Lesson 5.1 Understanding the Equal Sign

Lesson Overview

This lesson continues the development of students' understanding of the equal sign and their use of fundamental properties to reason about equations. Building on work in previous grades, it examines true/false and open equations using all four operations.

Vocabulary

equal sign

equation

Materials

pan balance or number balance (optional)

Lesson Objectives

Continue to develop a relational understanding of the equal sign by identifying equations as true or false and by solving missing value problems involving all operations..

Rationale for the Tasks

- The equations included in this lesson involve all operations and more complex relationships. Students' responses can reveal whether they have an operational or relational view of the equal sign (including whether they can use a compensation strategy) and whether they are able to recognize equations that reflect fundamental properties.
- The first open equation, 8 × 12 = ___ × 12, can reveal how students are thinking about the equal sign and equations. If a student responds:
  - 96, because 8 × 12 = 96," this indicates the student still holds an operational misconception of the equal sign.
  - "8, because on the left side 8 × 12 = 96, so I had to figure out what times 12 would make 96 on the right side," this indicates the student views the equal sign relationally but is treating the two sides of the equation separately.
  - "8, because then the two sides of the equation are both 8 × 12," this indicates the student views the equal sign relationally and is able to view the equation holistically and look structure across the equal sign.
- The other equations are designed to encourage students to look at relationships between expressions in an equation, to make use of the fundamental properties, and use their developing knowledge of multiplication and division.

Lesson at a Glance

Big Idea

Students continue to develop a relational understanding of the **equal sign** by identifying **equations** in different forms as true or false and by solving open equations. They transition from equations involving only addition, subtraction, and multiplication to include those involving division.

Jumpstart

How would you describe what this symbol means?

=

Explore and Discuss

For Task 1, ask students to work with a partner. Have students share their thinking with the class. Discuss why the equations are true or false.

In Task 2, ask students to trade their equations with a partner after writing their own. Select students to share equations with the class. Discuss why the equations are true or false.

Have students work with a partner for Task 3. Select students to share their thinking with the class. Discuss why the missing numbers make the equations true.

If students still demonstrate an operational view of the equal sign, continue to work with true/false and open equations to help them develop a relational view. Using tools such as a pan balance or number balance can help students visualize the equal sign as a symbol indicating two amounts are the same.

Review and Discuss

Find the missing value in the equation. Explain.

$100 \times \_\_\_ = 50 \times 40$

LESSON 4.1 Understanding the Equal Sign

Jumpstart

Write or display the Jumpstart on the board.

Listen for the different categories of responses students might make (for example, the equal sign means "put the answer" or the "total" vs. the equal sign means the equation is "balanced", or that one side "has the same amount as the other side"). Discuss the meaning of the equal sign as indicating that two quantities are the same amount or have the same value.

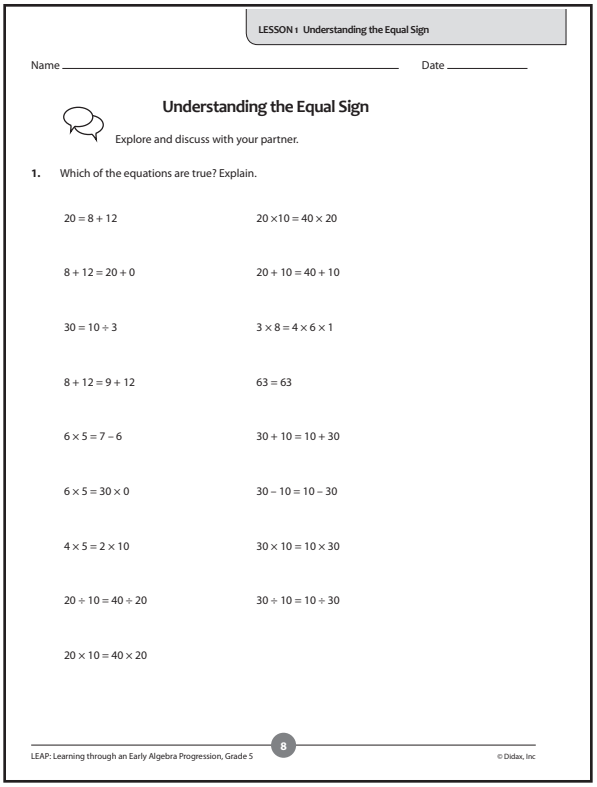
Jumpstart

How would you describe what this symbol means?

=

Explore and Discuss

1. Listen to students' responses. What strategies do they use? Do they compute each side and compare the values? Do they recognize fundamental properties? For example, do they recognize that 30 × 10 = 10 × 30 is true because this equation shows the Commutative Property of Multiplication? Use students' responses to reinforce the notion that the symbol '=' indicates that two quantities are the same amount or have the same value.



LESSON 1 Understanding the Equal Sign

Name _____ Date _____

Understanding the Equal Sign

Explore and discuss with your partner.

1. Which of the equations are true? Explain.

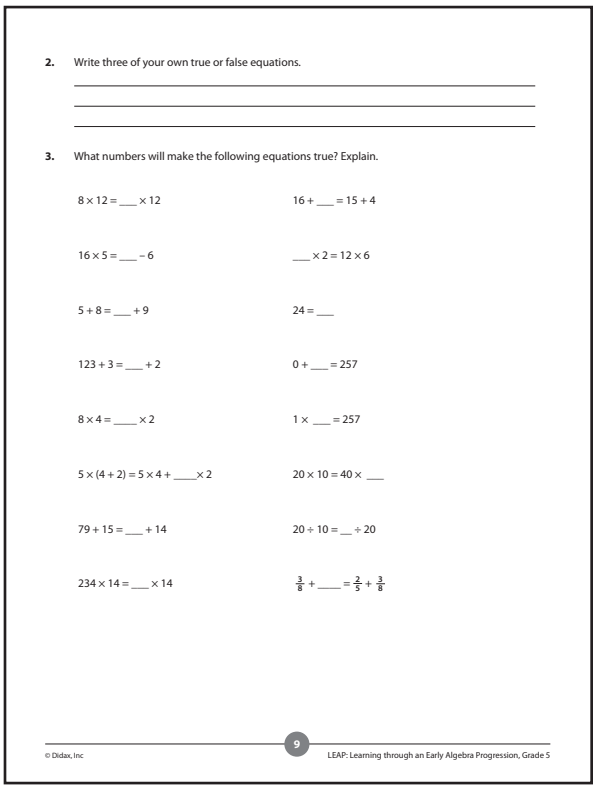
20 = 8 + 12	20 × 10 = 40 × 20
8 + 12 = 20 + 0	20 + 10 = 40 + 10
30 = 10 ÷ 3	3 × 8 = 4 × 6 × 1
8 + 12 = 9 + 12	63 = 63
6 × 5 = 7 − 6	30 + 10 = 10 + 30
6 × 5 = 30 × 0	30 − 10 = 10 − 30
4 × 5 = 2 × 10	30 × 10 = 10 × 30
20 ÷ 10 = 40 ÷ 20	30 ÷ 10 = 10 ÷ 30
20 × 10 = 40 × 20	

LEAP: Learning through an Early Algebra Progression, Grade 5 8 © Didax, Inc

Thinking about Student Responses

- Even though this is not students' first time considering the meaning of the equal sign, you may still observe students demonstrating the belief that the equal sign means "give the total" or "the answer comes next." For example, in Task 3, students might propose that the missing value in 5 + 8 = ___ + 9 is 13, the sum of 5 and 8. Such an operational view is often entrenched given students' repeated exposure over several years to equations in standard form, where operations are only on the left of the equal sign (for example, 4 + 10 = 14).
- When students first begin to view the equal sign relationally, they tend to solve open equations by computing the value of each side of the equation and trying to balance the two sides. For example, they will approach 8 × 4 = ___ × 2 in Task 3 by thinking, "I know 8 × 4 = 32, so now I need to figure out what times 2 is 32." Although this is correct thinking, students might also find the missing value using a structural approach that looks at relationships between quantities: "I know that 2 is half of 4, so the missing value must be twice 8."

Explore and Discuss



2. Write three of your own true or false equations.

3. What numbers will make the following equations true? Explain.

$8 \times 12 = \_\_\_ \times 12$

$16 + \_\_\_ = 15 + 4$

$16 \times 5 = \_\_\_ - 6$

$\_\_\_ \times 2 = 12 \times 6$

$5 + 8 = \_\_\_ + 9$

$24 = \_\_\_$

$123 + 3 = \_\_\_ + 2$

$0 + \_\_\_ = 257$

$8 \times 4 = \_\_\_ \times 2$

$1 \times \_\_\_ = 257$

$5 \times (4 + 2) = 5 \times 4 + \_\_\_ \times 2$

$20 \times 10 = 40 \times \_\_\_$

$79 + 15 = \_\_\_ + 14$

$20 \div 10 = \_\_ \div 20$

$234 \times 14 = \_\_\_ \times 14$

$\frac{3}{8} + \_\_\_ = \frac{2}{5} + \frac{3}{8}$

9

© Didax, Inc

LEAP: Learning through an Early Algebra Progression, Grade 5

2. **What do you notice about the quantities on each side of the equal sign when the equation is true?** You might have students trade true/false equations with a partner and then share some particularly interesting ones with the whole class. Remember to emphasize the notion that the equal sign indicates, in an arithmetic context, that two quantities are the same amount or have the same value.

3. Listen to students' responses and think about which demonstrate an operational view and which demonstrate a relational view of the equal sign. For example, students who fill in the first blank with "96" are demonstrating an operational view of the equal sign. Use students' responses to review the notion that the symbol '=' indicates two quantities have the same value.

What strategies did you use to find the missing number? For those students who are finding the missing value correctly, what strategies do you observe? Some students might compute on both sides of the equal sign. For example, in response to 16 – 5 = ___ – 6, these students might think, "16 – 5 is 11, so what number minus 6 is 11?" Other students might instead use a compensation strategy. For example, these students might think, "6 is one more than 5, so the missing number must be one more than 16." Discuss the strategies students use, developing their understanding of compensation strategies and use of fundamental properties and how these strategies can make equation solving more efficient.

Review and Discuss

Are the following equations true or false? Explain.

$100 \times \_\_\_ = 50 \times 40$

Review and Discuss

Write or display the Review and Discuss question on the board.

Listen to students' strategies. Sometimes, large numbers can push students to notice structure and not rely on computation. For example, students might think, "100 is twice as large as 50, so the missing number must be half as large as 40 to keep the equation balanced."

Addressing Common Difficulties

Students with an operational understanding of the equal sign believe that the equal sign means "the answer" or "give the total" or "the answer comes next."

To counter these difficulties, ensure that students understand that the equal sign means "balance." Listen for these descriptions of their thinking:

> "The equal sign means that whatever amount you have on the left is the same as what you have on the right."
>
> The equal sign shows us that both amounts on each side of the symbol are the same."

Teaching Support

Visualizing the Equal Sign

Do not be surprised if many students still demonstrate an operational view of the equal sign. Repeated work with true/false and open equations will help them develop a relational view. Using tools such as a pan balance or number balance can help students who need additional support visualize balancing equations.

SSL Supporting Struggling Learners

For more information, refer to Figure 1: Strategies to Support Struggling Learners.

Choosing Numbers Strategically

Number choice is important when helping students begin to examine equations structurally rather than as separate left-to-right computations. To encourage students to examine equations without rushing to compute, there are two approaches that can help:

"Obvious" equations such as $8 \times 12 = 12 \times 8$ might be recognized by students as true because the equation reflects the Commutative Property of Multiplication.

Equations such as $5 + 8 = \_\_\_ + 9$, where the numbers in the same position on either side of the equal sign are close in value, can encourage students to look for relationships to help them determine whether an equation is true or false.

Sharing Strategies

Encourage students to keep sharing their strategies. It is through listening to their classmates' explanations that students who are thinking operationally will begin to think relationally about the equal sign and students who are treating the two sides of an equation separately will begin to view equations holistically and look for relationships across the equal sign.

Mathematical Convention

The equal sign is a mathematical symbol used to represent the equivalence of two quantities or mathematical expressions.

Name _____ Date _____

Understanding the Equal Sign

Explore and discuss with your partner.

1. Which of the equations are true? Explain.

$20 = 8 + 12$

$20 \times 10 = 40 \times 20$

$8 + 12 = 20 + 0$

$20 + 10 = 40 + 10$

$30 = 10 \div 3$

$3 \times 8 = 4 \times 6 \times 1$

$8 + 12 = 9 + 12$

$63 = 63$

$6 \times 5 = 7 - 6$

$30 + 10 = 10 + 30$

$6 \times 5 = 30 \times 0$

$30 - 10 = 10 - 30$

$4 \times 5 = 2 \times 10$

$30 \times 10 = 10 \times 30$

$20 \div 10 = 40 \div 20$

$30 \div 10 = 10 \div 30$

$20 \times 10 = 40 \times 20$

2. Write three of your own true or false equations.

_____

_____

_____

3. What numbers will make the following equations true? Explain.

$8 \times 12 = \_\_\_ \times 12$

$16 + \_\_\_ = 15 + 4$

$16 \times 5 = \_\_\_ - 6$

$\_\_\_ \times 2 = 12 \times 6$

$5 + 8 = \_\_\_ + 9$

$24 = \_\_\_$

$123 + 3 = \_\_\_ + 2$

$0 + \_\_\_ = 257$

$8 \times 4 = \_\_\_\_ \times 2$

$1 \times \_\_\_ = 257$

$5 \times (4 + 2) = 5 \times 4 + \_\_\_\_ \times 2$

$20 \times 10 = 40 \times \_\_\_$

$79 + 15 = \_\_\_ + 14$

$20 \div 10 = \_\_ \div 20$

$234 \times 14 = \_\_\_ \times 14$

$\frac{3}{8} + \_\_\_\_ = \frac{2}{5} + \frac{3}{8}$