# Lesson 4.1 Understanding the Equal Sign

## Lesson Overview

This lesson continues to develop students' relational understanding of the equal sign and their use of fundamental properties to reason about equations.

Through continued exposure to equations with carefully chosen numbers, students can learn to recognize the structure underlying equations and use this structure to their advantage in developing efficient solution strategies.

**Vocabulary**

equal sign

equation

**Materials**

pan balance or number balance (optional)

## Lesson Objectives

- Continue to develop a relational understanding of the equal sign by identifying equations written in various formats) as true or false and by solving missing value problems.
- Extend from equations using only addition to those using multiplication as well.

## Rationale for the Tasks

The tasks are designed to encourage students to look for relationships across the equal sign and, when appropriate, use compensation strategies or apply the fundamental properties to reason about equations more efficiently.

Students' responses to the true/false equations can reveal whether they have an operational or relational view of the equal sign as well as whether they are looking for relationships across the equal sign, making use of the fundamental properties, and applying their understanding of multiplication.

The first open number sentence, $7 \times 15 = \_\_ \times 15$, can reveal how students are thinking about the equal sign and about equations in general. If a student responds:

- "105, because $7 \times 15 = 105$," this indicates the student still holds an operational view of the equal sign.
- "7, because on the left side $7 \times 15 = 105$, so I had to figure out what times 15 would make 105 on the right side," this indicates the student views the equal sign relationally and can correctly find the missing value, but is treating the two sides of the equation separately and does not recognize the relationship in the numbers. challenging form, so consistent exposure is key to building students' relational understanding of the equal sign.
- Students are asked to write their own true/false equations because this activity can be motivating for them while revealing the extent to which they are comfortable working with equations in various forms.

# Lesson at a Glance

## Big Idea

Students learn to think relationally about the equal sign by identifying equations as true or false and reasoning why and by solving open equations. They learn to recognize the structure underlying equations and use this structure to their advantage in developing efficient solution strategies.

### Jumpstart

How would you describe what this symbol means?

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## Explore and Discuss

For Task 1, ask students to work with a partner. Have students share their thinking with the class. Discuss why the equations are true or false.

After students write their own equations in Task 2, ask them to trade their equations with a partner. Select students to share some with the class. Discuss why the equations are true or false.

Have students work with a partner for Task 3. Ask students to share their thinking with the class. Discuss why the missing numbers make the equations true.

If students still demonstrate an operational view of the equal sign, continue to work with true/false and open number sentences to help them develop a relational view. Using tools such as a pan balance or number balance can help students visualize the equal sign as a comparison of two equivalent amounts.

## Review and Discuss

Are these equations true or false? Explain.

$10 \times 5 = 50 + 5$

$63 = 9 \times 7$

LESSON 4.1 Understanding the Equal Sign

# Jumpstart

**Write or display the Jumpstart question on the board.**

Listen to students' responses. Do students say that the equal sign means "the answer" or the "total"? Do they say the equal sign means the "equation is balanced," or that "one side has the same value as the other side"?

Help students understand that the quantities on either side of the equal sign are the same amount or have the same value.

**Jumpstart**

How would you describe what this symbol means?

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# Explore and Discuss

1. Listen to students' responses and think about how they relate to their definitions of the equal sign discussed in the Jumpstart. Use students' responses to develop the notion that the equal sign (=) indicates two quantities have the same value.
2. **What do you notice about the quantities on each side of the equal sign when the equation is true?** You might have students trade true/false equations with a partner and then share some particularly interesting ones with the whole class. Remember the big idea to emphasize is the notion that the equal sign indicates an equivalence relationship (that is, the two quantities are the same amount or have the same value), not a direction to compute or provide an answer. Focus on students' examples that use operations other than just addition.

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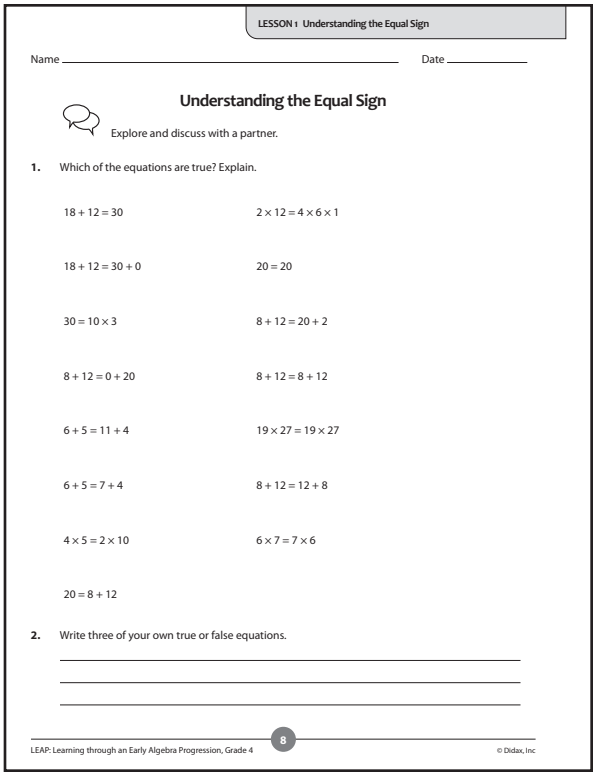

LESSON 1 Understanding the Equal Sign

Name _____ Date _____

**Understanding the Equal Sign**

Explore and discuss with a partner.

1. Which of the equations are true? Explain.

| | |
|---|---|
| $18 + 12 = 30$ | $2 \times 12 = 4 \times 6 \times 1$ |
| $18 + 12 = 30 + 0$ | $20 = 20$ |
| $30 = 10 \times 3$ | $8 + 12 = 20 + 2$ |
| $8 + 12 = 0 + 20$ | $8 + 12 = 8 + 12$ |
| $6 + 5 = 11 + 4$ | $19 \times 27 = 19 \times 27$ |
| $6 + 5 = 7 + 4$ | $8 + 12 = 12 + 8$ |
| $4 \times 5 = 2 \times 10$ | $6 \times 7 = 7 \times 6$ |
| $20 = 8 + 12$ | |

2. Write three of your own true or false equations.

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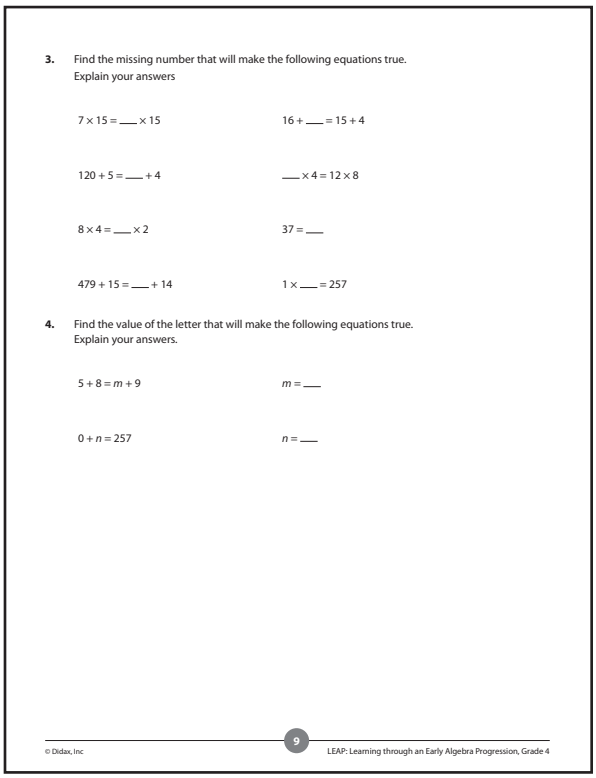

3. Find the missing number that will make the following equations true. Explain your answers

$7 \times 15 = \_\_ \times 15$ $16 + \_\_ = 15 + 4$

$120 + 5 = \_\_ + 4$ $\_\_ \times 4 = 12 \times 8$

$8 \times 4 = \_\_ \times 2$ $37 = \_\_$

$479 + 15 = \_\_ + 14$ $1 \times \_\_ = 257$

4. Find the value of the letter that will make the following equations true. Explain your answers.

$5 + 8 = m + 9$ $m = \_\_$

$0 + n = 257$ $n = \_\_$



3. **What strategies did you use to find the missing number?** Listen to students' responses and think about which demonstrate an operational view and which demonstrate a relational view of the equal sign. Students who respond with "105" as the missing value for the equation $7 \times 15 = \_\_ \times 15$ are demonstrating an operational view of the equal sign. Use students' responses to review the notion that the symbol = indicates two quantities have the same value.

   For those students who are finding the missing value correctly, what strategies do you observe? Some students might compute on both sides of the equal sign. For example, in response to $16 + \_\_ = 15 + 4$, these students might think, "15 + 4 is 19, so 16 plus what number is 19?" Other students might recognize that no computation is necessary and instead use a compensation strategy. For example, they might think, "16 is one more than 15, so the number in the blank must be one less than 4."

   Other items in this set encourage students to recall the fundamental properties. For example, since any number multiplied by 1 is that number, then the missing value in $1 \times \_\_ = 257$ must be 257.

4. **What strategies did you use to find the missing value *m*?** Students might recognize that no computation is necessary and instead use a compensation strategy. For example, they might think, "9 is one more than 8, so the value of *m* must be one less than 5."

## Thinking about Student Responses

- Even though this is not students' first time considering the meaning of the equal sign, you may still observe students demonstrating the belief that the equal sign means "give the total" or "the answer comes next." For example, students might propose that the missing value in $5 + 8 = \_\_ + 9$ is 13, the sum of 5 and 8. Such an operational view is often entrenched given students' repeated exposure over several years to equations of the form $a + b = c$. A relational view of the equal sign can take some time to develop.
- When students first begin to view the equal sign as a relational symbol, they tend to solve open equations such as the ones in this lesson by computing the value of each side of the equation and trying to balance the two sides. For example, they will approach $8 \times 4 = \_\_ \times 2$ by thinking, "I know $8 \times 4 = 32$, so now I need to figure out what times 2 is 32." Although this is correct thinking, students might also find the missing value using a structural approach that looks at relationships between quantities.

## Review and Discuss

Are these equations true or false? Explain.

$10 \times 5 = 50 + 5$

$63 = 9 \times 7$

## Review and Discuss

**Write or display the Review and Discuss question on the board.**

As before, listen to whether students interpret the equal sign as indicating that the quantities on both sides of the equation must have the same value. For $10 \times 5 = 50 + 5$, notice whether students are comfortable with operations on both sides of the equal sign. For $63 = 9 \times 7$, notice whether students are comfortable with no operation on the left side of the equal sign.

## Addressing Common Difficulties

Students with an operational understanding of the equal sign believe that the equal sign means "the answer" or "give the total" or "the answer comes next."

To counter these difficulties, ensure that students understand the equal sign means "balance." You could say:

- "The equal sign means that whatever amount you have on the left is the same as what you have on the right."
- "For $8 \times 4 = \_\_\_ \times 2$, I know $8 \times 4 = 32$, so now I need to figure out what number times 2 is 32. So, my answer is 16."

# Teaching Support

## Visualizing Equivalence

Do not be surprised if many students still demonstrate an operational view of the equal sign. Questioning students' strategies for finding missing values or determining if an equation is true or false, along with discussions of the meaning of the equal sign, will help them develop a relational view. Using tools such as a pan balance or number balance can help students visualize the equal sign as a comparison of two equivalent amounts.

**SSL Supporting Struggling Learners**

For more information, refer to Figure 1: Strategies to Support Struggling Learners.

## Choosing Numbers Strategically

Number choice is important when helping students begin to examine equations structurally rather than as separate left-to-right computations. If you would like to encourage students to examine equations without rushing to compute, there are two approaches that can help:

"Obvious" equations such as $7 \times 15 = 7 \times 15$ might be recognized by students as true because the left and right side are the same. They would not need to compute to find that the product on each side is 105.

Equations such as $30 \times 4 = \_\_\_ \times 2$, where the computation is more difficult but the numbers in the same position on either side of the equal sign have an obvious relationship, can encourage students to look for relationships to help them determine the missing value.

## Sharing Strategies

Encourage students to keep sharing their strategies. It is through listening to their classmates' explanations that students who are thinking operationally will begin to think relationally about the equal sign and students who are treating the two sides of an equation separately will begin to view equations holistically and look for relationships across the equal sign.

# Mathematical Convention

The equal sign is a mathematical symbol used to represent the equivalence of two quantities or mathematical expressions.

Name _____ Date _____

# Understanding the Equal Sign

Explore and discuss with a partner.

**1.** Which of the equations are true? Explain.

$18 + 12 = 30$

$2 \times 12 = 4 \times 6 \times 1$

$18 + 12 = 30 + 0$

$20 = 20$

$30 = 10 \times 3$

$8 + 12 = 20 + 2$

$8 + 12 = 0 + 20$

$8 + 12 = 8 + 12$

$6 + 5 = 11 + 4$

$19 \times 27 = 19 \times 27$

$6 + 5 = 7 + 4$

$8 + 12 = 12 + 8$

$4 \times 5 = 2 \times 10$

$6 \times 7 = 7 \times 6$

$20 = 8 + 12$

**2.** Write three of your own true or false equations.

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_____

_____

**3.** Find the missing number that will make the following equations true.
Explain your answers

$7 \times 15 = \_\_\_ \times 15$

$16 + \_\_\_ = 15 + 4$

$120 + 5 = \_\_\_ + 4$

$\_\_\_ \times 4 = 12 \times 8$

$8 \times 4 = \_\_\_ \times 2$

$37 = \_\_\_$

$479 + 15 = \_\_\_ + 14$

$1 \times \_\_\_ = 257$

**4.** Find the value of the letter that will make the following equations true.
Explain your answers.

$5 + 8 = m + 9$

$m = \_\_\_$

$0 + n = 257$

$n = \_\_\_$