LEAP

Learning through an Early Algebra Progression

The first early algebra curriculum for students in grades 3–5.

TERC Because math and science build futures

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This curriculum was developed by TERC, the University of Texas at Austin, and the University of Wisconsin–Madison, and was funded in part by grants from the National Science Foundation and the U.S. Department of Education.

LEAP
Learning through an Early Algebra Progression

THE IMPORTANCE OF EARLY ALGEBRA

"Students who complete Algebra II are more than twice as likely to graduate from college compared with students with less math preparation."

—National Math Advisory Panel Report, 2008

Can Early Algebra Impact Future Success?

We all know that algebra can be the gatekeeper to future success in mathematics, in college and beyond. All too often, students reach middle and high school with little understanding of algebraic concepts. As a result, many students are unsuccessful in algebra, which impacts achievement in college and future career opportunities. What if, instead, students were encouraged to build their algebraic understanding over time, beginning in the elementary grades?

How to Teach Early Algebra?

Shifting the teaching and learning of algebra concepts to the early grades raises significant questions.

- What does early algebra look like in the elementary grades?
- Would young children be capable of thinking in ways that have traditionally been viewed as possible only for older students?
- How do we support teachers in building classrooms that foster deep algebraic thinking?

The LEAP program is a culmination of years of research to answer questions such as these. It is a supplemental program entirely focused on building early algebraic thinking. It uses a series of student-focused activities to help children build their understanding of key algebraic concepts and practices over time. The program is structured so that algebraic concepts develop across grades using a connected curriculum that reflects a progression of increasingly sophisticated ideas.

The Big Ideas

The LEAP program focuses on three big ideas:

- Equivalence, Expressions, Equations, and Inequalities
- Generalized Arithmetic
- Functional Thinking

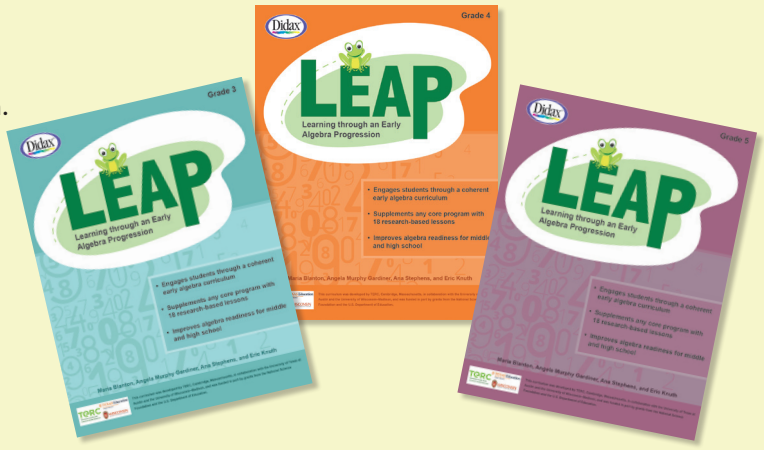
The chart below shows these ideas progress across the three grade levels.

Big Ideas in the Grades 3–5 LEAP Curriculum

Big Idea Equivalence, Expressions, Equations, and Inequalities		
Grade 3	**Grade 4**	**Grade 5**
Understanding the Equal Sign Lessons 1 and 2	**Understanding the Equal Sign** Lesson 1	**Understanding the Equal Sign** Lesson 1
Variables, Expressions, and Equations Lessons 7, 8, and 9	**Variables, Expressions, and Equations** Lessons 7, 8, and 9	**Variables, Expressions, and Equations** Lessons 9 and 11
	Properties of Equations Lesson 10	**Properties of Equations** Lesson 7 and 8
Big Idea Generalized Arithmetic		
Grade 3	**Grade 4**	**Grade 5**
Properties of Operations Lessons 3, 4, 10, and 11	**Properties of Operations** Lessons 2, 3, 4, and 11	**Properties of Operations** Lessons 2, 3, 4, 10, and 12
Relationships in Arithmetic Lessons 5 and 6	**Relationships in Arithmetic** Lessons 5 and 6	**Relationships in Arithmetic** Lessons 5 and 6
Big Idea Functional Thinking		
Grade 3	**Grade 4**	**Grade 5**
Finding a Relationship Lessons 12, 13, and 15	**Finding a Relationship** Lesson 12	
Reasoning with Relationships Lessons 17 and 18	**Representing and Reasoning with Relationships** Lessons 13, 14, 15, 16, 17, and 18	**Representing and Reasoning with Relationships** Lessons 13, 14, 15, 16, and 19
Constructing and Interpreting Graphs Lessons 14 and 16		**Constructing and Interpreting Qualitative Graphs** Lessons 17 and 18
Big Ideas All Three Big Ideas		
Grade 3	**Grade 4**	**Grade 5**
	Using "Letters" in Math Lesson 19	**Using "Letters" in Math** Lesson 20

The LEAP Program

- Designed to be integrated with any curriculum.
- Currently offered for Grades 3–5.
- Each grade contains 18–20 one-hour lessons.
- Lessons are taught throughout the year.
- Instruction is divided into four main areas: Plan, Teach, Support, and Assess.



How to Implement LEAP in Your Math Class

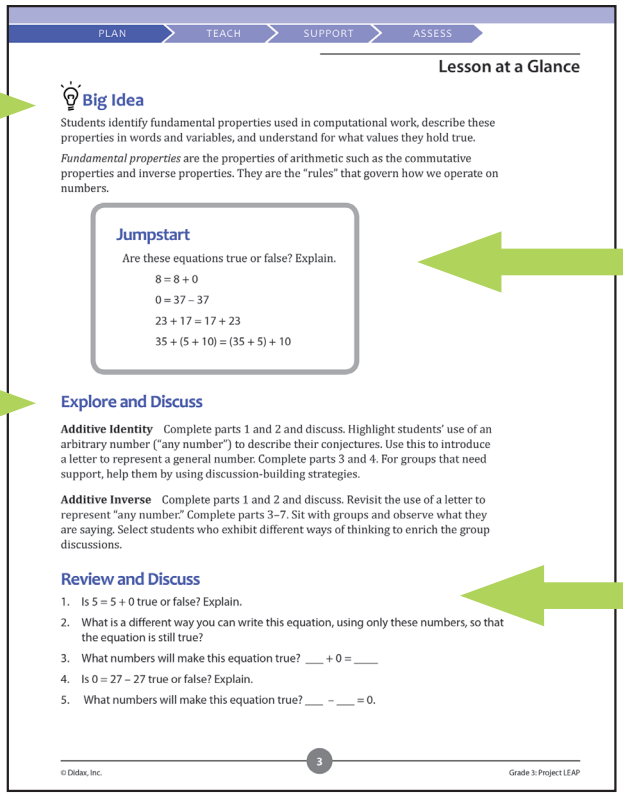
The LEAP lessons should be taught throughout the school year. Each lesson lasts about one hour and is designed to fit within the daily math instructional period. Each lesson is designed using the same structure of Jumpstart, Explore & Discuss, and Review & Discuss.

LEAP Lesson at a Glance

(from Grade 3)

Big Idea
Shows teachers key concepts of lesson

Explore & Discuss
Small-group investigations to explore concepts that engage students in algebraic thinking practices.
– 35 minutes.



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Lesson at a Glance

Big Idea

Students identify fundamental properties used in computational work, describe these properties in words and variables, and understand for what values they hold true.

Fundamental properties are the properties of arithmetic such as the commutative properties and inverse properties. They are the "rules" that govern how we operate on numbers.

Jumpstart

Are these equations true or false? Explain.

8 = 8 + 0
0 = 37 – 37
23 + 17 = 17 + 23
35 + (5 + 10) = (35 + 5) + 10

Explore and Discuss

Additive Identity Complete parts 1 and 2 and discuss. Highlight students' use of an arbitrary number ("any number") to describe their conjectures. Use this to introduce a letter to represent a general number. Complete parts 3 and 4. For groups that need support, help them by using discussion-building strategies.

Additive Inverse Complete parts 1 and 2 and discuss. Revisit the use of a letter to represent "any number." Complete parts 3–7. Sit with groups and observe what they are saying. Select students who exhibit different ways of thinking to enrich the group discussions.

Review and Discuss

1. Is 5 = 5 + 0 true or false? Explain.
2. What is a different way you can write this equation, using only these numbers, so that the equation is still true?
3. What numbers will make this equation true? ___ + 0 = ___
4. Is 0 = 27 – 27 true or false? Explain.
5. What numbers will make this equation true? ___ – ___ = 0.

© Didax, Inc. 3 Grade 3: Project LEAP

Jumpstart
Whole-class activity to review prior content and prepare students' thinking
– 15 minutes.

Review & Discuss
Whole-class discussions to summarize and formatively assess student thinking
– 10 minutes.

Built-in Assessment

- Assessments are provided every 4–5 lessons.
- Each assessment is a formative tool that helps teachers determine how students are progressing.

Professional Development

Teachers will learn the research behind the LEAP program and why early algebra is so important. They will learn how to conduct all of the components of a LEAP lesson as well as ideas for incorporating LEAP into their current curriculum.

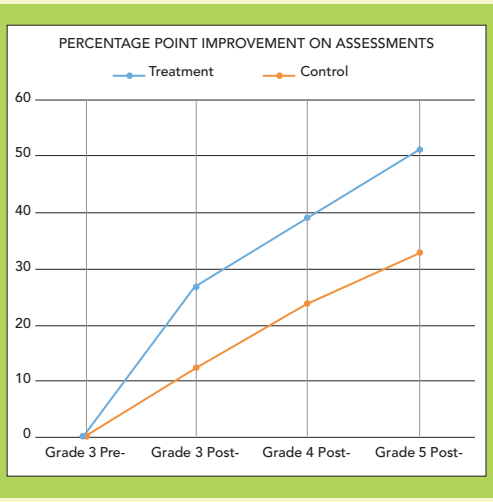
As teachers develop more confidence with the core practices of LEAP, they will develop "algebra eyes and ears" that will bring out the algebraic features of all math lessons.

Contact **leap@didax.com** for more information.

About the Research

The LEAP program is based on over a decade of research in elementary classrooms. All lessons have undergone years of testing in authentic settings in order to understand how children make sense of lesson tasks and activities and how to support teachers in successfully implementing the program. Lessons are packed with research-based insights into how children think about particular concepts, the difficulties they might have, and how teachers can address these.

"Through experimental studies, we have found that students who are taught the LEAP curriculum as part of their regular math instruction significantly outperform (their peers) who receive only regular, arithmetic-focused instruction on growth in understanding of core algebraic concepts and practices, including those in at-risk settings" *(Blanton et al., 2018)*.



A study of the effectiveness of the LEAP curriculum was conducted in a diverse population of students in grades 3–5. Forty-six schools in three school districts participated. Students in treatment schools were taught the early algebra intervention by classroom teachers during regular mathematics instruction. Students in control schools received only regular mathematics instruction. Results show that during Grade 3, treatment students, including those in at-risk settings, improved at a significantly faster rate than control students and maintained their advantage throughout the study.

Blanton, M., Stroud, R., Stephens, A., Gardiner, A., Stylianou, D., Knuth, E., Isler-Baykal, I., Strachota, S. (2019). Does Early algebra matter?: The effectiveness of an early algebra intervention in grades 3–5. American Educational Research Journal 56(5), 1930–1972, DOI: 10.3102/0002831219832301

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This curriculum was developed by TERC, the University of Texas at Austin, and the University of Wisconsin–Madison, and was funded in part by grants from the National Science Foundation and the U.S. Department of Education.

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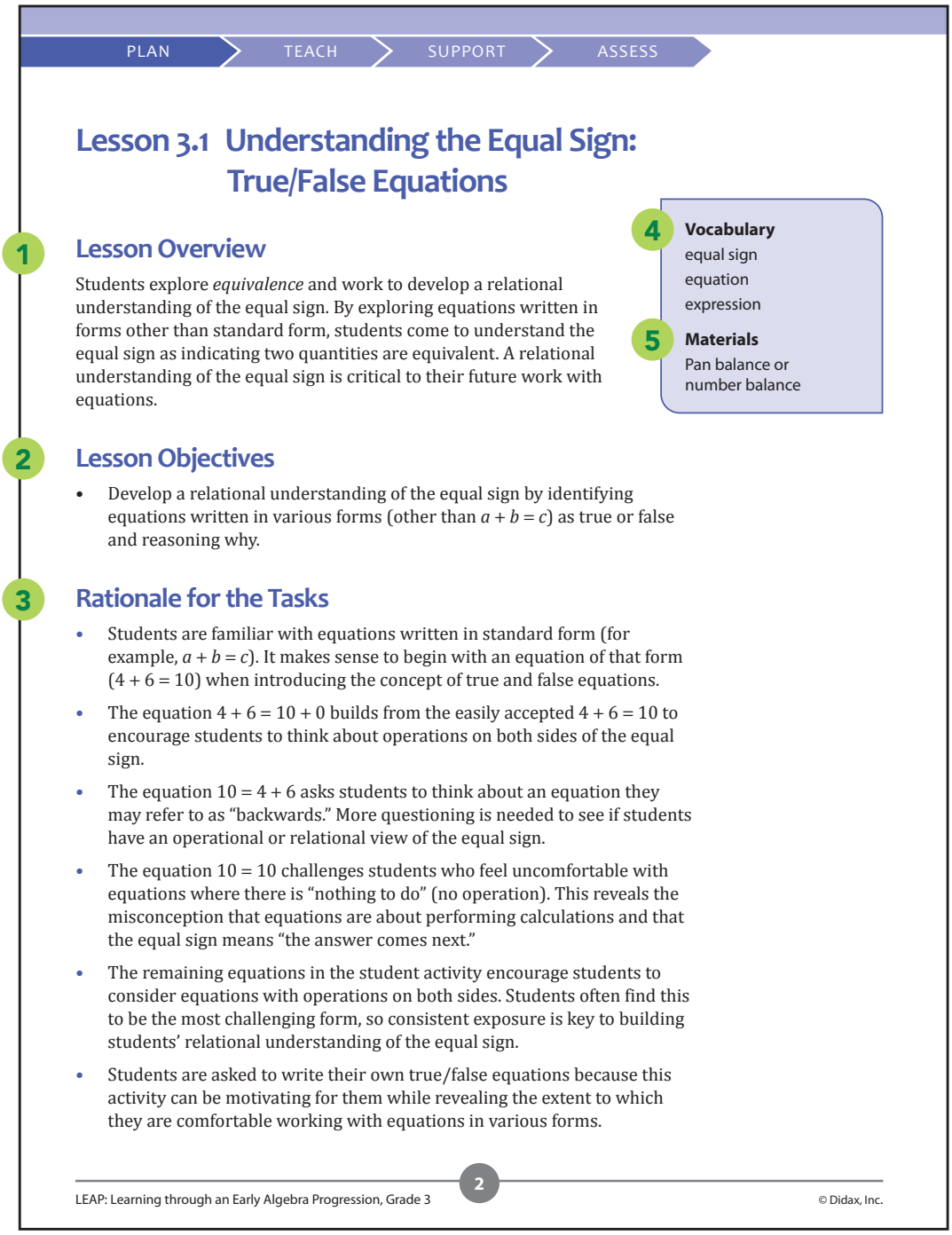
Sample LEAP Lesson

Grade 3 – Understanding the Equal Sign

1. **Lesson Overview** provides a brief description of the lesson's focus.
2. **Lesson Objectives** identify the specific learning goals addressed in the lesson.
3. **Rationale for the Tasks** describes why the tasks are important and how they are designed to address student learning.
4. **Lesson Vocabulary** highlights important terms used in this lesson.
5. **Lesson Materials List** lets the teacher know exactly which materials they need for this lesson.



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Lesson 3.1 Understanding the Equal Sign: True/False Equations

1 Lesson Overview

Students explore *equivalence* and work to develop a relational understanding of the equal sign. By exploring equations written in forms other than standard form, students come to understand the equal sign as indicating two quantities are equivalent. A relational understanding of the equal sign is critical to their future work with equations.

4 Vocabulary
equal sign
equation
expression

5 Materials
Pan balance or number balance

2 Lesson Objectives

- Develop a relational understanding of the equal sign by identifying equations written in various forms (other than $a + b = c$) as true or false and reasoning why.

3 Rationale for the Tasks

- Students are familiar with equations written in standard form (for example, $a + b = c$). It makes sense to begin with an equation of that form ($4 + 6 = 10$) when introducing the concept of true and false equations.
- The equation $4 + 6 = 10 + 0$ builds from the easily accepted $4 + 6 = 10$ to encourage students to think about operations on both sides of the equal sign.
- The equation $10 = 4 + 6$ asks students to think about an equation they may refer to as "backwards." More questioning is needed to see if students have an operational or relational view of the equal sign.
- The equation $10 = 10$ challenges students who feel uncomfortable with equations where there is "nothing to do" (no operation). This reveals the misconception that equations are about performing calculations and that the equal sign means "the answer comes next."
- The remaining equations in the student activity encourage students to consider equations with operations on both sides. Students often find this to be the most challenging form, so consistent exposure is key to building students' relational understanding of the equal sign.
- Students are asked to write their own true/false equations because this activity can be motivating for them while revealing the extent to which they are comfortable working with equations in various forms.

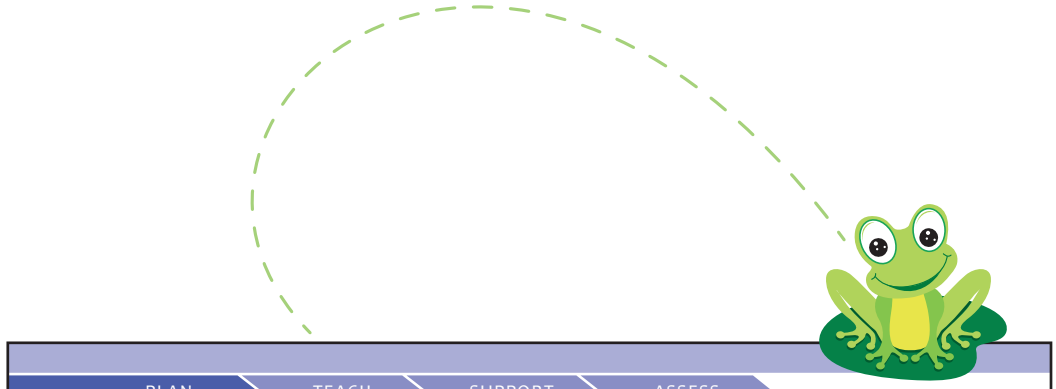
2

LEAP: Learning through an Early Algebra Progression, Grade 3

Sample LEAP Lesson

Grade 3 – Understanding the Equal Sign

1 **Lesson at a Glance** page provides an overview of what will be taught in the lesson.



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Lesson at a Glance 1

Big Idea

Students learn to think relationally about the equal sign by identifying equations as true or false and reasoning why.

Jumpstart

How would you describe what this symbol means?

=

Explore and Discuss

Have students complete Tasks 1 and 2 and discuss in small groups what makes an equation true or false. Have students complete Task 3 and discuss in their groups what constitutes a good working definition of the equal sign. Do students see the equal sign as an operational symbol, or have they begun to develop a relational understanding of the equal sign?

Sit with groups and observe what they are saying. Select students who exhibit different ways of thinking to enrich whole-group discussions. Help groups that need support by using discussion-building strategies (see Teaching Support, page 7).

Review and Discuss

Are these equations true or false? Explain.

11 + 4 = 15 + 2

23 = 17 + 6

Sample LEAP Lesson

Grade 3 – Understanding the Equal Sign

1. **Jumpstart questions** help engage students by recalling information from an earlier lesson or previewing new concepts that will come later.
2. **Explore and Discuss** are student-focused small group activities that help students explore and build an understanding of the concepts. Questions in bold blue font are important for teachers to ask during discussion to enrich conversations and develop student's understanding.
3. **Student pages** appear in the teacher book alongside the relevant instruction. Blackline masters of the pages are included with each lesson.

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LESSON 3.1 Understanding the Equal Sign: True/False Equations

1 Jumpstart

Write or display the Jumpstart question on the board.

Listen to students' responses. Do students say that the equal sign means "the answer" or the "total"? Do they say the equal sign means the "equation is balanced," or that "one side has the same value as the other side"?

Help students understand that the quantities on either side of the equal sign are the same amount or have the same value.

Jumpstart

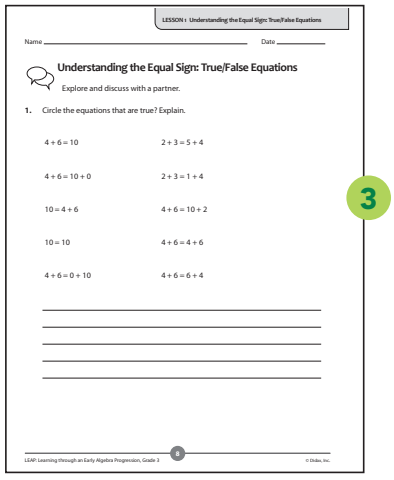
How would you describe what this symbol means?

=

2 Explore and Discuss

1. **What can you say about all the equations that are true?**
   Think about how students' responses relate to their definitions of the equal sign discussed in the Jumpstart. Use these responses to reinforce the notion that the equal sign (=) indicates that two quantities have the same value.

   You may want to show the equal sign as the middle of a pan balance, where the objects on either side have the same weight. You might also show it as the middle of a number balance, where the expressions indicated on either side have the same value. Tools such as a pan balance or number balance can help students explore equivalence in concrete ways and can increase understanding. Gesturing to convey balance can also support students in picturing what it means for two quantities to be equivalent.



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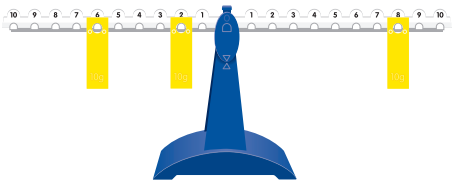

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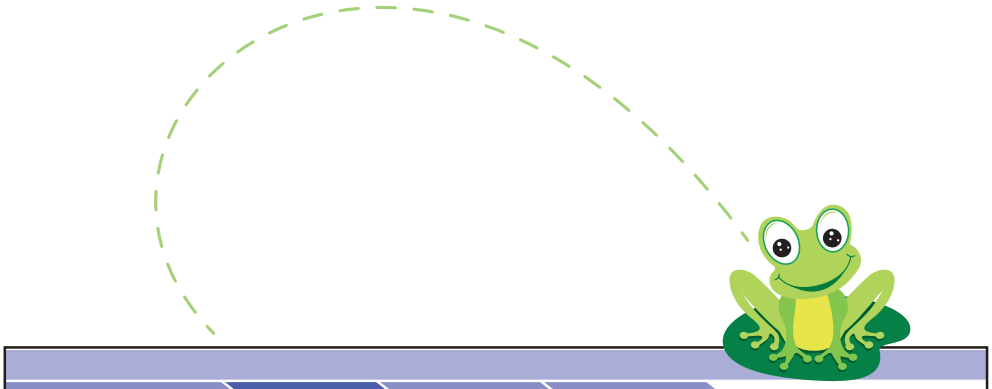
Sample LEAP Lesson

Grade 3 – Understanding the Equal Sign

1 Thinking About Student Responses uses research into childrens' algebraic thinking to provide teachers with insight into how student work and discussion reveals how students are progressing in their understanding.



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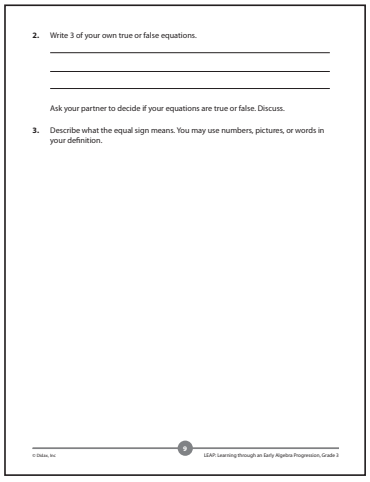
2. **How did you determine whether your partner's equations were true or false?**
   Have students trade their true/false equations with a partner and discuss. Share some of the equations with the whole class, including those that are in forms other than standard form (4 + 6 = 10). Make sure to include those with operations on both sides of the equal sign (such as 4 + 6 = 5 + 5), with operations on only one side of the equal sign (such as 8 = 3 + 5 or 10 + 2 = 12), and with no operation at all (such as 9 = 9).
3. **How did you describe what the equal sign means?**
   At this point, you should hear students beginning to talk relationally about the equal sign. Do they say, for example, "The equal sign means that whatever amount you have on the left, you have that same amount on the right" or "The equal sign means the sides are balanced"?

   Listen for students who still hold an operational view of the equal sign—that is, they think the equal sign means they need to find the answer.

   Notice whether students are comfortable writing equations in different forms other than standard form.



2. Write 3 of your own true or false equations.

Ask your partner to decide if your equations are true or false. Discuss.

3. Describe what the equal sign means. You may use numbers, pictures, or words in your definition.

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1 Thinking about Student Responses

Students' understanding of the equal sign as an operational symbol ("the answer comes next") often leaves them unsure about any equation that is not in a standard form.

For example, they may say that 8 = 8 is false because there is "nothing to do," or that 9 = 5 + 4 is "backwards" because the operation is to the right of the equal sign. They may also ignore the last number in the expression on the right side of the equal sign, saying, for example that 2 + 3 = 1 + 4 is false because 2 + 3 ≠ 1 and 4 + 6 = 10 + 2 is true because 4 + 6 = 10. They might also say 4 + 6 = 10 + 2 can be rewritten as 4 + 6 = 10 + 2 = 12.

In contrast, students who hold a relational view will likely compute each side of the equation separately and then compare the quantities. For example, they would reason that 2 + 3 = 1 + 4 is true because 2 + 3 and 1 + 4 are each equal to 5.

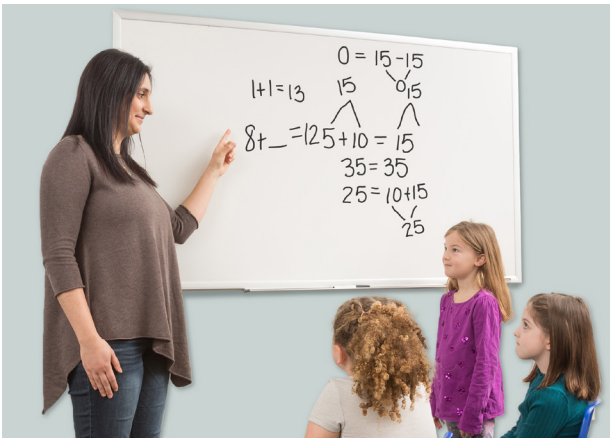
Even very young students can develop a relational understanding of the equal sign when given the opportunity to reason about true/false equations in various forms.

Sample LEAP Lesson

Grade 3 – Understanding the Equal Sign

1 **Review and Discuss Prompts** give teachers a quick tool to check student understanding and provide just-in-time support for students who are struggling.

2 **Addressing Common Difficulties** helps teachers recognize and respond to student misconceptions



PLAN | TEACH | SUPPORT | ASSESS

LESSON 3.1 Understanding the Equal Sign: True/False Equations

1

Review and Discuss

Are the following equations true or false? Explain.

11 + 4 = 15 + 2

23 = 17 + 6

Review and Discuss

Write or display the Review and Discuss question on the board.

- Listen for students who still think operationally about the equal sign. Such students might say, "11 + 4 = 15 + 2 is true because 11 + 4 = 15." You could ask, "What about the 2? What role does it play in the equation?"
- Do students think that equations cannot have a single value to the left of the equal sign? Listen for students who say, "23 = 17 + 6 is backwards." Help students understand that 23 = 17 + 6 is equivalent to 17 + 6 = 23.
- Students are beginning to think relationally about the equal sign when they understand that an equation is true only if the values of the quantities or expressions on both sides are the same.

2

Addressing Common Difficulties

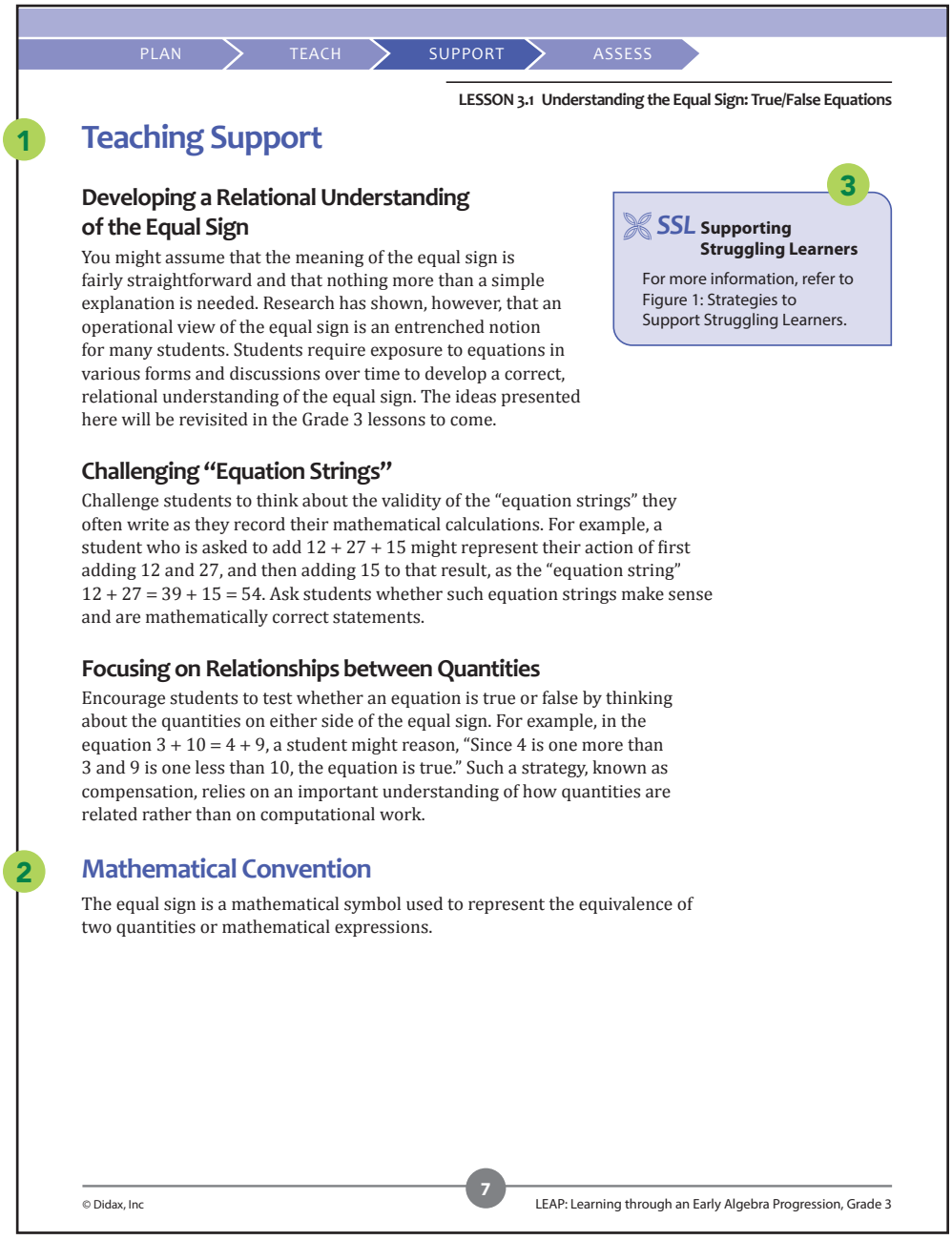
Many students with an operational understanding of the equal sign believe that the equal sign means, "gives the total" or "the answer comes next." Students might say, "2 + 3 = 5 = 1 is a true equation" because they add the expression on the left side and ignore the + 1 in the expression on the right.

To counter these difficulties, ensure that students understand the equal sign means balance. You could say, "The equal sign means that whatever amount you have on the left is the same as what you have on the right," or "2 + 3 = 5 + 1 is not a true equation because 2 + 3 is 5, but 5 + 1 is 6 so this equation isn't balanced." For students who need more support, use tools such as a number balance to explore equivalence in more concrete ways.

Sample LEAP Lesson

Grade 3 – Understanding the Equal Sign

1. **Teaching Support** provides a deeper look at the concepts and strategies students are learning, ensuring that the big ideas in each lesson are emphasized.
2. **Mathematical Conventions** help teachers and students understand accepted practices in mathematics.
3. **Support for Struggling Learners** is built in to every lesson, including strategies that will help all students to develop a deeper understanding of new concepts.



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LESSON 3.1 Understanding the Equal Sign: True/False Equations

1 **Teaching Support**

Developing a Relational Understanding of the Equal Sign

You might assume that the meaning of the equal sign is fairly straightforward and that nothing more than a simple explanation is needed. Research has shown, however, that an operational view of the equal sign is an entrenched notion for many students. Students require exposure to equations in various forms and discussions over time to develop a correct, relational understanding of the equal sign. The ideas presented here will be revisited in the Grade 3 lessons to come.

3

SSL **Supporting Struggling Learners**

For more information, refer to Figure 1: Strategies to Support Struggling Learners.

Challenging "Equation Strings"

Challenge students to think about the validity of the "equation strings" they often write as they record their mathematical calculations. For example, a student who is asked to add 12 + 27 + 15 might represent their action of first adding 12 and 27, and then adding 15 to that result, as the "equation string" 12 + 27 = 39 + 15 = 54. Ask students whether such equation strings make sense and are mathematically correct statements.

Focusing on Relationships between Quantities

Encourage students to test whether an equation is true or false by thinking about the quantities on either side of the equal sign. For example, in the equation 3 + 10 = 4 + 9, a student might reason, "Since 4 is one more than 3 and 9 is one less than 10, the equation is true." Such a strategy, known as compensation, relies on an important understanding of how quantities are related rather than on computational work.

2 **Mathematical Convention**

The equal sign is a mathematical symbol used to represent the equivalence of two quantities or mathematical expressions.

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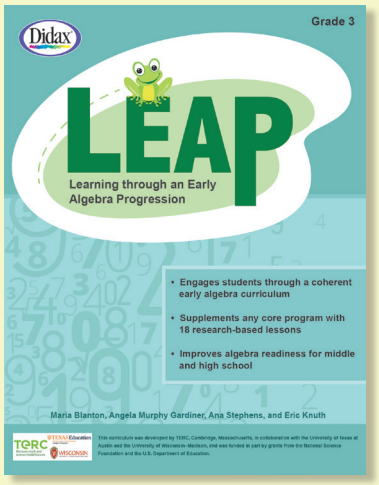
The first early algebra curriculum for students in grades 3–5.

$39.99 each



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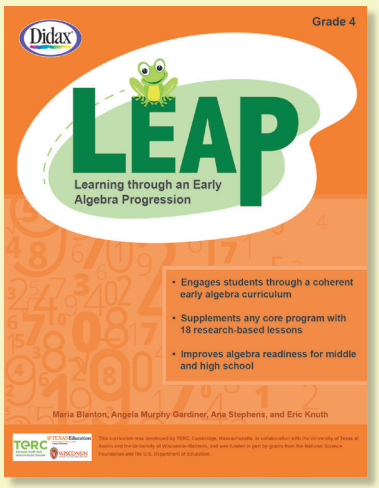

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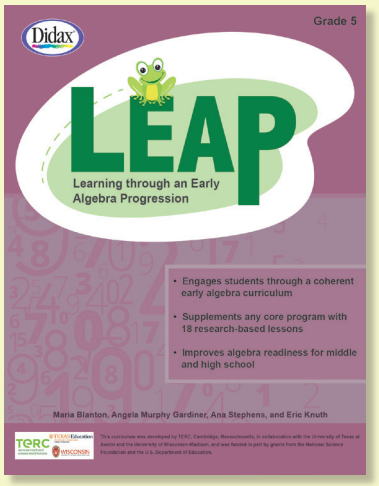

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#211923 Grade 5

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