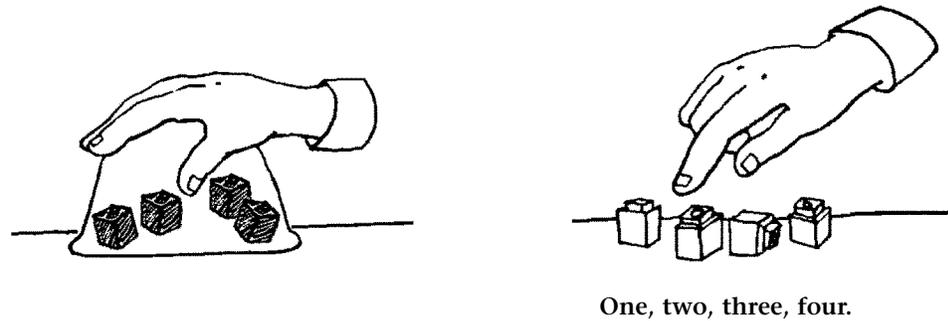
Variation: If no xylophone is available, just count with the children as they slide counters from the pile. Then say, “Push them back” as a signal for children to return the counters to the pile.

Materials: Counters • Margarine tubs (1 per child)

This activity helps children focus on one-to-one correspondence as they count along with you. The children drop counters into their margarine tubs as they practice counting to a designated number. Then you or a student says “Dump,” and everyone empties his or her tub and counts again. For example:



Dump.



Repeat the activity several times.

1-3 Making Towers

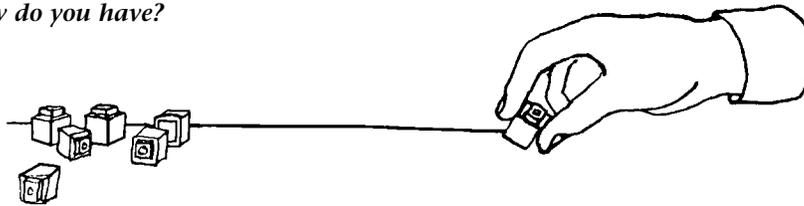
Small-Group Activity

Materials: Connecting cubes

In this activity, the children get practice in counting to a designated number as you direct them to make several towers of the same height. For example:

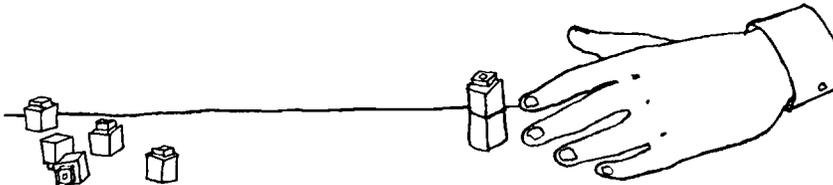
Get one cube.

How many do you have?



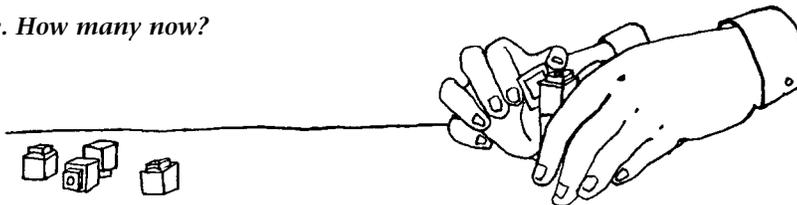
One.

Get one more. How many do you have now?



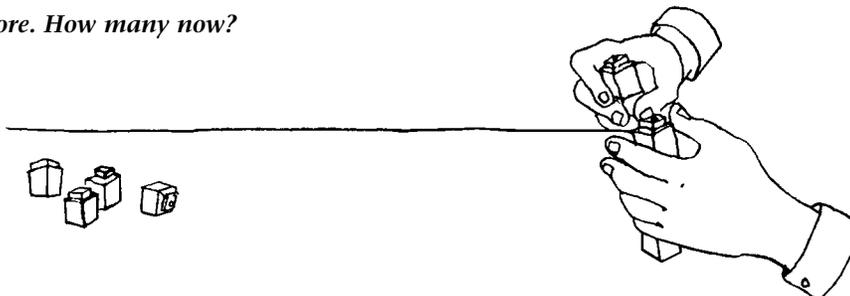
One, two.

Get one more. How many now?



One, two, three.

Get one more. How many now?



One, two, three, four.

When the children have counted as far as you want them to, repeat the process, having them make another tower. Make several towers with them so that they get a lot of practice with the sequence they need to work on.