Grades
5–12

PEMDice™

Order of Operations Game

by Eric Olson

Didax

Order Number 211293

395 Main Street
Rowley, MA 01969
www.didax.com

Contents

Math Standards Addressed by PEMDICE ™

5.NF.B.4.A Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = (ac)/(bd)$).

6.EE.A.2.C Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6\,s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.

HSA.SSE.A.1. Interpret expressions that represent a quantity in terms of its context.*

HSA.SSE.A.1.A Interpret parts of an expression, such as terms, factors, and coefficients.

HSA.SSE.A.1.B Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)n$ as the product of P and a factor not depending on P.

HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

What is PEMDICE ™?

PEMDICE ™ is a fun, easy-to-learn math game that develops proficiency with the order of mathematical operations. The game is simple and engaging, yet it presents multiple challenges that inspire facility, creativity, and ultimately mastery of these key skills.

The simple order in which mathematical operations are performed is an essential thing for any math student to know.

These steps are represented by the acronym PEMDAS[1]:

Parentheses → **E**xponents → **M**ultiplication/**D**ivision (left right) → **A**ddition/**S**ubtraction (left right)

Why the Correct Order of Operations Is Important

If the correct order of operations is not understood, even the best students will struggle to compute a solution or solve an algebraic problem.

For example, what is:

$$3 \times 4 - 2?$$

Depending on which operation is done first, it is either:

$$12 - 2 = 10, \qquad \text{or} \qquad 3 \times 2 = 6.$$

These options get even more complicated as math students encounter exponents, roots, and various grouping symbols. The steps represented by PEMDAS are the rules of “math grammar,” and it is essential that they become second nature for all math students.

Learning the Game

PEMDICE™ is a simple game to learn—and one that can become infinitely complex in its variety and play. It has several variations that easily can be adapted for use at many levels, in many learning environments, and in a great diversity of classroom situations.

PEMDICE™ can be played as a form of solitaire, in pairs, in small groups, or even with an entire class.

In its essence, it is 15 dice that include numbers, operators, an equals sign, and a set of parentheses. With each toss, a new combination of dice is an opportunity for learners to create mathematical statements that are true according to the rules of the order of operations.

Unlike many games, PEMDICE™ can be played with a "shake of the dice," to form as many equations as possible in the time allowed.

Understanding Key Mathematical Concepts

Players learn the importance of grouping numbers, the value of multiplying by 1 (in an alternate form), and that multiplying a complicated expression by 0 yields 0.

A game of PEMDICE™ can be either competitive or collaborative, and it allows a teacher to adjust the rules on the fly as students progress.

PEMDICE™ for a Multi-Level Approach to Learning

There are enough variations of the game to keep students working at appropriate levels. Students can start with the 12 simplest dice for a beginner's version, then add the advanced dice as knowledge and experience allows.

Students can practice playing a game in which a student rolls the dice three times. On the second and third throws, he/she rolls just a few of the dice, but is restricted to the numbers and operators from the first roll. Challenge your students by saying: **"Is possible to use all the dice in one big equation?"** and **"What are some of the tricks that you can learn as you become an expert?"**

A classroom teacher will find PEMDICE™ useful in many areas of the curriculum, or whenever the order of operations lesson is being taught. Playing the game will reinforce many key ideas and make them more engaging at the same time.

Have students play PEMDICE™ for a few minutes when they have free time or are in need of a challenge. It naturally strengthens many skills and builds confidence. *Warning!—the game can be addictive!* Some kids have been known to play for fun.

See the practice games on the following pages for a glimpse of the possibilities and practices of PEMDICE™

The Dice

The dice consist of the following numbers and operators. There are two sets of dice included in this package.

PEMDICE ™	
Dice per set	15
Constant Dice	
Equals: =, =, =, =, =,=	1
Parentheses: (, (, (,),),)	2
Numerical Dice	
Even numbers: 2, 2, 4, 4, 6, 6	1
Odd numbers: 3, 3, 5, 5, 7, 7	1
Odd and even: 2, 3, 4, 5, 6, 7	1
8’s, 9’s, 10’s: 8, 8, 9, 9, 10, 10	1
Multiples of 10’s*: 20, 30, 40, 50, 60, 100	1
Fractions *: 1/2, 1/2, 1/3, 1/3, 1/4, 1/4	1

Operator Dice	
Plus and minus: +, +, +, −, −, −	1
Multiplication and Division: ×, ×, ×, ÷, ÷, ÷	1
Operators, mixed +,+, −, −, ×, ÷ and +, −, ×, ×, ÷, ÷	2
Square roots, cube roots, Powers, Factorial*: $\sqrt{}$, $\sqrt[3]{}$, ∧, $\wedge^2$, $\wedge^3$, !	2

* A more elementary version of the game can be played without the tens die and the fraction die. The tens, fractions, powers, and roots dice can be added later on, as students begin to learn those concepts.

Using PEMDICE™ in the Classroom

PEMDICE™ can be used in the classroom in a variety of ways.

- As an enrichment activity for an individual or small groups of two or three when free time is available
- With an entire class when a teacher is presenting a specific lesson—for example, a lesson on square roots.
- As a way to energize your class, or when you want to reinforce your students' skills on the order of operations.
- As a competitive challenge for a few individual students or for the entire class: **“Who can be the first to make a 10-dice equation?”**
- As a collaborative goal: **“Let's all play together until we can make 1000 points!”** (The more, and longer equations are, the more points we score.)

Members of a class can work towards various objectives with their own set of PEMDICE™ or by sharing a set in small groups. The ability to manipulate the dice and form equations is very important, so it is best to work with four or fewer students on a single set of dice.

Flexibility

A key feature of PEMDICE™ is that it allows the teacher a great deal of flexibility to fine-tune any of the several games from this booklet.

These games fit a variety of educational situations—and it is easy to make up your own games to fit a particular lesson. Adjustments can be easily made as each round is played.

By increasing or decreasing the time allowed, the level of urgency can be adjusted to fit the situation. By allowing students to roll the dice again, or even choose the value of certain dice, the game can be made much easier for students first learning the rules.

As students develop certain tricks and a facility for creating multiple and longer equations, the scoring bar can be raised and more competition introduced where appropriate.

Teachers should celebrate creative solutions in which dice are combined in unusual ways to form an equation by snapping a picture of their solution and posting it on a leader board.

Game Rules for PEMDICE™

There are only a few rules about what is allowed! Of course the rules of PEMDAS apply to PEMDICE™.

- Numbers and operators can be used in *any combination*, as long as the equation is *mathematically true*.
- The number 6 cannot be used as a 9.
- The subtraction sign (−) cannot be used as the number 1.
- Two or more numbers can be combined to make a multi-digit number (the 10 cube in front of a 2 cube is 102).
- A number could be placed next to a set of parentheses to indicate multiplication.
- You cannot use the "+" cube in front of a number to start an equation, but you could use a subtraction sign (−) to indicate a negative number.
- Because of the commutative law, equations in which added or multiplied quantities are simply switched are not considered to be different equations, and do not score any points.
- If dice are rearranged and a different statement is made using the same numbers in a similar way, it *is* a new equation: $3 + 4 = 7$ is different from $-3 + 7 = 4$.
- Exponents of exponents are done top to bottom. $2\wedge^3\wedge^2$ is 2^9, not 8^2
- The factorial symbol is done first. It must be used next to a number or parenthesis. $2\wedge^3!$ = is $2\wedge^6 = 64$

See the "Sample Games" section for more examples of these and other rules interpretations**.**

Helpful Hints

Success with PEMDICE™ requires a good understanding of the mathematical "order of operations," so be sure to follow PEMDAS! Also, it is very helpful to quickly identify pairs of numbers that can be combined into another, possibly simpler number, so that by working with the combined numbers longer equations can be made.

For example: if you have:

÷, 6, and 3,

put them together, look at what is left and think of building an equation with a 2.

$$6 \div 3 = 2$$

The same applies with the parentheses. If you group a 4 and a 2 as (4 + 2) with your division by 3, then you will quickly have a longer equation as a block to work with.

Consider:

$$(4 + 2) \div 3$$

So that "(4 + 2) ÷ 3" is really 7 cubes and can be used to quickly make a variety of equations with the remaining cubes.

How to Determine the Score in PEMDICE™

Each equation is scored by counting the dice used in the equation and then multiplying that number by itself. The longer the equation, the higher the score.

For example, 5 × 6 = 30 would earn a score of 25 since it uses 5 dice.

The score for three equations of six dice, each made in a single round would be:

$$6 \times 6 = 36 \text{ points for each,}$$

$$\text{and therefore a total of 108 points } (3 \times 36 = 108).$$

A single equation with eleven cubes scores 121 points!

$$11 \times 11 = 121 \text{ points}$$

Teachers can easily adjust the scoring to emphasize a particular skill: **"You will score 50 extra points if you can use the square-root symbol!"** or **"Double your score if you use the parentheses."**

Learners from the elementary level to high school will quickly develop a series of strategies that help them to play the game effectively.

Object of the Game

The object of the game is to score the most points. Points are earned by making true mathematical equations using any allowed configurations of the dice. Points are awarded for each equation according to the square of the number of cubes used in the equation. (See How to Determine the Score in PEMDICE™. Note: the parentheses only count *if* the equation computes differently when the parentheses are removed.)

Game Setup

Players take turns, using the following rules. A time limit is commonly agreed upon, based on the player's experience and level of expertise—typically 3–5 minutes.

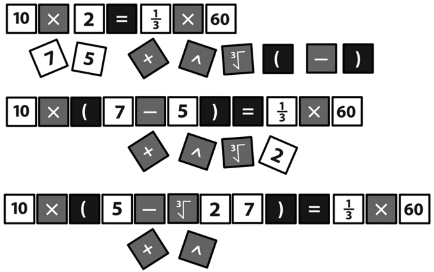
The player places the 15 dice in the container and then shakes and spills them onto a table. After arranging the dice into two groups (numbers and operators) for easy comparison, the player records the configuration of dice at the top of his or her score sheet for later reference.

Play

- Each player makes as many true equations as possible.
- Each equation is scored and recorded on the score sheet to count, unless it is the last one formed as the timer runs out.
- The game is played for a predetermined amount of time, typically 20 minutes, or an agreed upon number of rounds, until a winning total of points is reached. Incorrect equations do not accumulate any points.
- The player who has the most points at the end of the game wins, and is declared "Mathter of the Realm."

Sample Game

This is what a game with three equations look like when played.

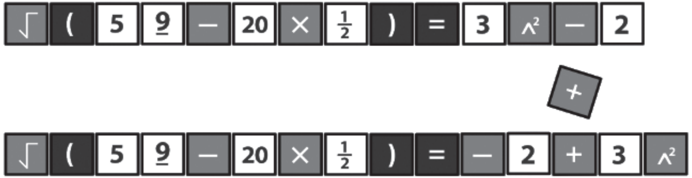


Notice that if the parentheses were removed from the last two equations, the result does not equate.

It is possible to rearrange a few key cubes to make a totally different equation (the two equations that score 100 points are similar, but they are not just making the same statement in a different form).

As mentioned, in the two previous equations if the parentheses are removed the equations are altered (and are no longer true). An effective strategy is to create variations on an equation to quickly make more points.

It is possible to use all 15 cubes in a great variety of ways. Sometimes it is possible to use a trick like the one shown below to use that last, extra, operator. By thinking this way, players develop a quicker ability to work with negative numbers—and increase their score!



The Basic Game

Number of Players: 2 or more

Dice Needed: All 15 dice: 6 numerical dice, 3 dice that have the same markings on all six faces (parentheses and equals), and 6 dice with a mixture of mathematical operations: (+, -, ÷, ×, √ , 3√ , ^, ^2, ^3, and !).

Score Sheets: Score sheets for recording equations are provided to make play easier. See pages 13–15.

Game 1 Beginner's Version

This is a variation on **The Basic Game.** (See page 8.)

This game is played by spilling the cubes and recording the operators—but then allowing some or all of the number cubes to be manipulated and changed into numbers that make a true equation. (You can also try keeping the numbers and changing only the operators.)

This game is fast and easy—and gets each player acquainted with the cubes. The game can then progress gradually to become the standard game by spilling the dice and allowing any selection of dice to be tossed three more times. The game progresses further by limiting the tosses and/or the successive number of dice being tossed.

Game 2 Any Operators

This game is a variation on **Game 1: Beginner's Version**.

In this game, only the numbers are spilled.

This is a lightning game that has the objective of making one equation that uses all 15 cubes as quickly as possible.

You may select any of the mathematical operations symbols that you like, but you must use the numbers you have rolled.

The first person to use all their dice shouts "PEMDICE!" and wins that round.

Game 3 Choose Your Numbers

This is another variation on **The Basic Game** and has the same rules, except this version does not have the element of chance favoring the player who tosses an easier combination of dice.

Each player, after arranging his or her dice into two groups (numbers and operators) for easy comparison, then elects to play with either her own set of numbers or her opponent's set. The player also has the option of using either her own set of operators or her opponent's. (If both players elect to play with the same set of numbers (or operators), the unused set of dice is reconfigured to match the one selected.)

One key strategy of this competitive version of PEMDICE™ is to choose the most advantageous sets of numbers and operators.

If players so choose, they can write their choice on the score sheet in order to avoid giving away their strategy, and reveal them when ready. The game is then played as above.

Game 4 The Longest Equation

This is a variation on **The Basic Game.** (See page 8.)

Player spills the cubes and manipulates them into the single longest possible equation.

The first player to use all the cubes shouts "PEMDICE!"and wins the round.

If, at the end of six minutes, no one has used all the cubes, the player with the longest equation wins the round. In the event of a tie, the round is replayed. Play the best three out of five rounds.

Game 5 Take No Prisoners!

This is a variation on **The Basic Game.** (See page 8.)

To add an extra level of competitiveness, players can “challenge” equations on each other’s answer sheet.

If Player 1 challenges an equation on Player 2’s score sheet and detects an error made by Player 2, Player 2 gets no points, and Player 1 is awarded Player 2's points.

If a player challenges an opponent and is incorrect, then the challenger loses that number of points.

Game 6 No Cube Left Behind!

In this game, the numbers are spilled and the object is to be the first to use each cube at least once in your list of equations. Any number of equations is allowed.

Game 7 Solitaire, Basic Game

Play or practice any of the games listed here to improve the player’s skill level .

Establish a record for the fastest time to use all 15 dice for one equation, to make 500 points, or to make the highest score possible in a set amount of time.

Additionally, you could simply go for the most creative way to use all the cubes—can you incorporate a three-digit number in your equation? Can you use an exponent of an exponent?

Game 8 Solitaire, Around the Horn!

Try to complete each of the challenges below in 20 minutes or less. With each spill, select any of the options to try to do: (Hint: When you roll the dice and get difficult operators, elect to play Game 2.)

1. Playing by standard rules, achieve a score of 300. Use all the cubes playing Game 2 (Any Operators) rules.
2. Use all the cubes playing the "Game 6 (No Cube Left Behind)" rules.
3. Use all the cubes in a single, magnificent equation!

A Note About PEMDAS

While "PEMDAS" has grown steadily in its use over the last few decades in the US and around the world (where it goes by other acronyms like "BODMAS"), the spread of computers has lead to PEMDAS being increasingly common. However, teachers must be careful to teach it correctly and emphasize that multiplication/division are considered equal and are not done one before the other, but rather left to right—as are addition/subtraction. PEMDICE™ can help anyone learning these rules develop a consistent, accurate second nature regarding the order of work when dealing with equations.

Additional Resources

[1] For more information on the Order of Operations and PEMDAS, please visit: http://mathworld.wolfram.com/PEMDAS.html

PEMDICE™ Score Sheet

Spill the dice and list them below to keep track of your starting cubes. Put the numbers in the first 6 boxes, listing them from lowest to highest, then list the 6 operations dice.

Record each (correct!) equation in the boxes provided below the dotted line. Record your score to the right of the boxes.

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